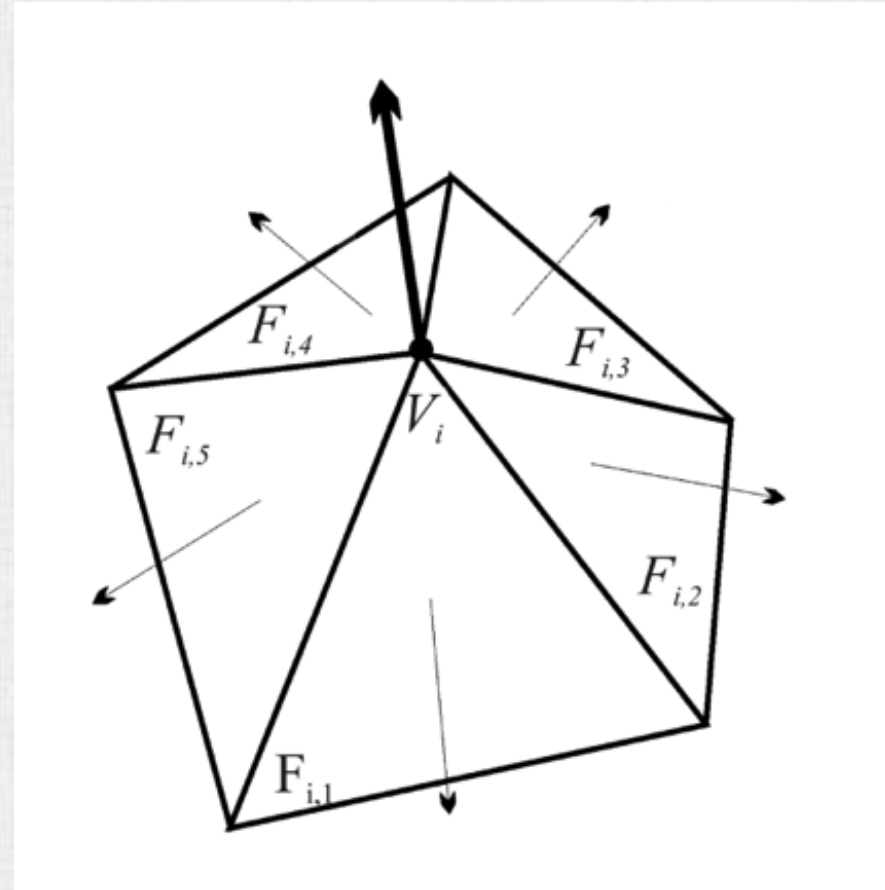
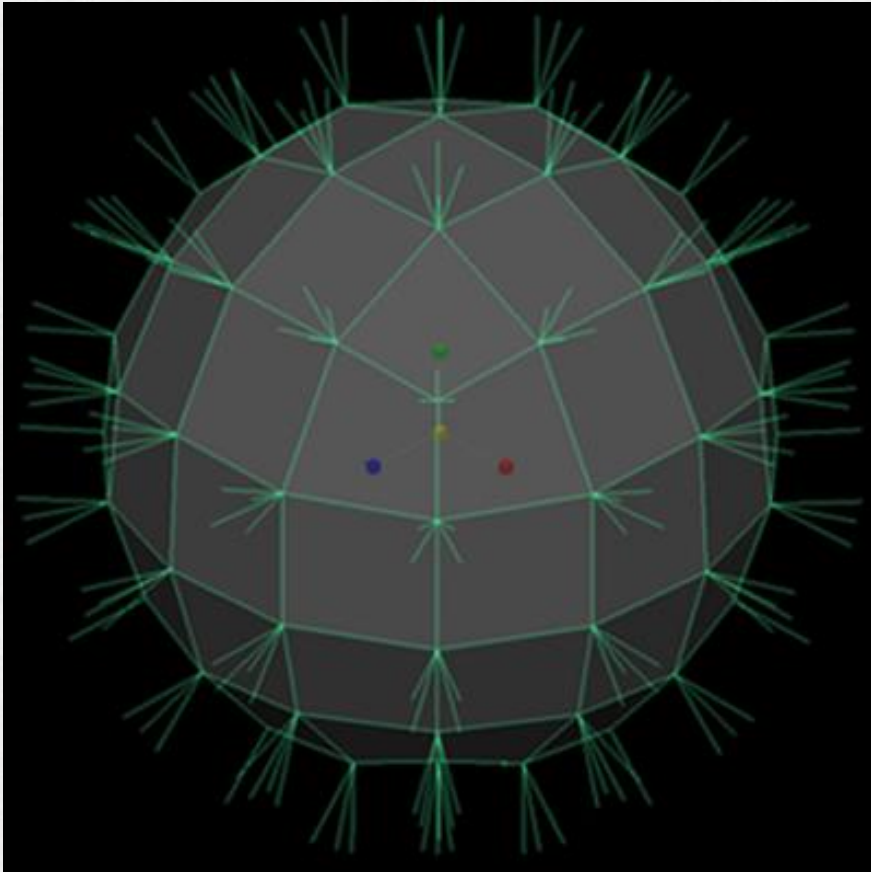


More Texture Mapping



Recall: Averaged Vertex Normals

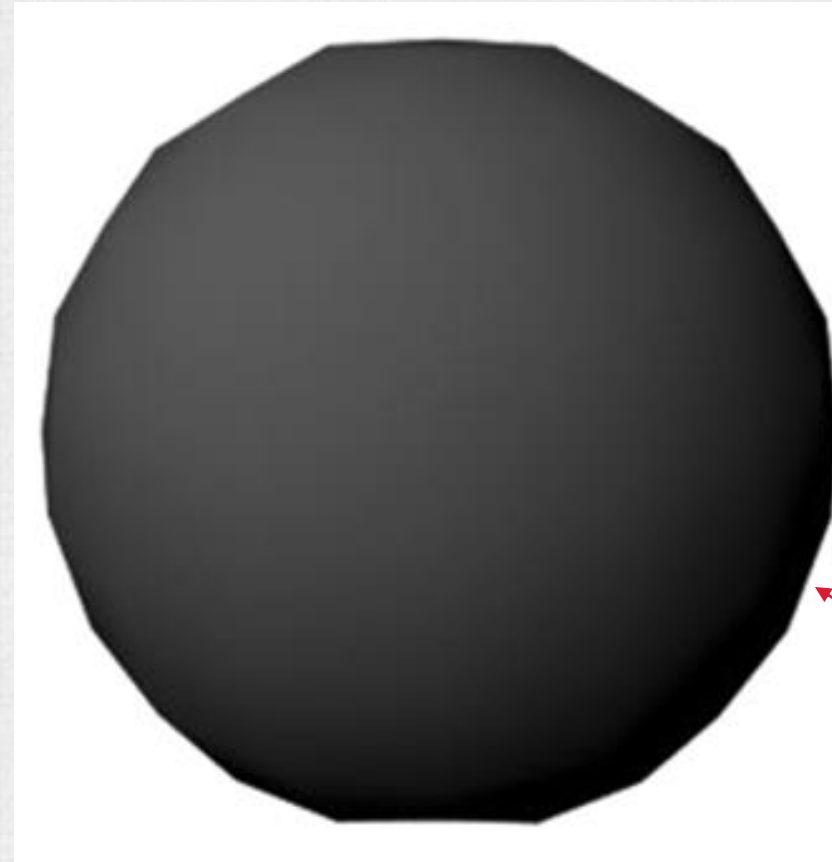
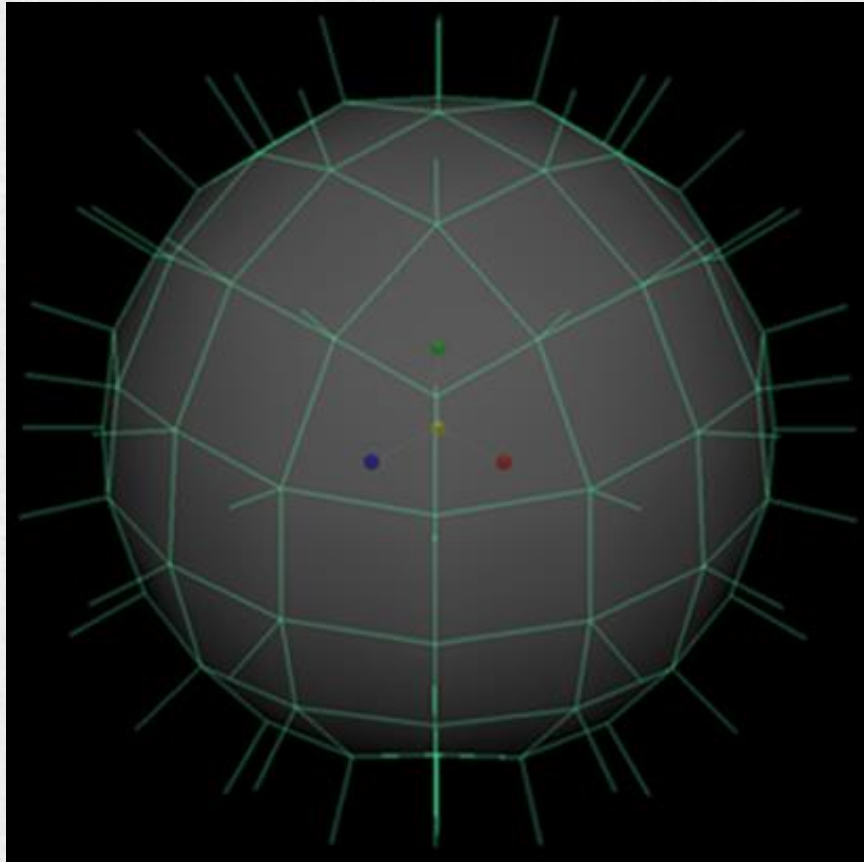
- Each vertex belongs to a number of triangles, each with their own normal
- Averaging those normals (weighted averaging, based on: area, angle, etc.) gives a unique normal for each vertex



