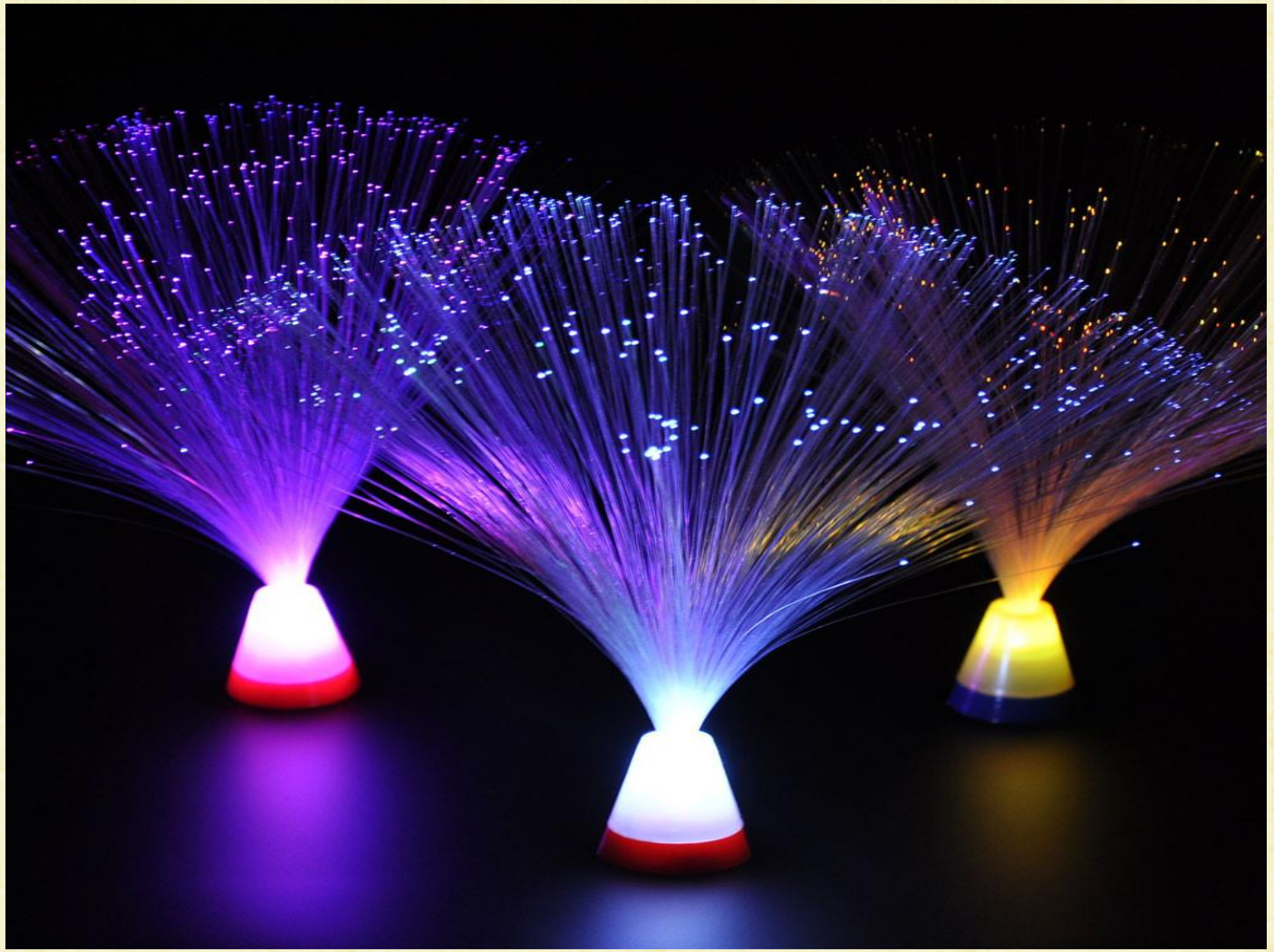
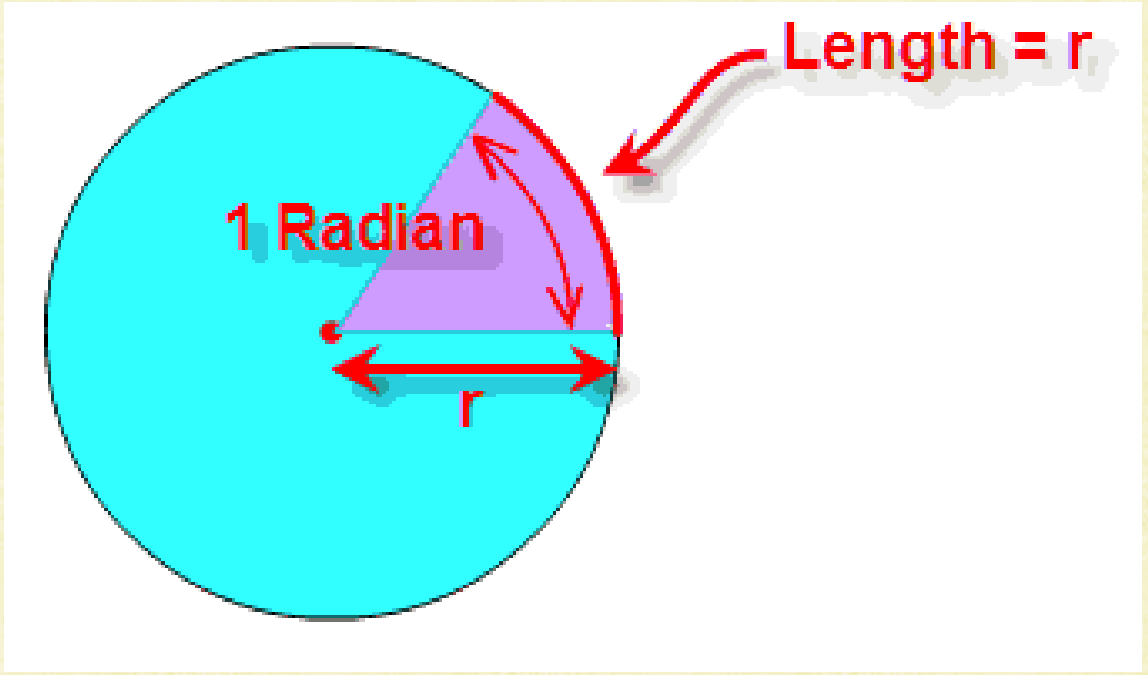
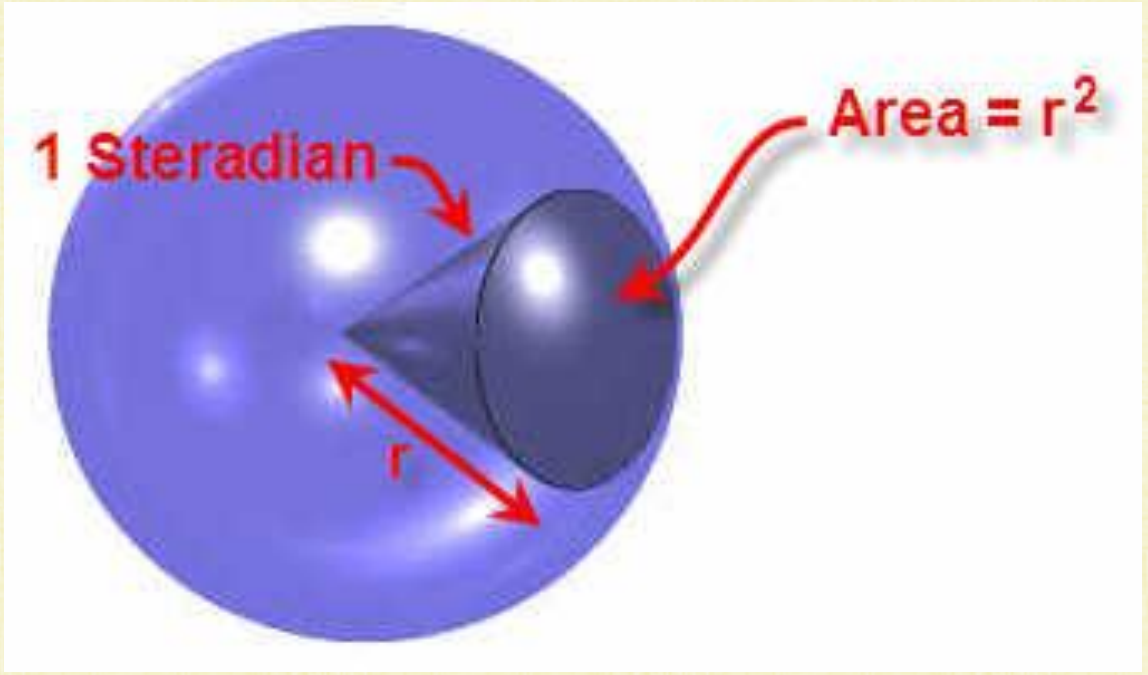
Optics