Recall: Smooth Shading

- Use barycentric weights to interpolate averaged vertex normals to the interior of a triangle:

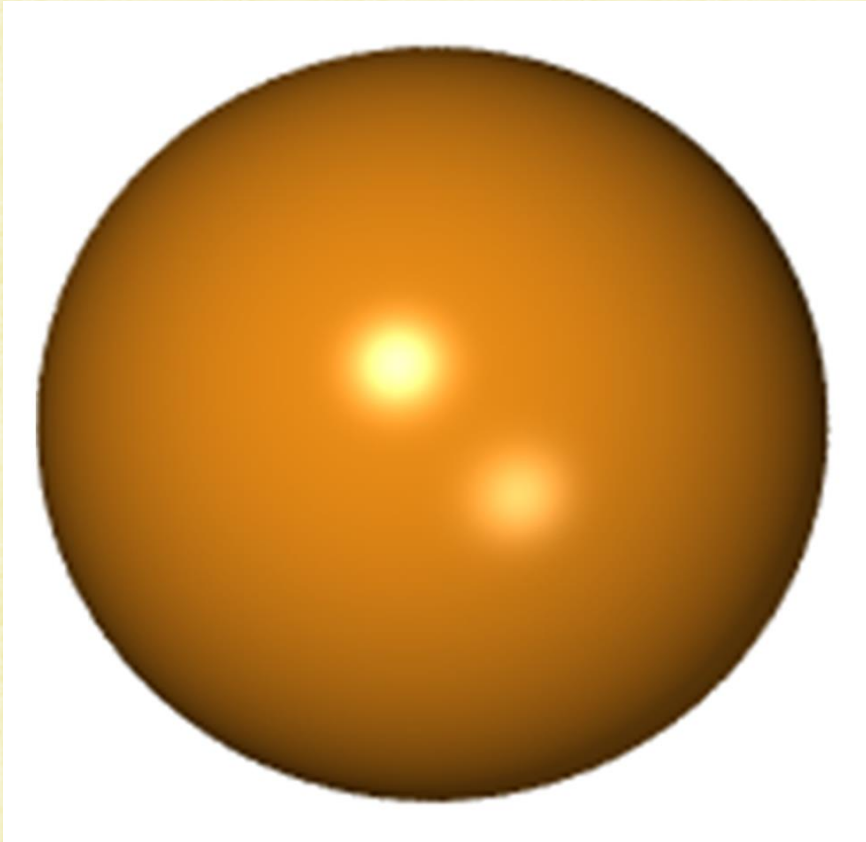
$$\hat{N}_p = \frac{\alpha_0 \hat{N}_0 + \alpha_1 \hat{N}_1 + \alpha_2 \hat{N}_2}{\|\alpha_0 \hat{N}_0 + \alpha_1 \hat{N}_1 + \alpha_2 \hat{N}_2\|_2}$$



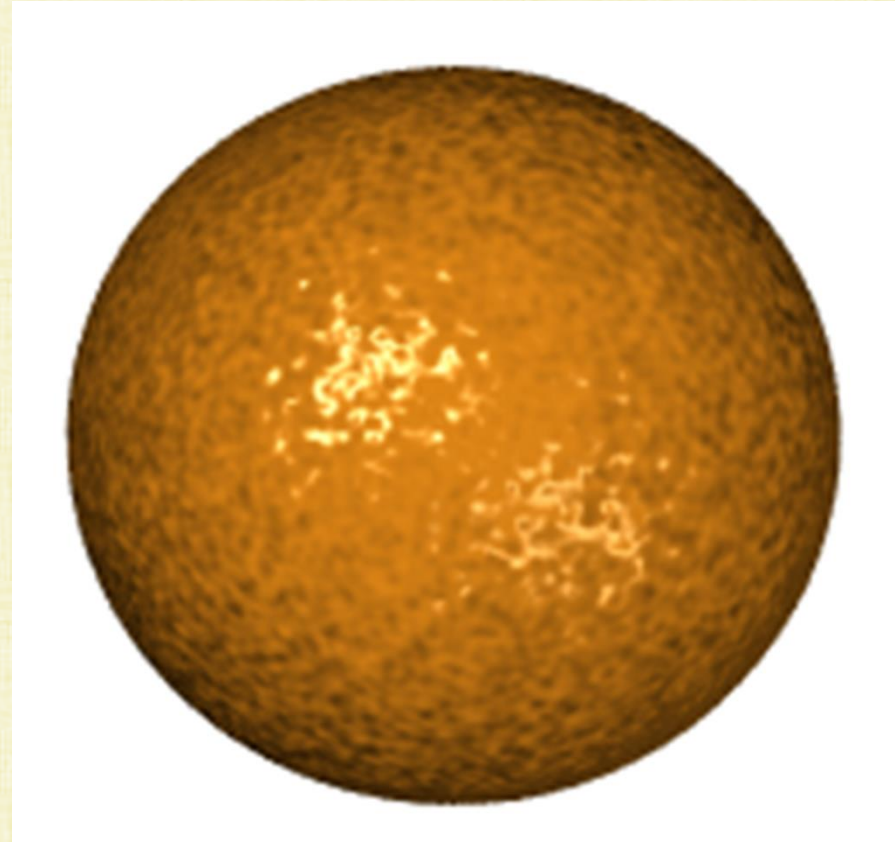
faceted
silhouette

Perturbing the Normal

- Store a normal vector in the texture (instead of a color)
- A perturbed normal can be used to make “fake” geometric details



using real normal



using fake normal

Normal Map

- A normalized vector has each component in $[-1,1]$, so can convert back and forth to a color format via:

$$(R, G, B) = 255 \frac{\vec{N} + (1,1,1)}{2} \quad \text{and} \quad \vec{N} = \frac{2}{255} (R, G, B) - (1,1,1)$$



normal mapping on a plane
(note the variation in specular highlights created by variation of the normal)

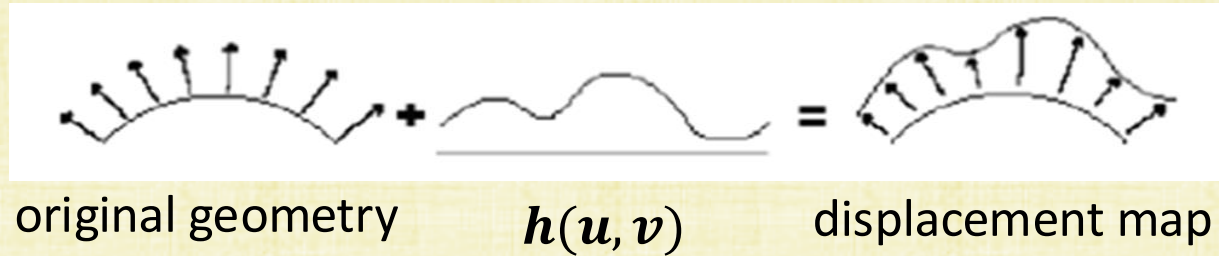
Bump Map (variation of a Normal Map)

- Single-channel (grey-scale) height map h_{ij} , representing the height at location (u_i, v_j)
- The tangent plane at a point (u_i, v_j, h_{ij}) is: $-\frac{\partial h(u_i, v_j)}{\partial u} (u - u_i) - \frac{\partial h(u_i, v_j)}{\partial v} (v - v_j) + (h - h_{ij}) = 0$
- So, the outward pointing (non-unit) normal is: $\left(-\frac{\partial h(u_i, v_j)}{\partial u}, -\frac{\partial h(u_i, v_j)}{\partial v}, 1 \right)$
- Compute the partial derivatives via finite differences: $\frac{\partial h(u_i, v_j)}{\partial u} = \frac{h_{i+1, j} - h_{i-1, j}}{u_{i+1} - u_{i-1}}$ and $\frac{\partial h(u_i, v_j)}{\partial v} = \frac{h_{i, j+1} - h_{i, j-1}}{v_{j+1} - v_{j-1}}$



Displacement Mapping

- Subdivide geometry at render time, and use a height map $h(u, v)$ to perturb vertices in the normal direction
- Pros: self-occlusion, self-shadowing, **detailed silhouettes**
- Cons: expensive, requires adaptive tessellation, still need bump/normal map for sub-triangle detail



bump map



displacement map

Displacement Mapping



bump map



displacement map

Inferencing Displacement Maps

- Train a **neural network** to inference a texture (based on the shape of a skeleton)
- Use the texture as a displacement map to drive cloth deformations



inferred texture image



vertices look up colors



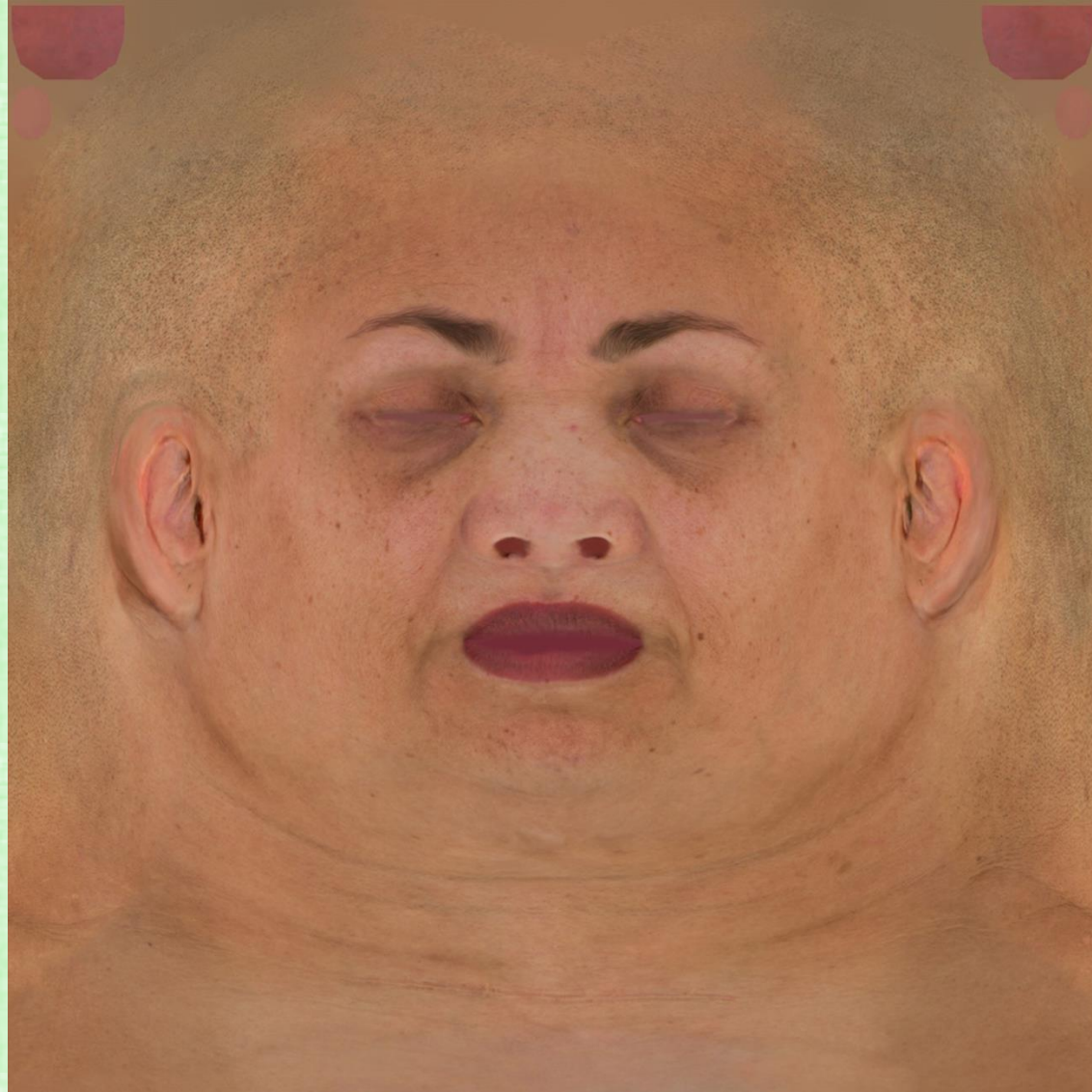
displace vertices to get
folding & wrinkling