Solid Angle

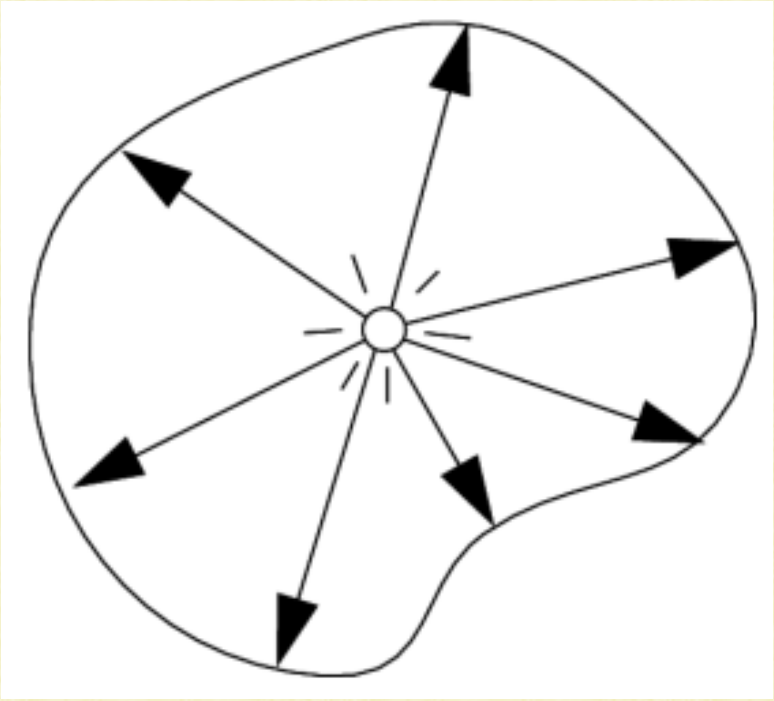
- 3D angle, measured in steradians
- 2D angles have $\theta = \frac{l_{arc}}{r}$, and 3D solid angles have $\omega = \frac{A_{on\ the\ sphere\ surface}}{r^2}$
- Circumference of a circle is $C = 2\pi r$, so a circle has 2π radians
- Surface area of a sphere is $4\pi r^2$, so a sphere has 4π steradians



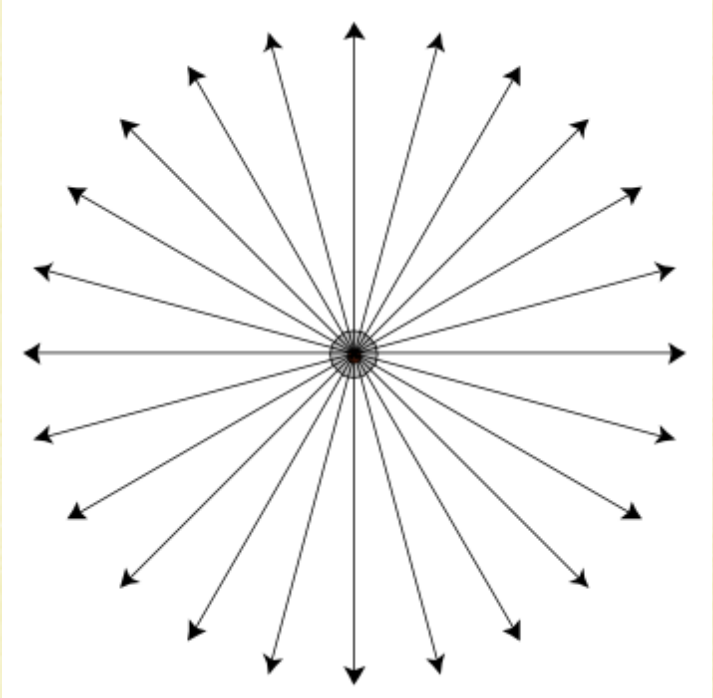


Radiant Intensity from a Point Light Source

- Power per unit solid angle $I = \frac{d\Phi}{d\omega}$
  - Φ is light power (in watts = joules per second)
- Anisotropic light source: I varies across the light (as a function of ω)
- Isotropic light source: integrate $d\Phi = I d\omega$ to obtain $\Phi = I \int_{sphere} d\omega = 4\pi I$