Texture Acquisition via Imaging



Light Stage

Texture Acquisition via Imaging



Face Texture

A Democratized Approach using Gaussian Splatting



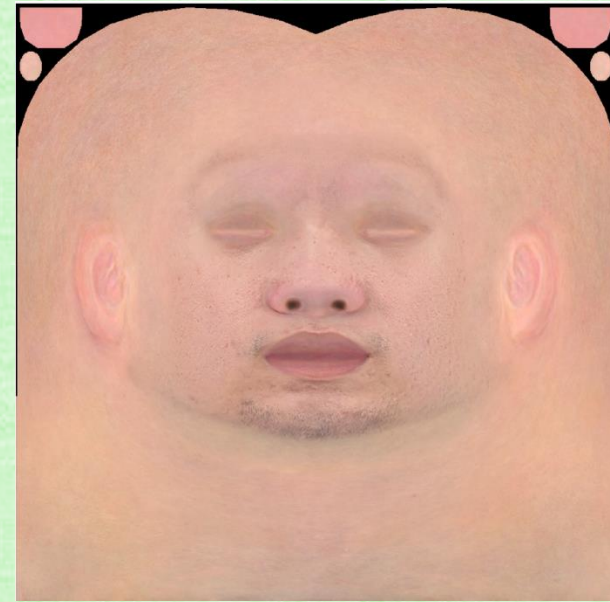
cell phone images



de-lit images
via portrait editing

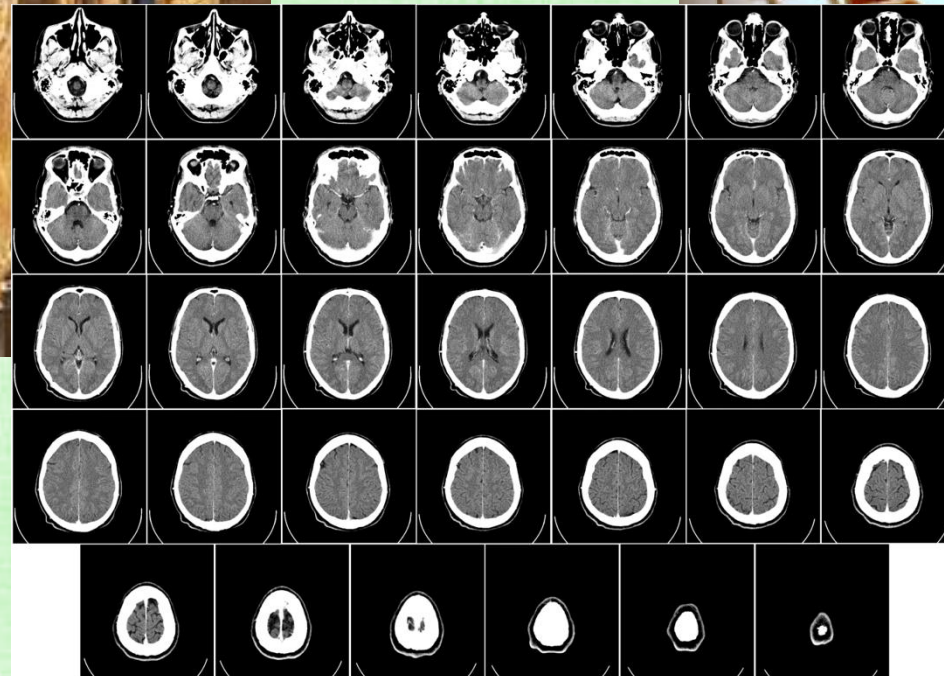
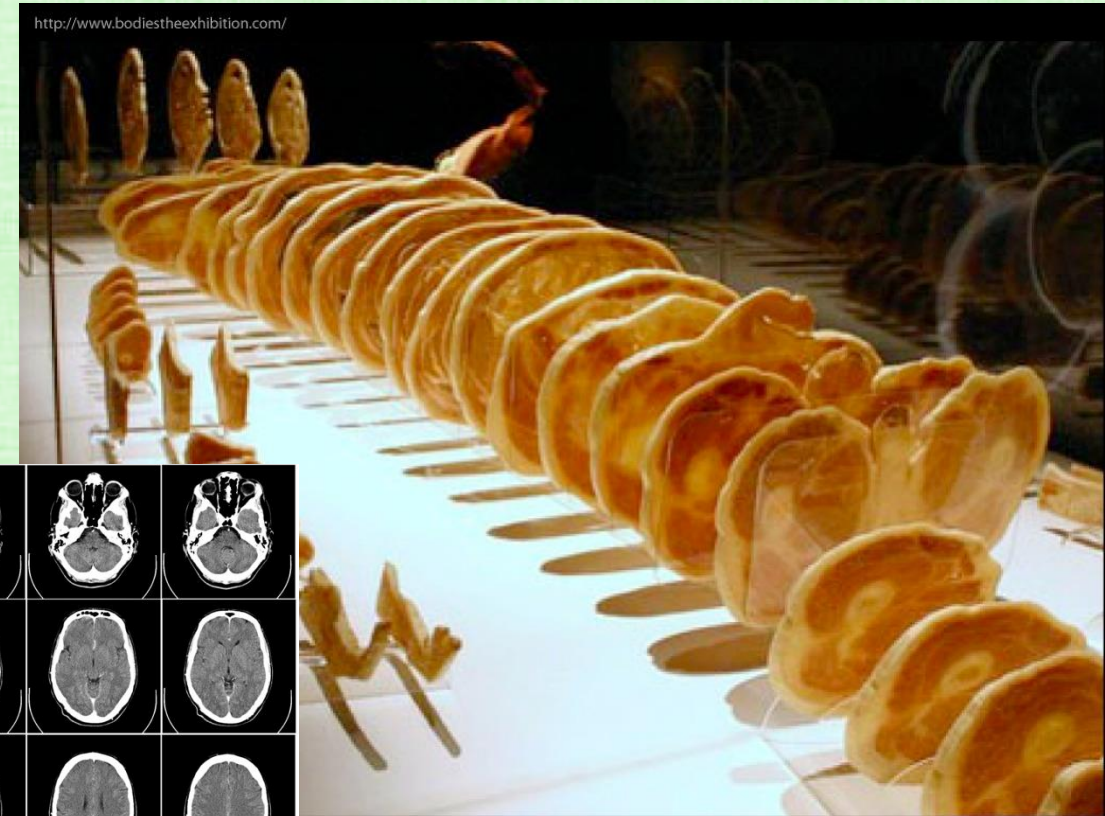
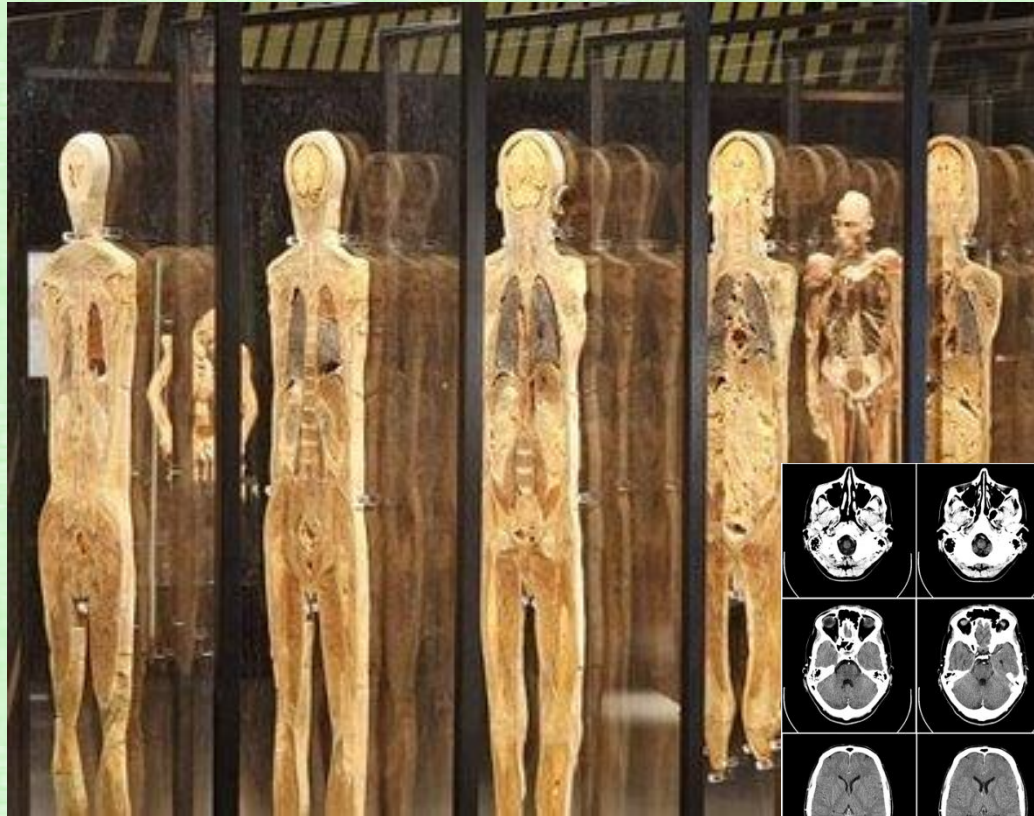


de-lit texture



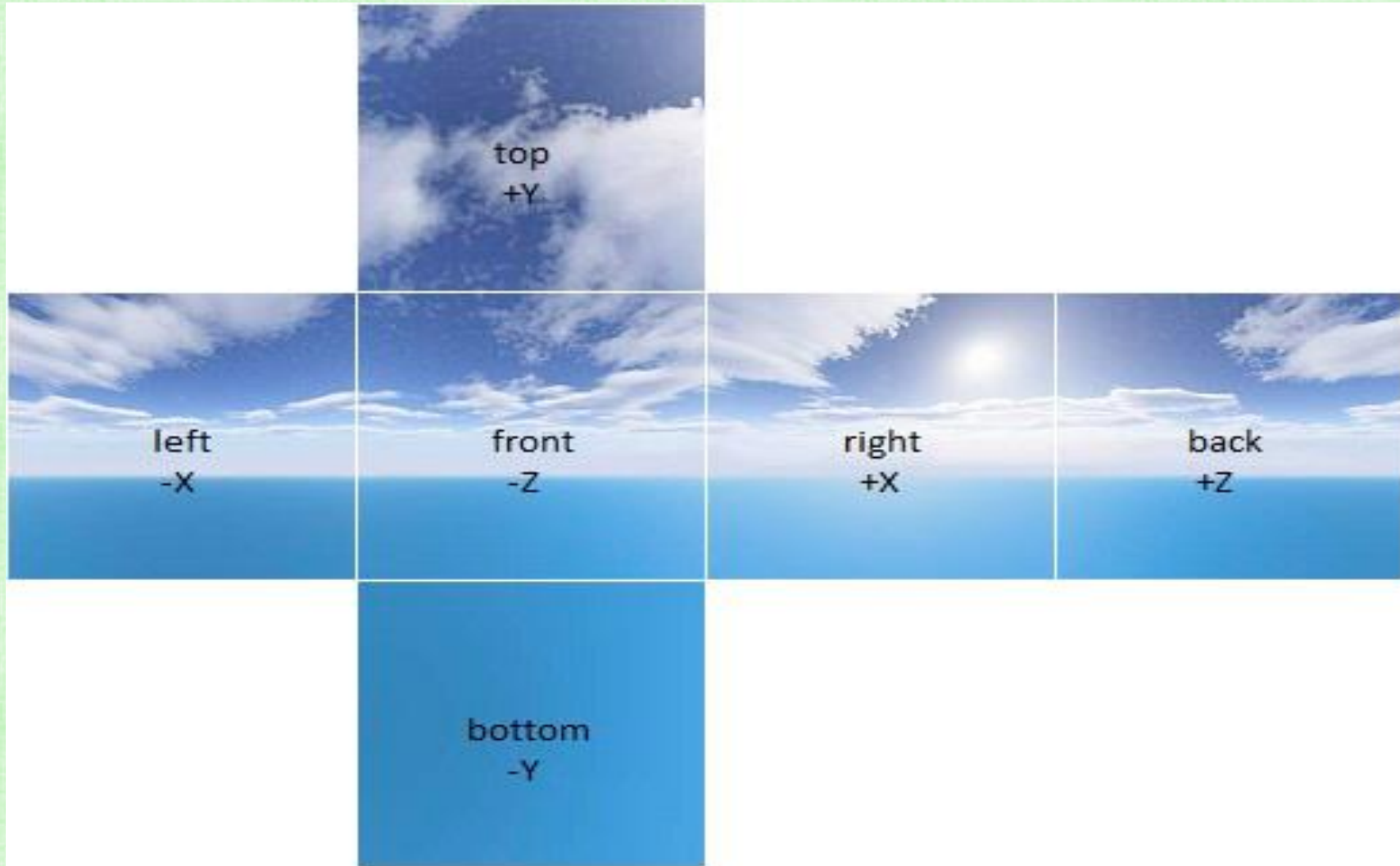
in-painted texture
(removing hair/eyebrows, etc.)

Texture Acquisition via Medical Imaging



Sky Boxes

- Take pictures of the sky, and use them as textures on the inside of a cube (or hemisphere, etc.)



Recall: Measuring Incoming Light

- Light Probe: a small reflective chrome sphere
- Photograph it, in order to record the incoming light (at its location) from all directions



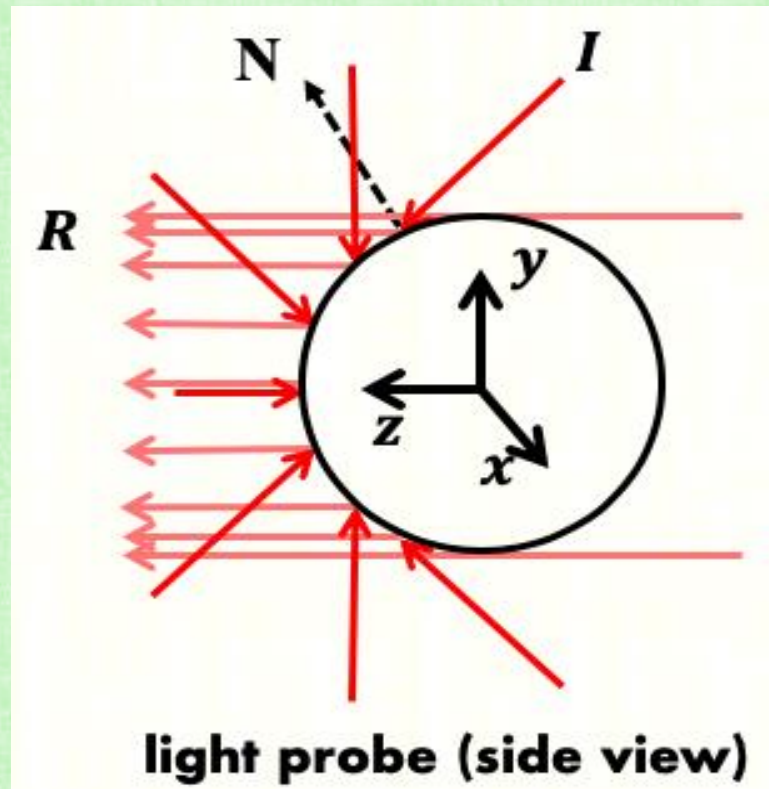
Recall: Using the Measured Incoming Light

- The measured incoming light can be used to render a synthetic object with realistic lighting



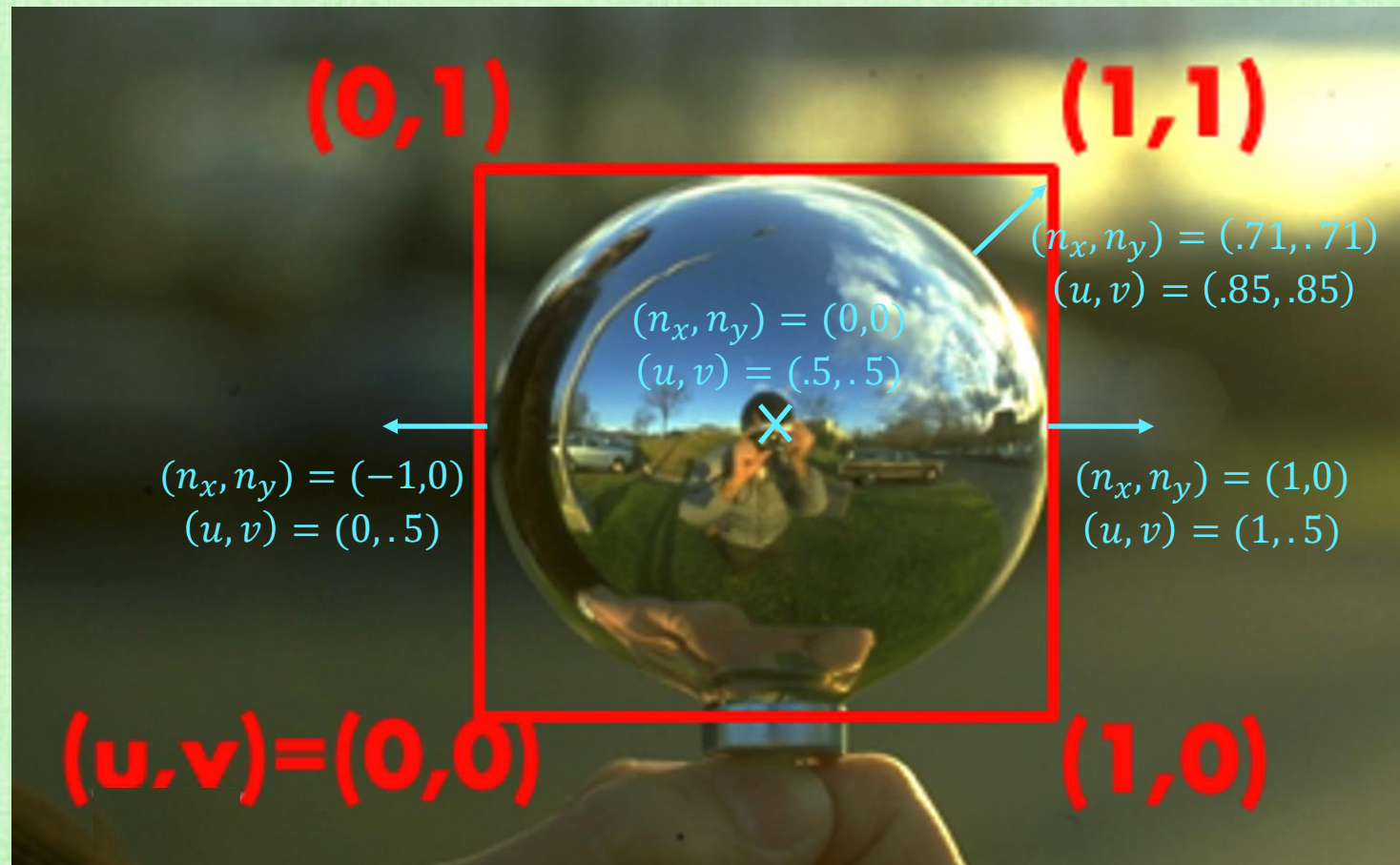
Environment Mapping

- Place a coordinate system at the center of the sphere; then, the surface normal is: $N = \frac{(x, y, z)}{\sqrt{x^2 + y^2 + z^2}}$
- Let R be the direction from the light probe to the camera
- I and R are equal-angle from N (because of mirror reflection)
- Thus, N has a one-to-one correspondence with I (i.e., N encodes the incoming light direction)



Environment Mapping

- Compute the normal $(\mathbf{n}_x, \mathbf{n}_y, \mathbf{n}_z)$ at the point on the geometry that is being rendered
- Use \mathbf{n}_x and \mathbf{n}_y (in the interval $[-1, 1]$) to obtain texture coordinates $(\mathbf{u}, \mathbf{v}) = \frac{1}{2}(\mathbf{n}_x + 1, \mathbf{n}_y + 1)$
- Then, the incoming light can be “looked up” in the texture (which is a picture of the chrome sphere)

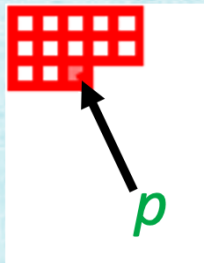


Environment Mapping

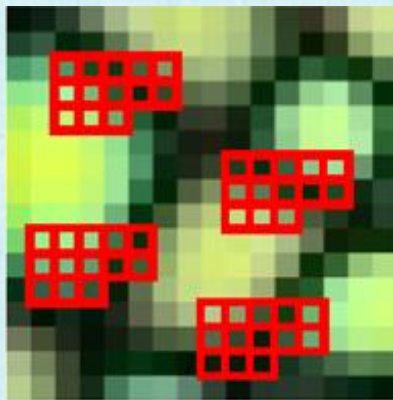


Texture Synthesis: Pixel Based

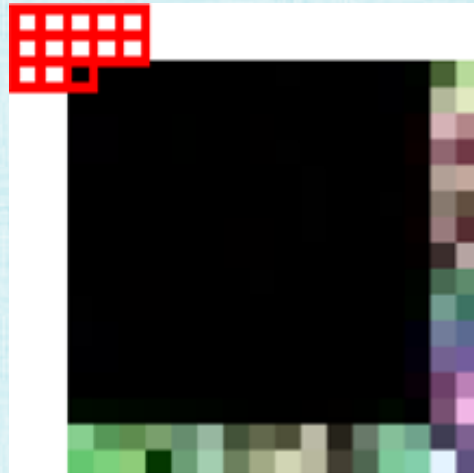
- Create a larger texture (one pixel at a time) from a small sample (using its structural content)
- Generate the texture in a raster scan ordering
- To generate the texture for pixel p :
 - Compare p 's neighboring pixels in the (red) **stencil** to all potential choices in the sample
 - Choose the stencil position with the smallest difference to fill pixel p
- When the **stencil** needs values outside the domain, use periodic boundaries (so, fill the last few rows/columns with random values)