anisotropic light source

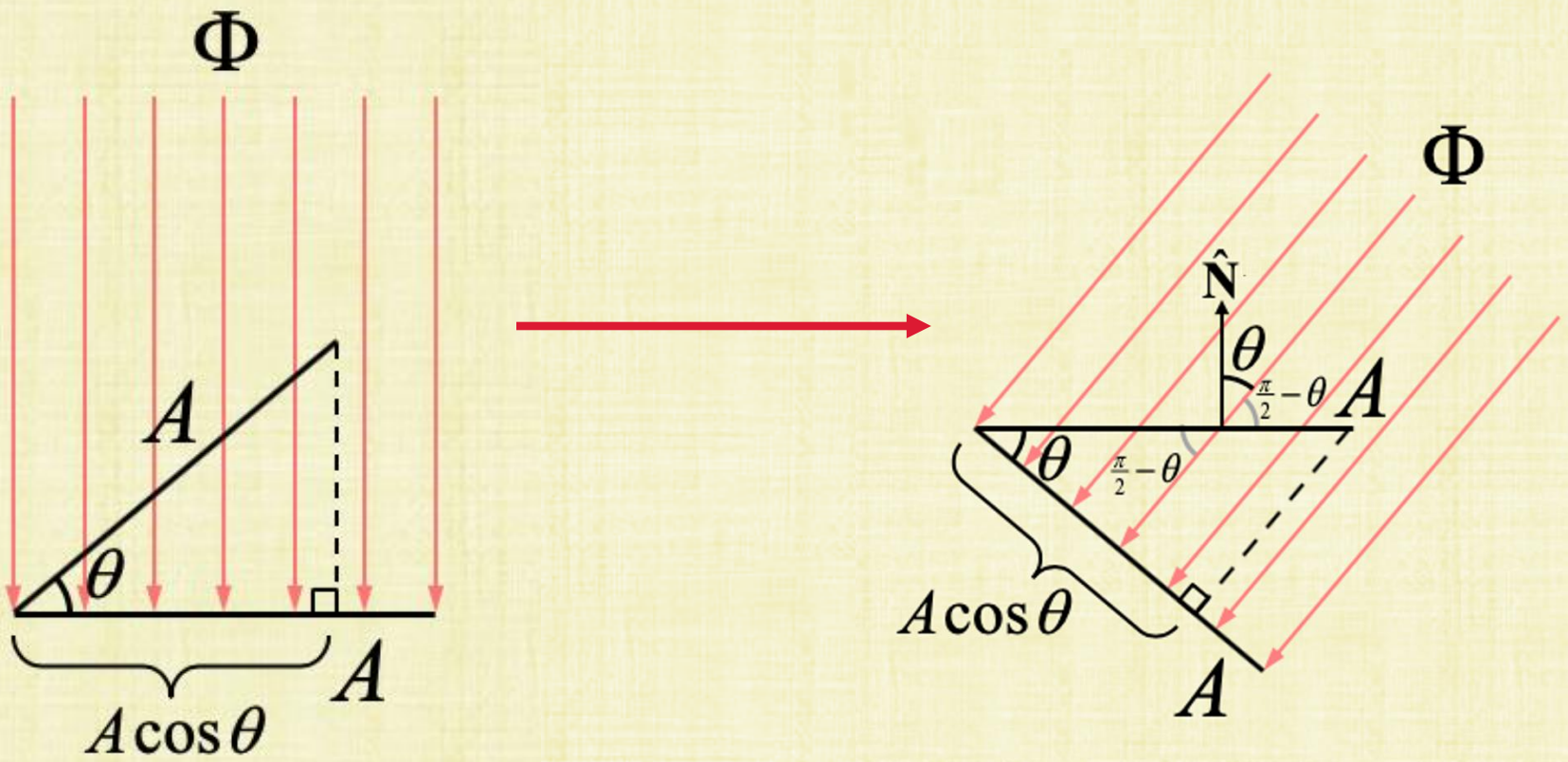


isotropic light source

Irradiance onto a Surface

- Power per unit surface area $E = \frac{d\Phi}{dA}$
- Given $E_{flat} = \frac{\Phi}{A}$, note that $E_{tilted} = \frac{\left(\frac{A\cos\theta}{A}\right)\Phi}{A} = E_{flat}\cos\theta$
- Irradiance decreases as you tilt the surface, since less photons hit it

This is the same $\cos\theta$ that we used for diffuse shading

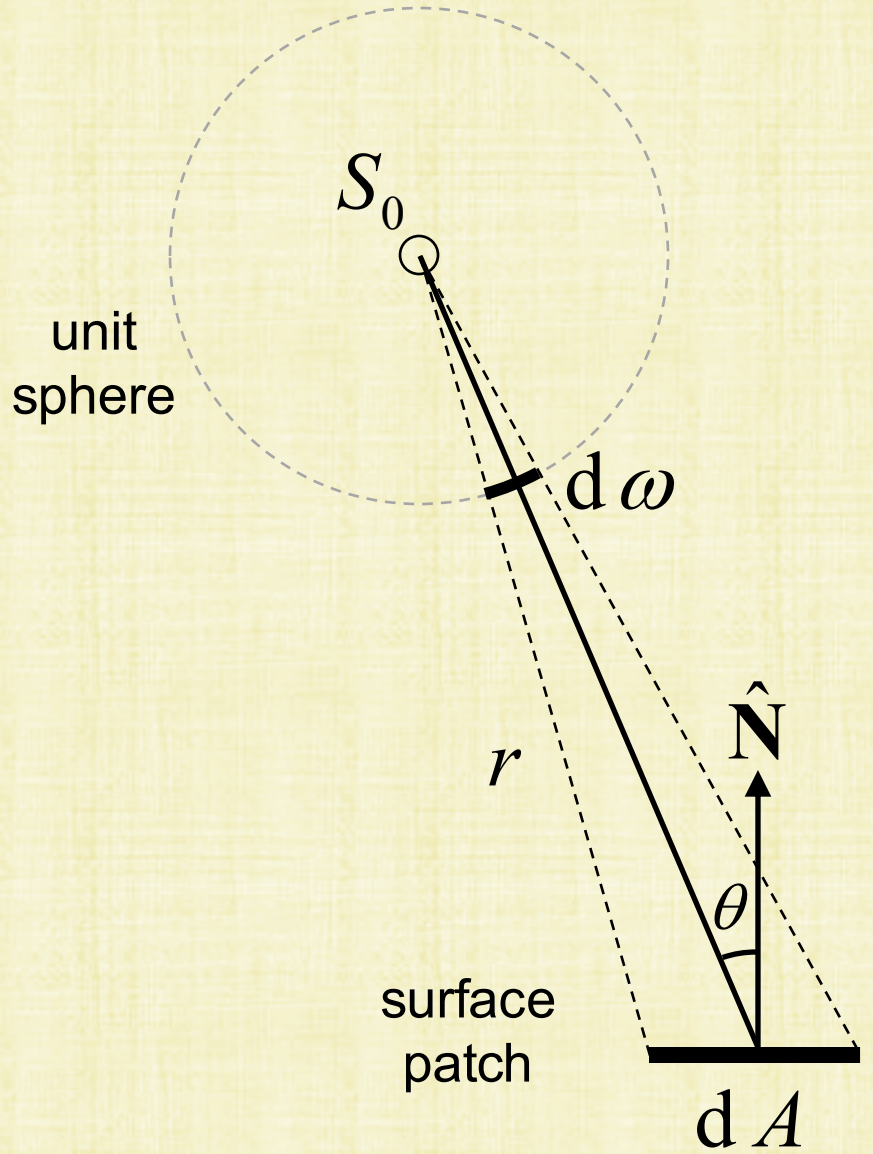


Solid Angle vs. Cross-Sectional Area

- Definition of solid angle: $d\omega = \frac{dA_{sphere}}{r^2}$
- From the previous slide: the (orthogonal) cross-sectional area of the surface patch is $dA \cos\theta$
- So, given a sphere of radius r (in the figure):

$$d\omega = \frac{dA \cos\theta}{r^2}$$

- The solid angle decreases as the surface tilts away from the light (increasing θ, decreasing $\cos\theta$)
- The solid angle decreases as the surface is moved further from the light (increasing r)



Radiance from an Area Light Source

- Light power is emitted per unit area (not from a single point)
- The emitted light goes in various directions (measured via solid angles)

- Break an area light up into (infinitesimally) small area chunks:
  - Each area chunk emits light into each of the solid angle directions
    - i.e. radiant intensity per area chunk
  - Each emitted direction has a cosine term (similar to irradiance)

- Radiance – radiant intensity per area chunk

$$L = \frac{dI}{dA\cos\theta_{light}} \left(= \frac{d^2\Phi}{d\omega\, dA\cos\theta_{light}} = \frac{dE}{d\omega\cos\theta_{light}} \right)$$