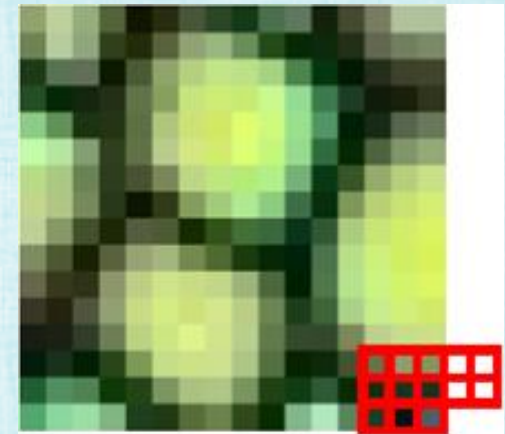
stencil



texture sample



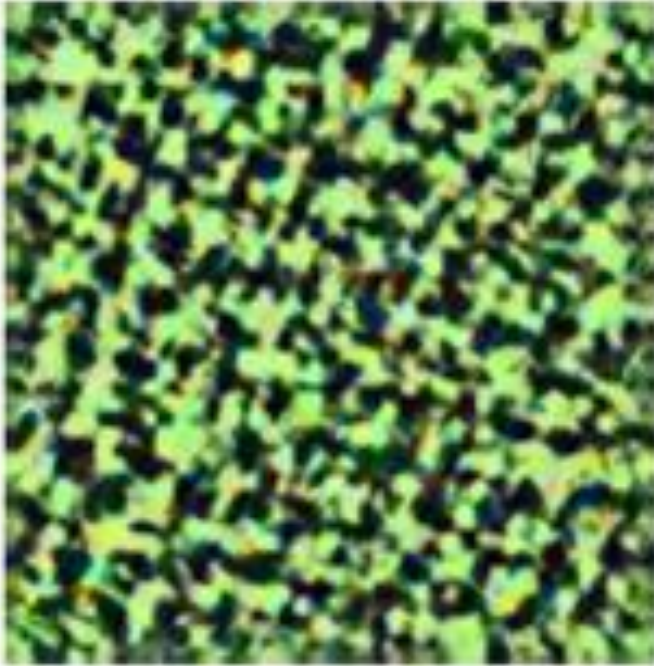
raster scan ordering (with randomly generated periodic boundaries)



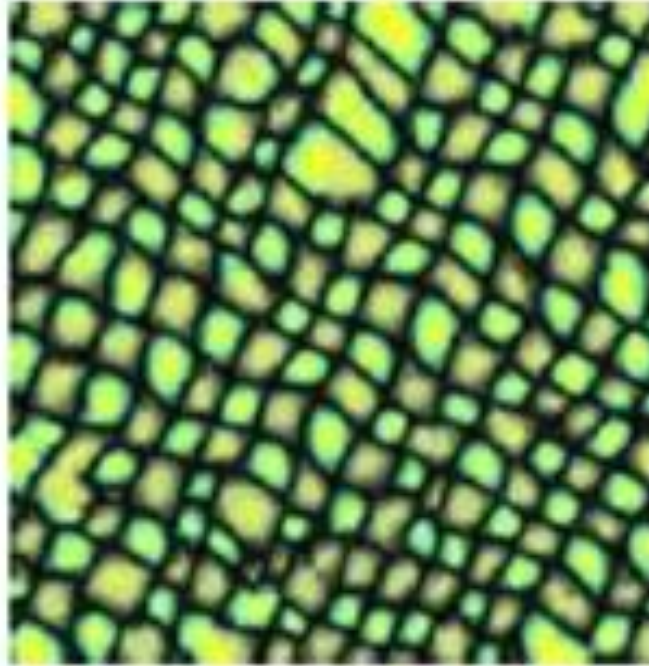
Texture Synthesis: Pixel Based



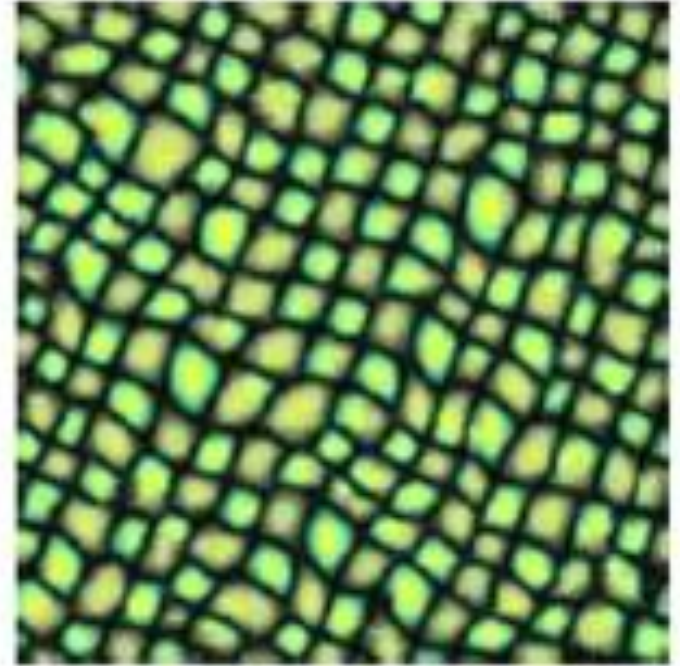
Sample



Heeger and Bergen



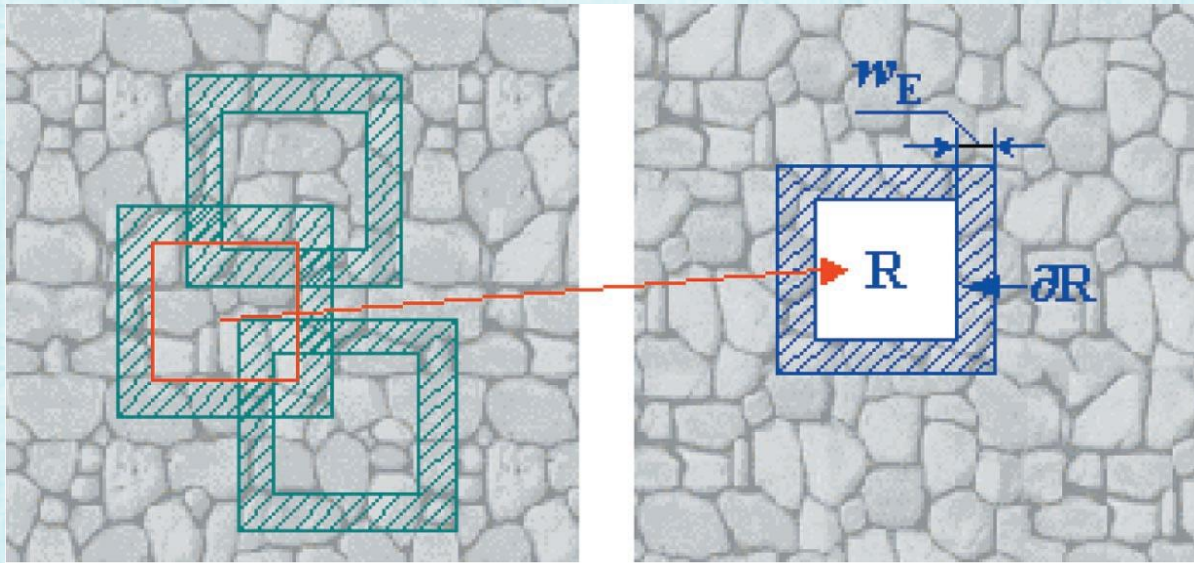
Efros and Leung



Wei and Levoy

Texture Synthesis: Patch Based

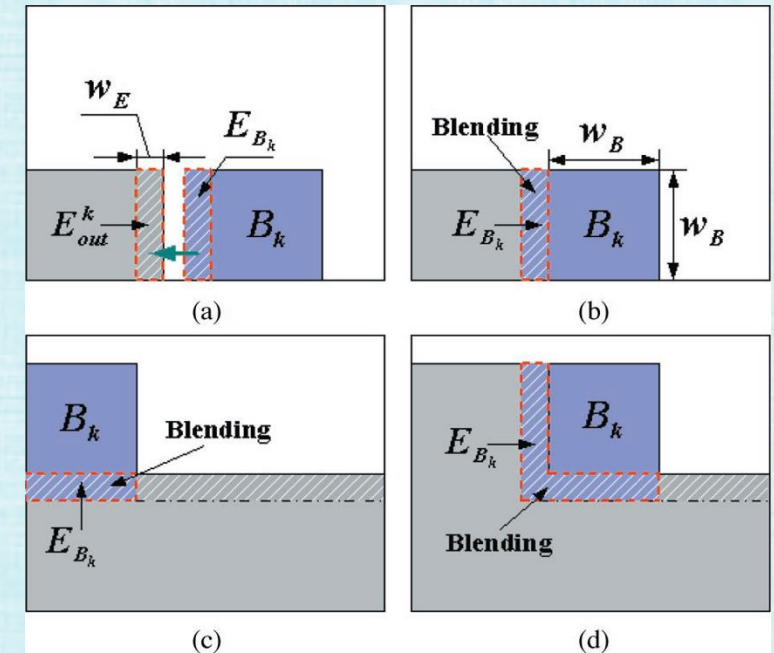
- For each patch:
 - Search the original sample to find the candidate that best matches the overlap boundaries
 - Choose the best candidate
 - Blend overlapped regions to remove “seams”



sample

texture

matching
boundary
regions



Texture Synthesis: Patch Based



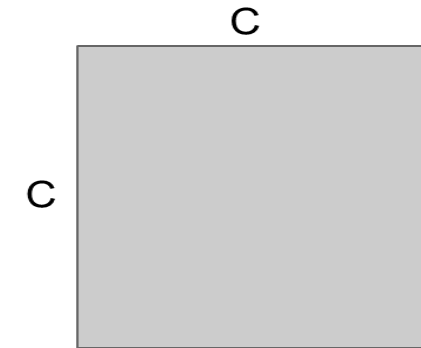
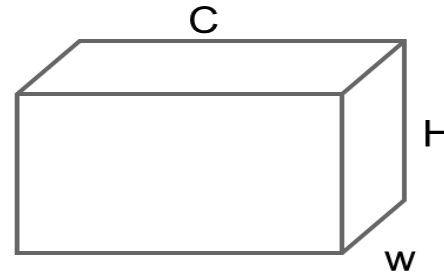
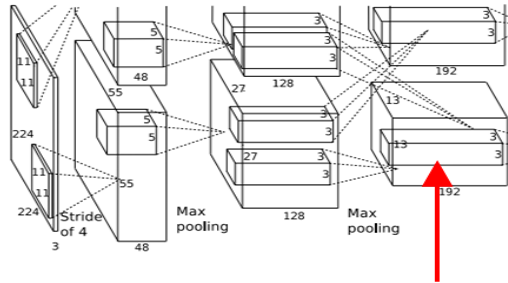
Texture Synthesis: AI Methods

CS231N

Neural Texture Synthesis: Gram Matrix



[This image](#) is in the public domain.



Each layer of CNN gives $C \times H \times W$ tensor of features; $H \times W$ grid of C -dimensional vectors

Outer product of two C -dimensional vectors gives $C \times C$ matrix measuring co-occurrence

Average over all HW pairs of vectors, giving **Gram matrix** of shape $C \times C$

Efficient to compute; reshape features from

$C \times H \times W$ to $=C \times HW$

then compute $G = FF^T$

Texture Synthesis: AI Methods

CS231N

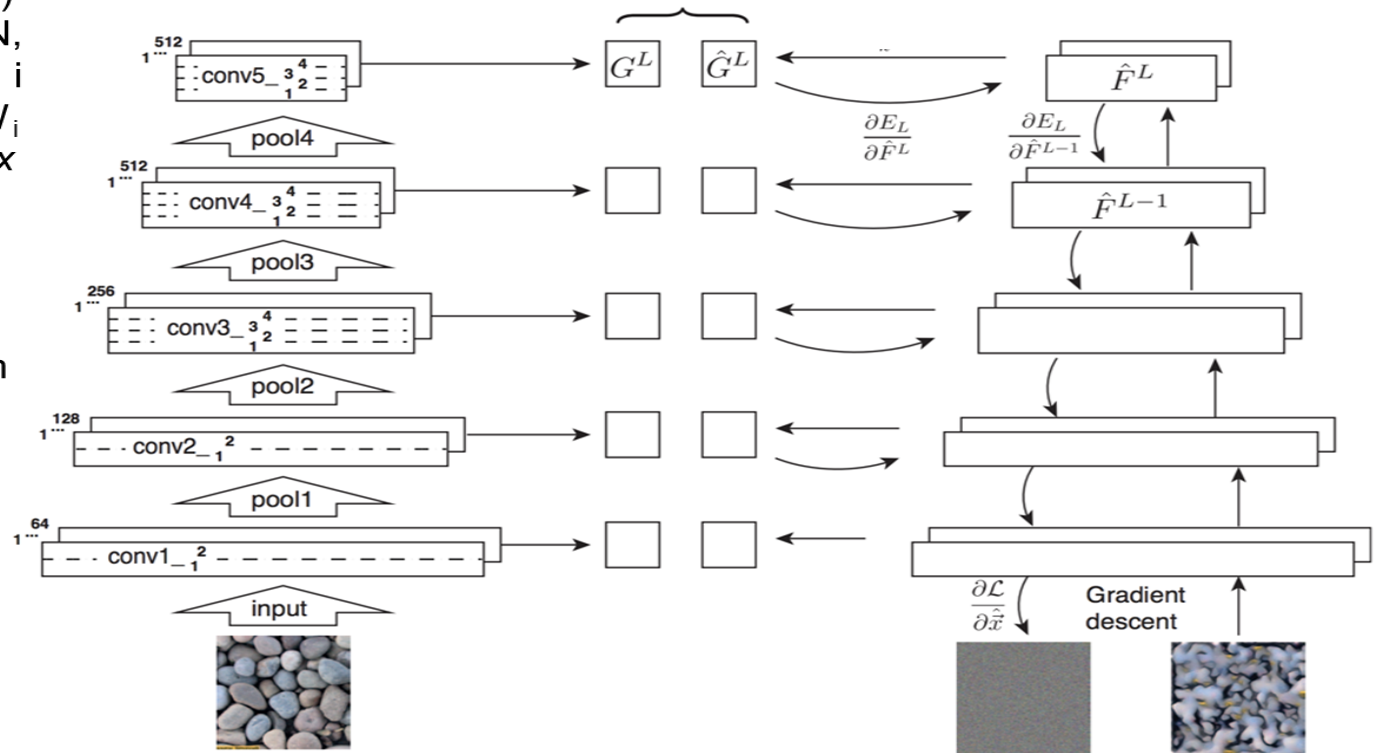
Neural Texture Synthesis

1. Pretrain a CNN on ImageNet (VGG-19)
2. Run input texture forward through CNN, record activations on every layer; layer i gives feature map of shape $C_i \times H_i \times W_i$
3. At each layer compute the *Gram matrix* giving outer product of features:

$$G_{ij}^l = \sum_k F_{ik}^l F_{jk}^l \text{ (shape } C_i \times C_i \text{)}$$

4. Initialize generated image from random noise
5. Pass generated image through CNN, compute Gram matrix on each layer
6. Compute loss: weighted sum of L2 distance between Gram matrices
7. Backprop to get gradient on image
8. Make gradient step on image
9. GOTO 5

$$E_l = \frac{1}{4N_l^2 M_l^2} \sum_{i,j} (G_{ij}^l - \hat{G}_{ij}^l)^2 \quad \mathcal{L}(\vec{x}, \hat{\vec{x}}) = \sum_{l=0}^L w_l E_l$$