Objects act as Area Lights

- Light comes from all visible objects in the world (not just from light sources)
- Each area chunk of each object acts as a source of light (i.e. as a light source)

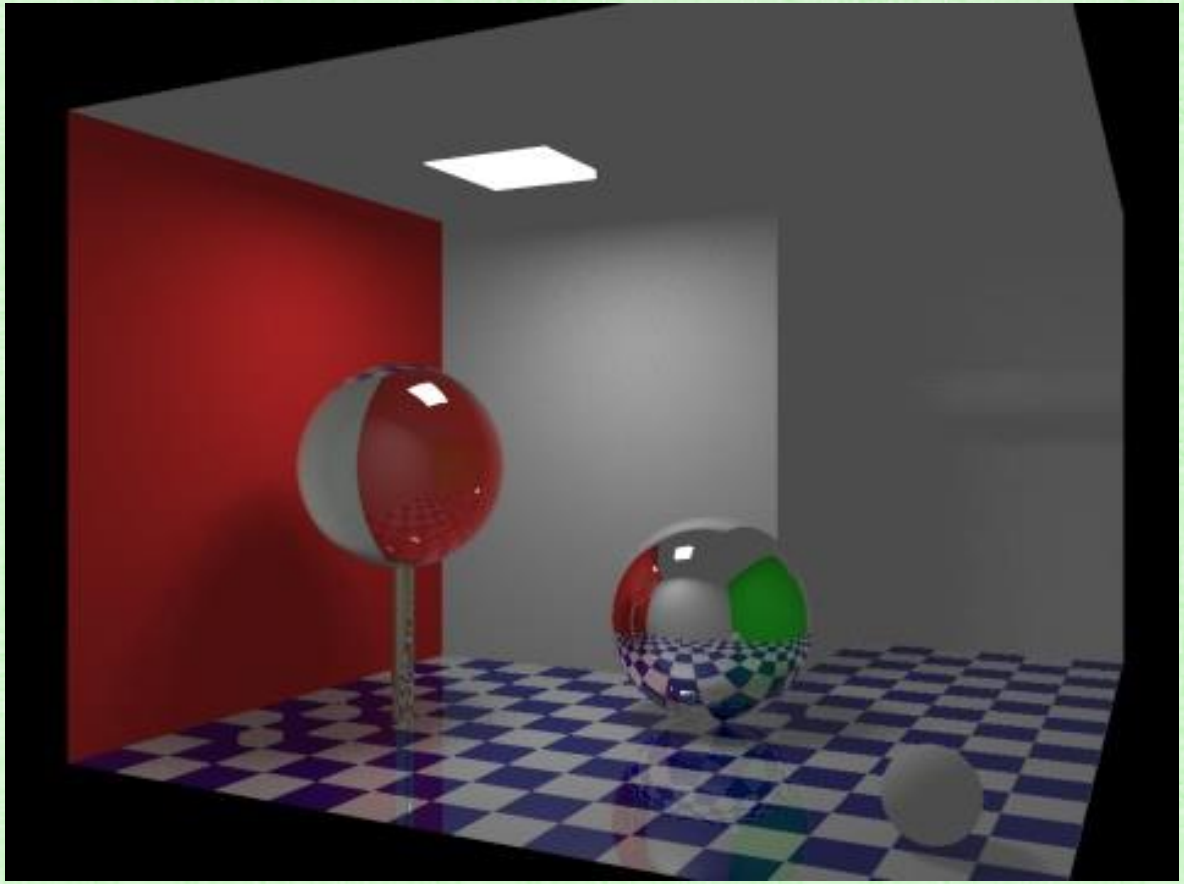
- A tree “shines” light onto the car; then, the car “shines” light onto the camera:

Objects act as Area Lights

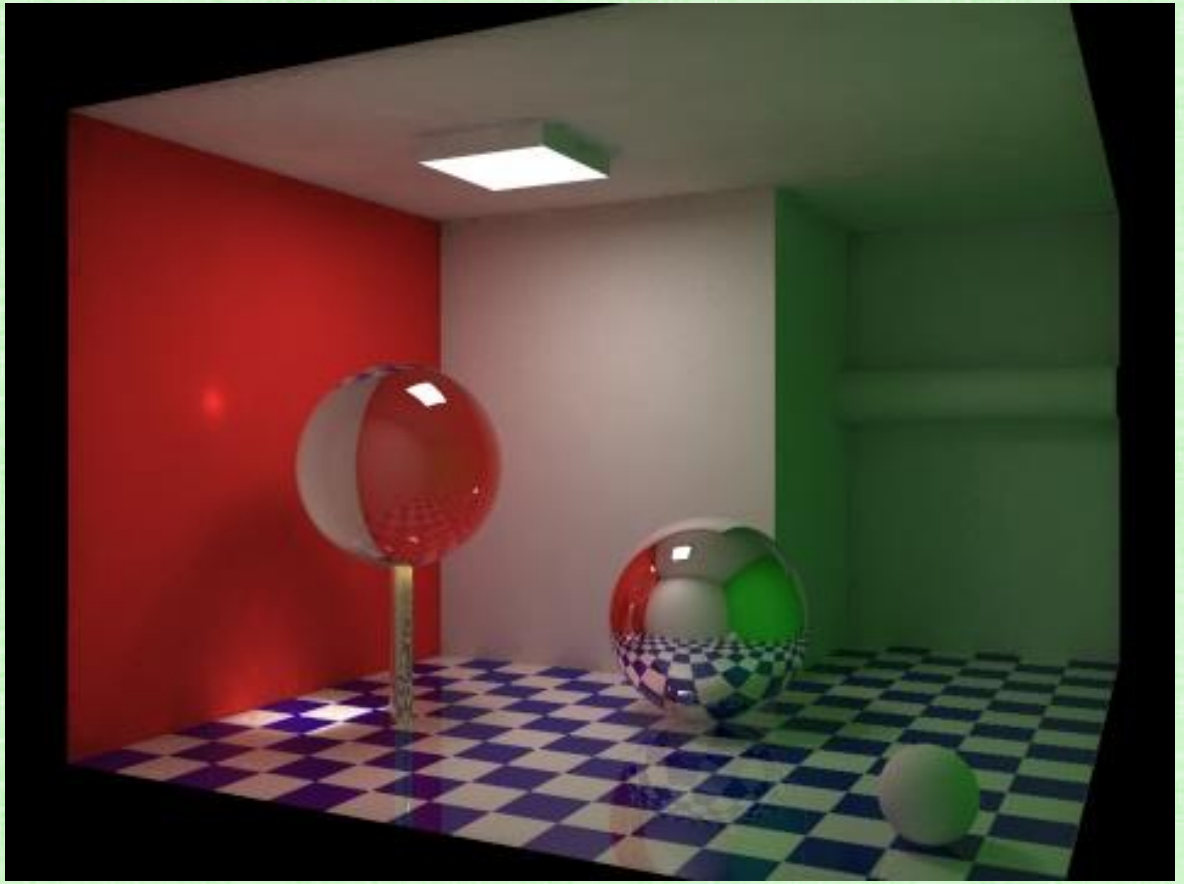
• The red paper “shines” red light onto the statue (this is called color bleeding); then, the statue “shines” red light onto the camera:

Objects act as Area Lights

- It’s not good enough to only look for light along shadow rays (even though, it’s pretty good)



using light emitted along shadow rays (only)



using light emitted along shadow rays
and light emitted from objects

Measuring Incoming Light

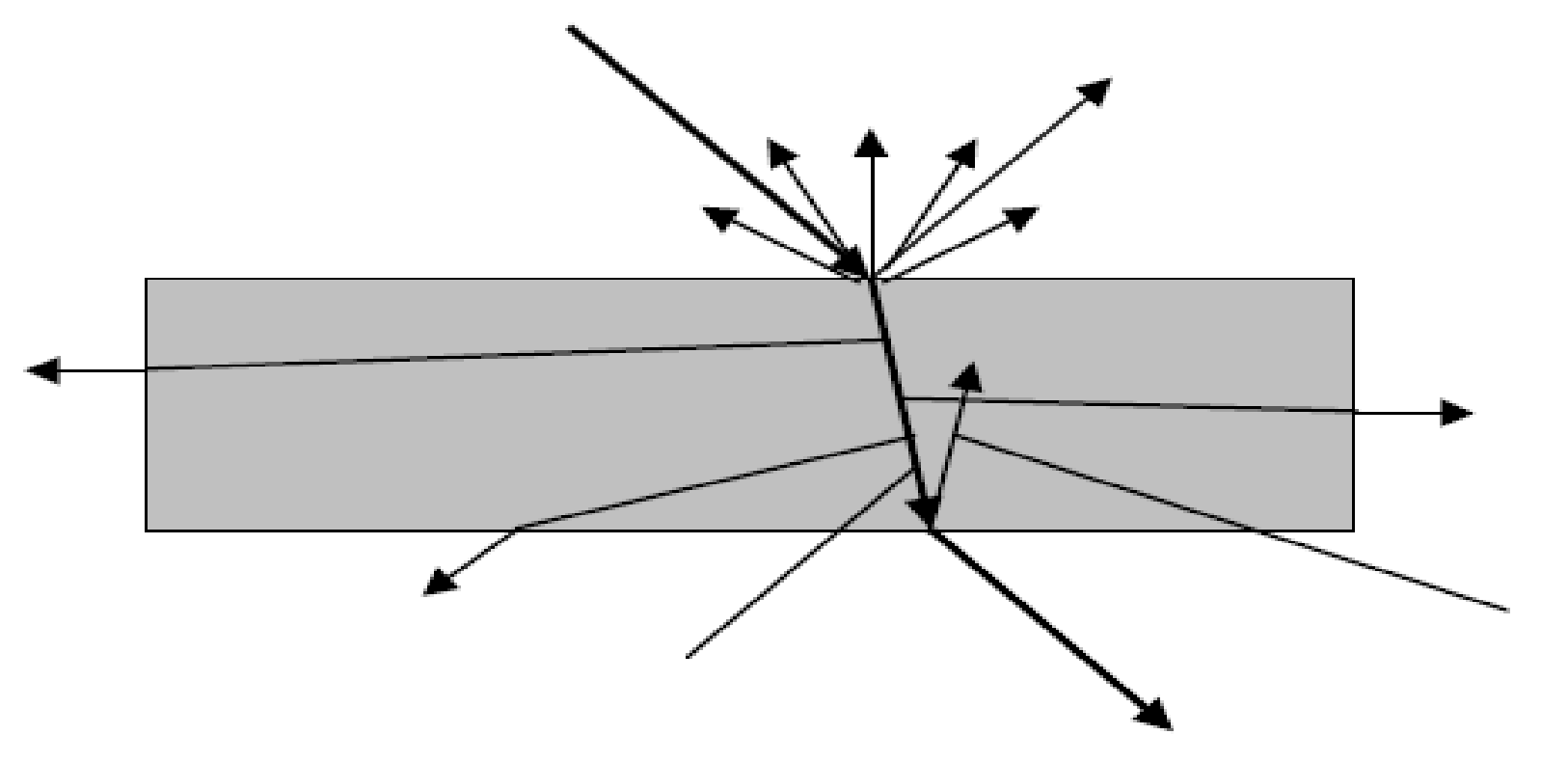
- Light Probe: a small reflective chrome sphere
- Photograph it, in order to record the incoming light (at its location) from all directions

Using the Measured Incoming Light

- The measured incoming light can be used to render a synthetic object with realistic lighting

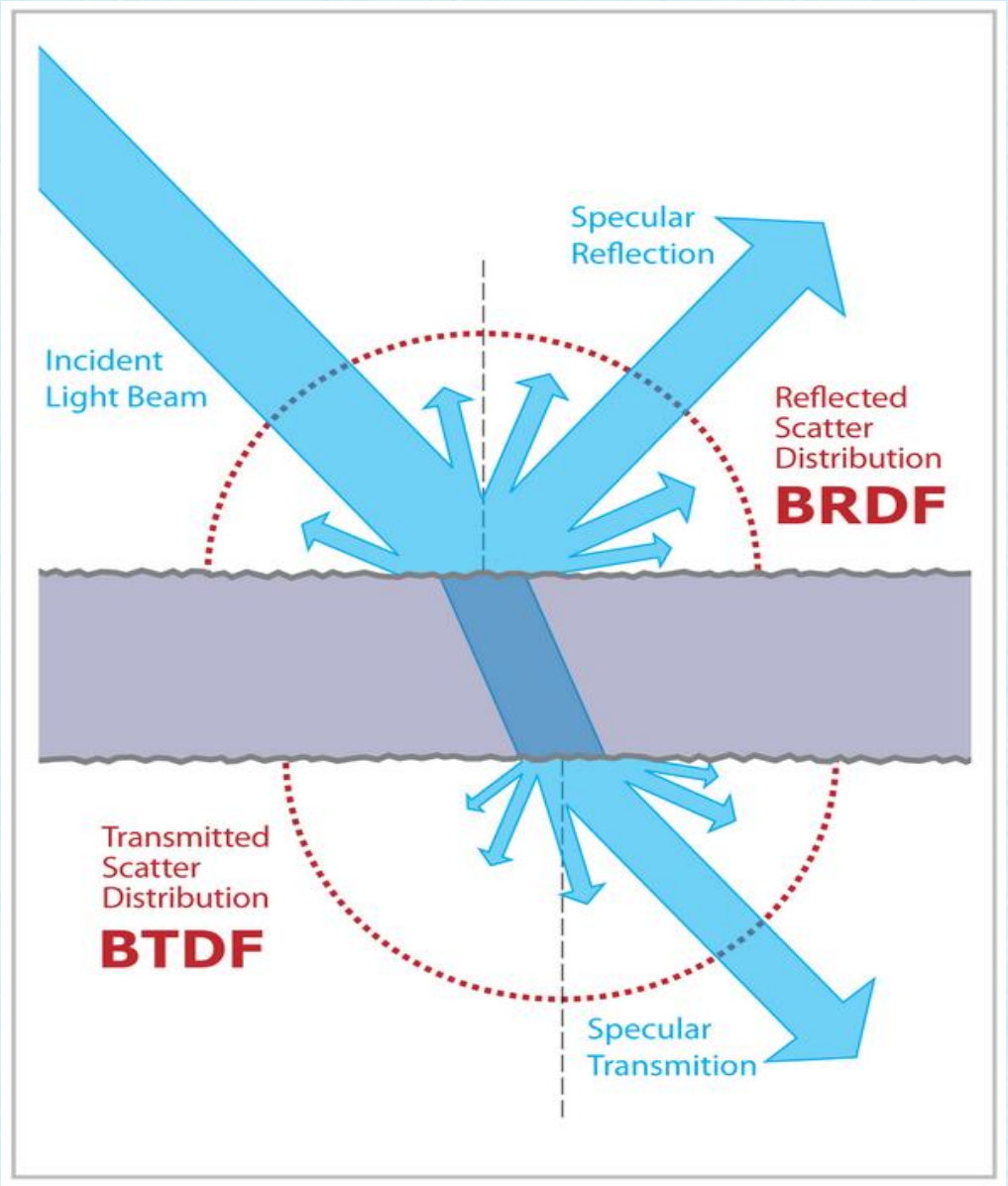
Light/Object Interactions

- When light hits a material, it may be: absorbed, reflected, transmitted
- As light passes through a material, it may be: absorbed, scattered
- When exits a material, it may be: absorbed, reflected, transmitted

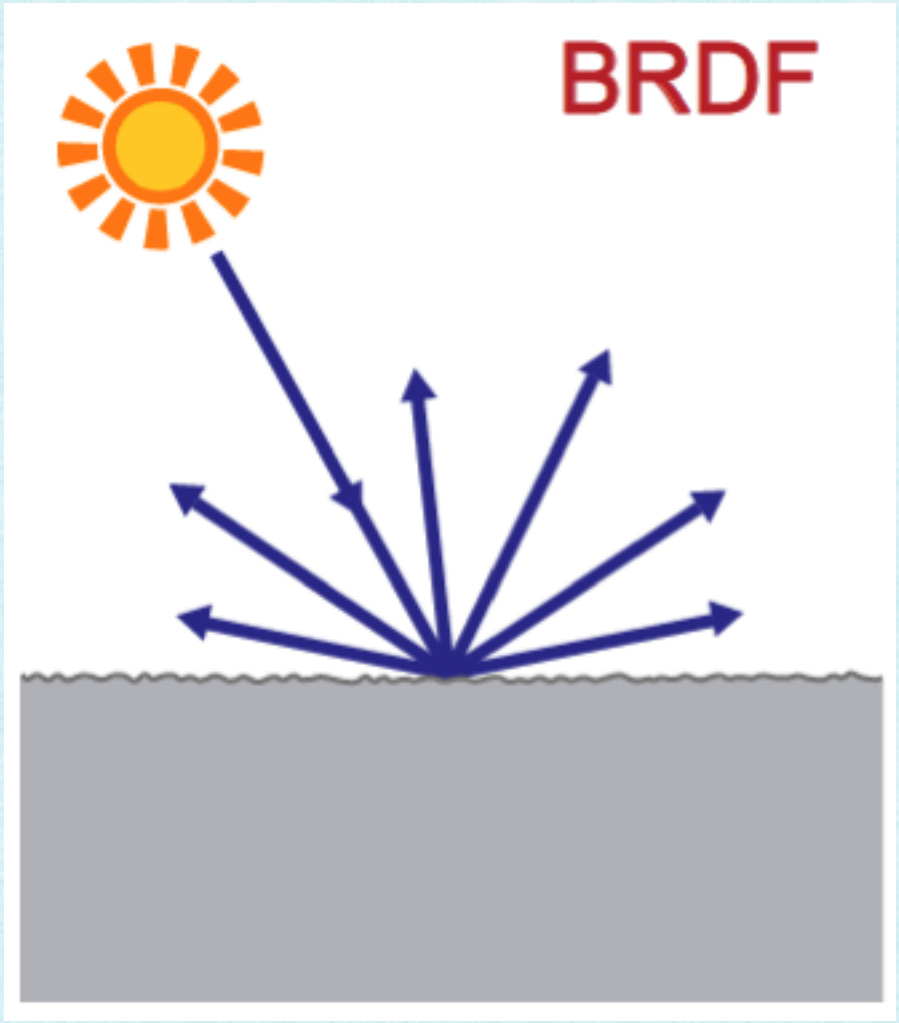


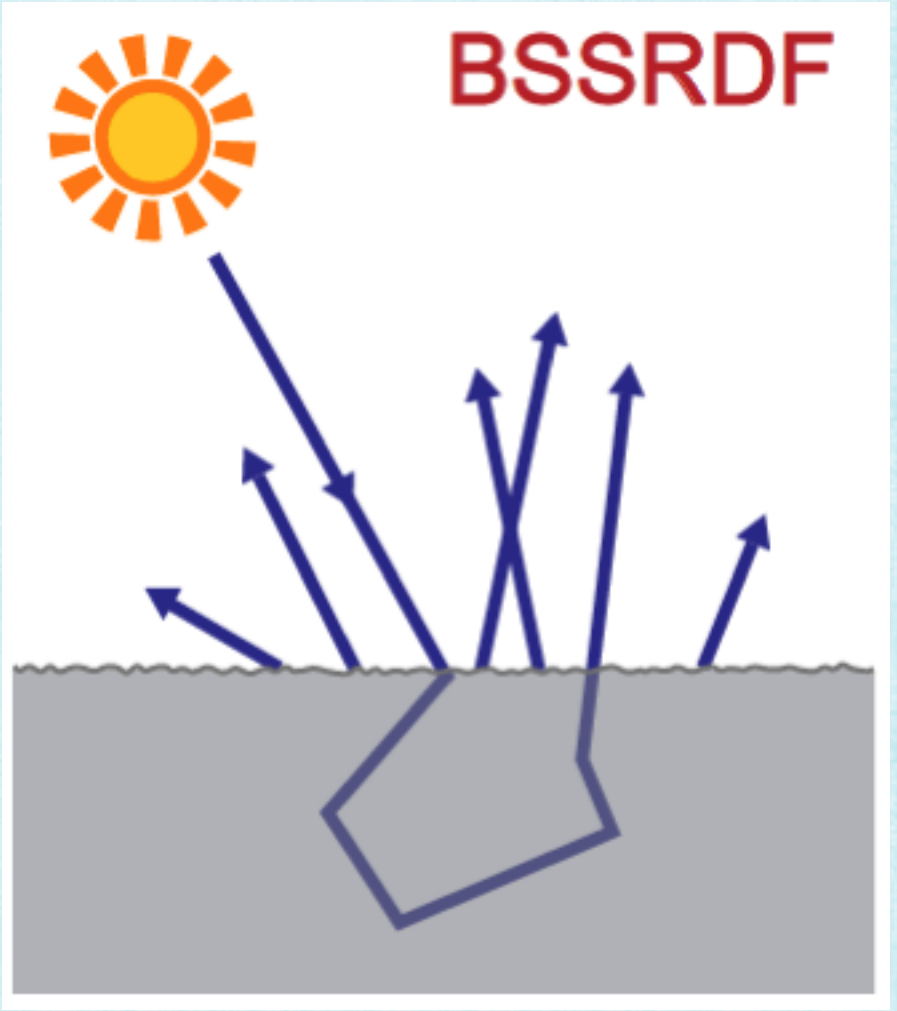
Engineering Approximations

- BRDF
  - Bidirectional Reflectance Distribution Function
  - models how light is reflected
- BTDF
  - Bidirectional Transmittance Distribution Function
  - models how light is transmitted
- BSSRDF
  - Bidirectional Surface Scattering Reflectance Distribution Function
  - combined reflection/transmission model



Opaque (BRDF) vs. Translucent (BSSRDF)





Opaque (BRDF) vs. Translucent (BSSRDF)

Opaque (BRDF) vs. Translucent (BSSRDF)

Opaque (BRDF) vs. Translucent (BSSRDF)

BRDF (objects act as area lights)

- $BRDF(\lambda, \omega_i, \omega_o, u, v)$
  - λ is the wavelength (we'll cheat with R, G, B as usual)
  - (u, v) are the coordinates on the object's surface (we'll cheat with a texture)
  - $\omega_i(\theta_i, \phi_i)$ and $\omega_o(\theta_o, \phi_o)$ are the incoming/outgoing light directions
    - Each parameterized via the 2D surface of a hemisphere: θ and ϕ in polar coordinates

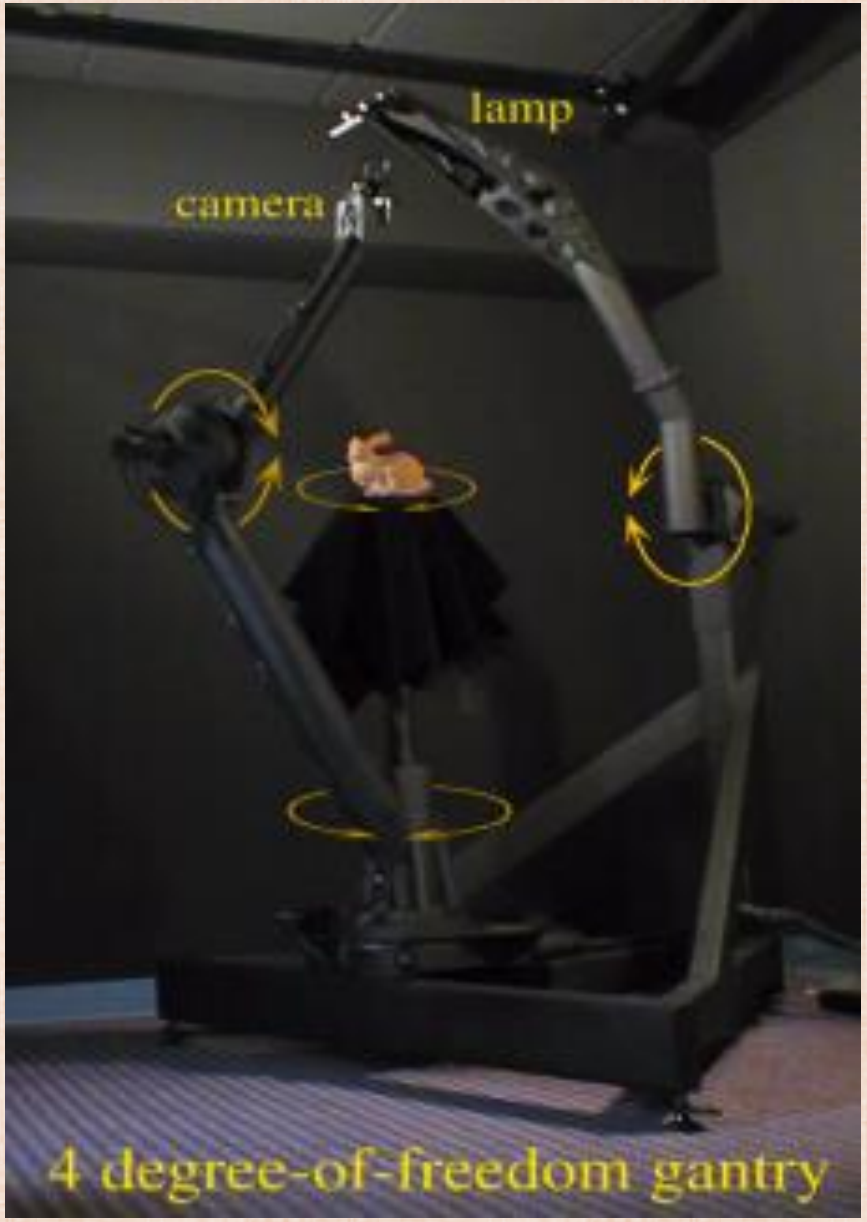
- Consider: $BRDF_R(\omega_i, \omega_o)$, $BRDF_G(\omega_i, \omega_o)$, $BRDF_B(\omega_i, \omega_o)$
  - Each is a 4D function, i.e. functions of 4 variables $\theta_i, \phi_i, \theta_o, \phi_o$

- Specifically: $BRDF(\omega_i, \omega_o) = \frac{dL_o(\omega_o)}{dE_i(\omega_i)}$
- The outgoing light emitted from a surface patch (acting as an area light), as a fraction of the incoming light hitting that surface patch (irradiance)

Measuring and Approximating BRDFs

- Can measure 4D BRDF data with a gonioreflectometer (to obtain a 4D table of values)

- Engineering Approximations:
  - Blinn-Phong Model – simplest and general purpose (plastic)
  - Cook-Torrance Model – better specular (metal)
  - Ward Model – anisotropic (brushed metal, hair)
  - Oren-Nayar Model – non-Lambertian (concrete, plaster, the moon)
  - Etc.



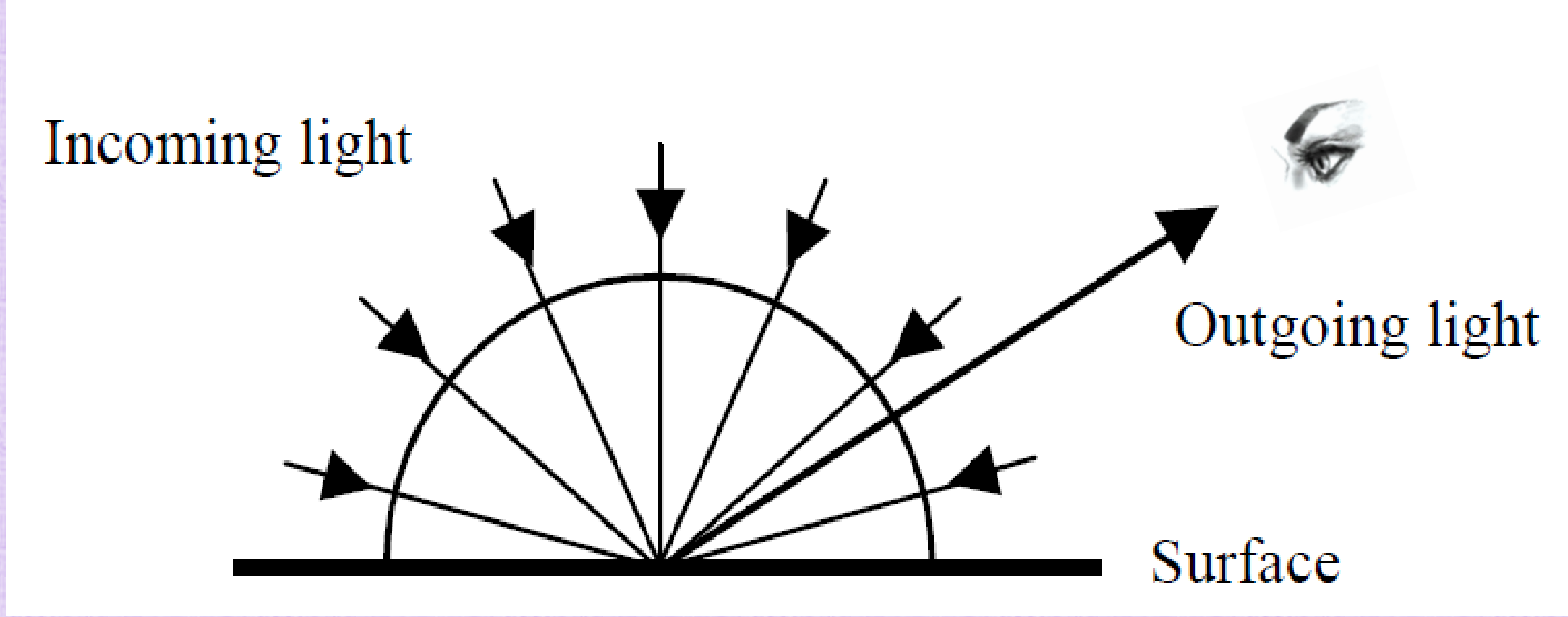
The Lighting Equation

- Given a point on an object:
  - Light from every incoming direction ω_i hits that point
  - For each incoming direction ω_i, light is reflected outwards in every direction ω_o
  - The BRDF indicates what fraction of the light from an incoming direction ω_i is reflected in each of the outgoing directions ω_o

- Light is reflected in all outgoing directions, allowing us all to see the same spot on an object
- But, we all see different light; so, it can (and often does) look differently to each of us
- To render a synthetic scene, need to figure out what light each pixel on the camera's film sees

It's an Integral

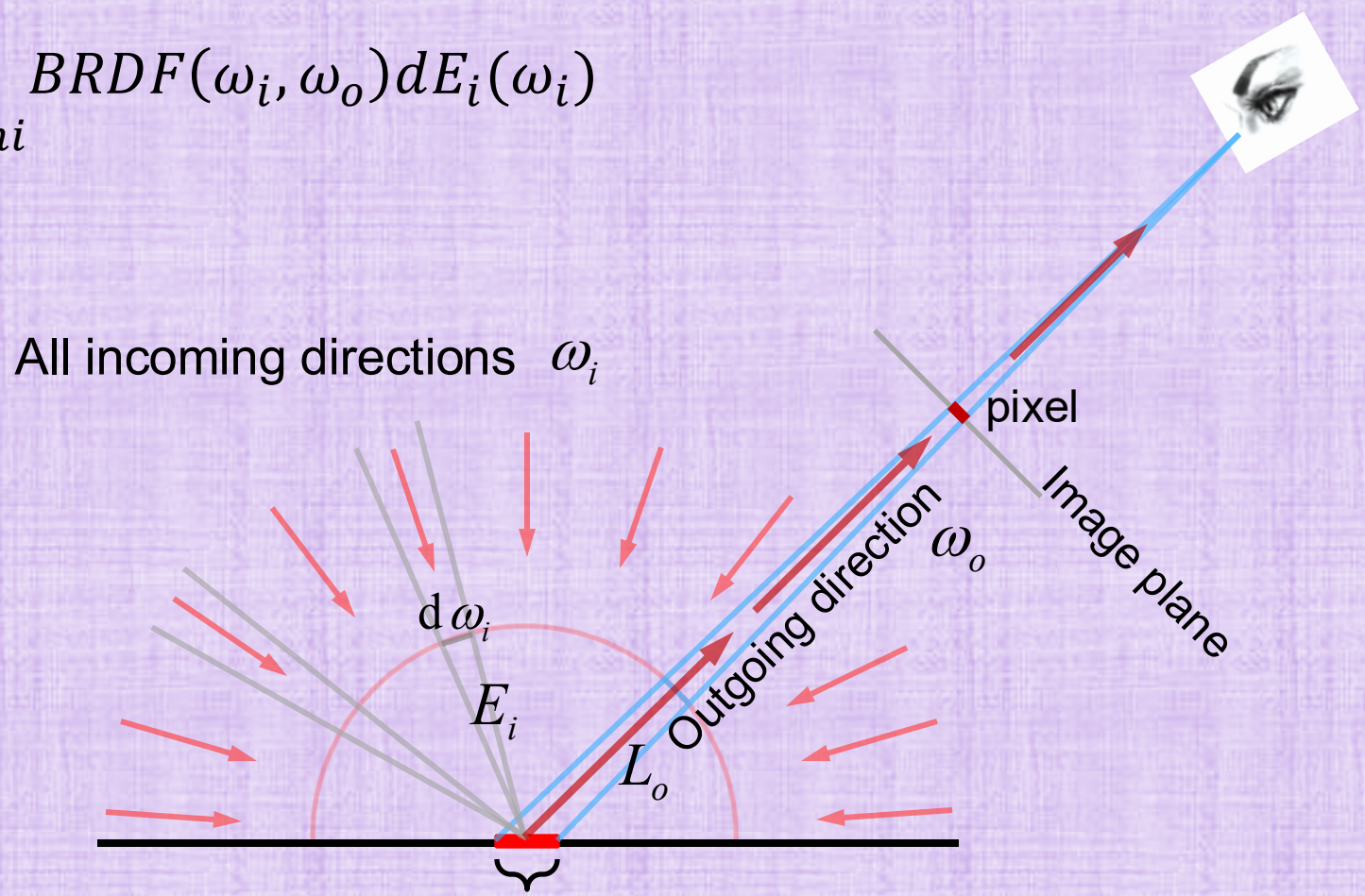
• The total amount of light reflected in a single outgoing direction (towards a pixel) is the sum of the of the light reflected in that direction due to light incoming from every direction: $L_o(\omega_o) = \sum_{i \in hemi} L_{o\ due\ to\ i}(\omega_i, \omega_o)$



The Lighting Equation

- For each pixel, integrate the BRDF across all incoming directions for every point in the pixel's un-projected area (which acts as an area light)

$$L_o(\omega_o) = \int_{i \in hemi} BRDF(\omega_i, \omega_o) dE_i(\omega_i)$$



(Radiance only) Lighting Equation

- Multiplying the BRDF by an incoming irradiance gives the outgoing radiance

$$dL_{o\ due\ to\ i}(\omega_i, \omega_o) = BRDF(\omega_i, \omega_o) dE_i(\omega_i)$$

- For even more realistic lighting, we'll bounce light all around the scene
- It's tedious to convert between E and L, so use $L = \frac{dE}{d\omega \cos\theta}$ to obtain:

$$dL_{o\ due\ to\ i}(\omega_i, \omega_o) = BRDF(\omega_i, \omega_o)\ L_i \cos\theta_i\ d\omega_i$$

- Then,

$$L_o(\omega_o) = \int_{i \in hemi} BRDF(\omega_i, \omega_o) L_i \cos\theta_i\ d\omega_i$$

Pixel Color

- Power per unit area hitting a pixel (irradiance):

$$E_i = \int L_i \cos\theta_i \, d\omega_i$$

obtained from integrating $dE = L \cos\theta \, d\omega$

- Assume that the film is small and almost orthogonal to the incoming light, so that $\cos\theta \approx 1$ can be ignored
- Assume L is approximately constant across a (very) small pixel; then,

$$E_{pixel} \approx L_{pixel,ave} \int d\omega_i = L_{pixel,ave} \omega_{pixel}$$

- If the film is small, so that all the pixels have a similar solid angle; then,

$$E_{pixel} \approx \left(\frac{\omega_{film}}{\#\ pixels} \right) L_{pixel,ave}$$

- Thus, can store L instead of E (and uniformly scale by constant later)