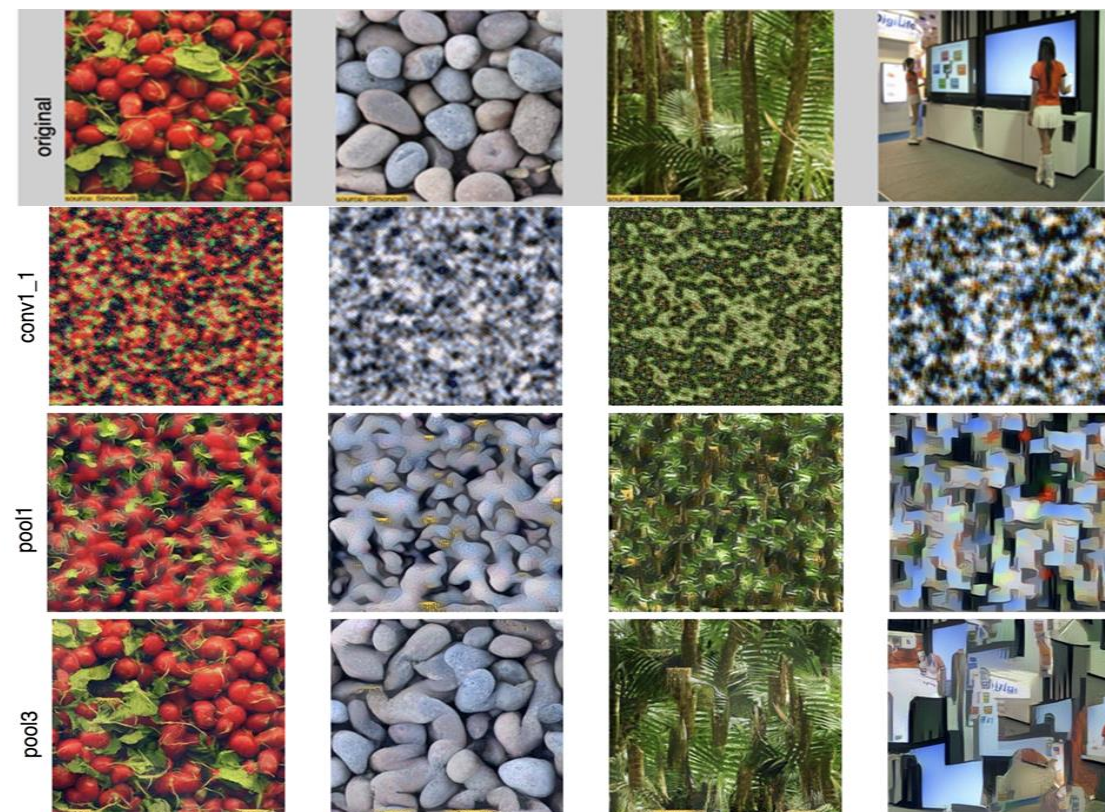
Gatys, Ecker, and Bethge, "Texture Synthesis Using Convolutional Neural Networks", NIPS 2015
Figure copyright Leon Gatys, Alexander S. Ecker, and Matthias Bethge, 2015. Reproduced with permission.

Texture Synthesis: AI Methods

CS231N

Neural Texture Synthesis

Reconstructing texture
from higher layers recovers
larger features from the
input texture



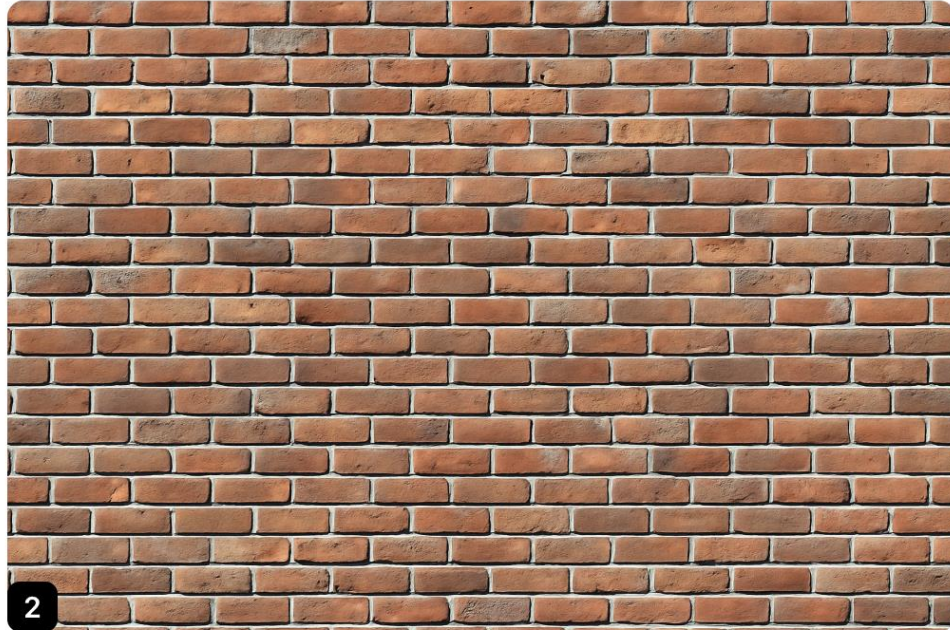
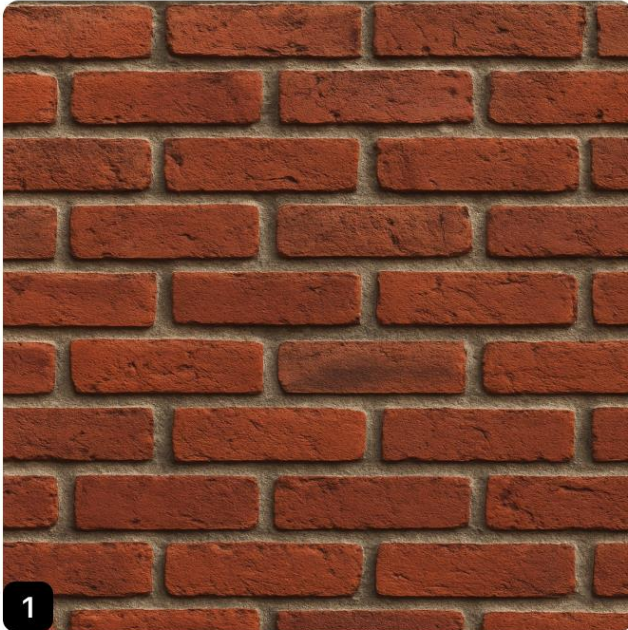
Gatys, Ecker, and Bethge, "Texture Synthesis Using Convolutional Neural Networks", NIPS 2015
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Texture Synthesis: ChatGPT

generate a brick texture

Which image do you like more?

[Skip](#)

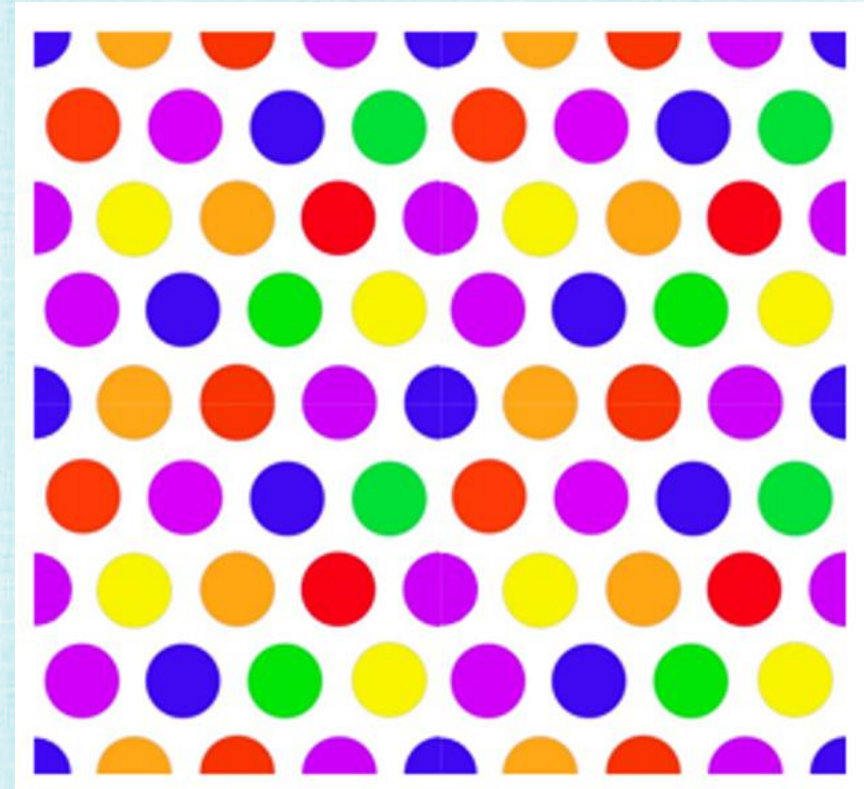
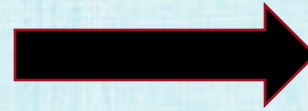
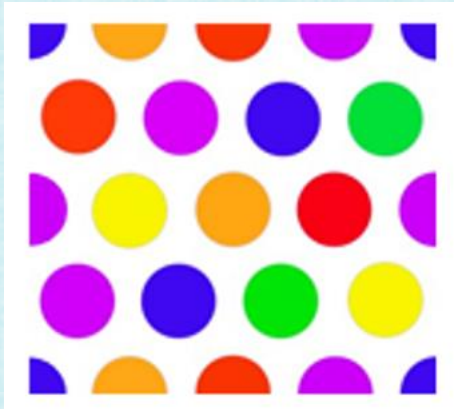
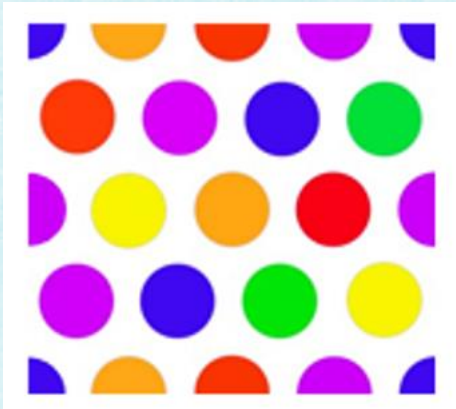
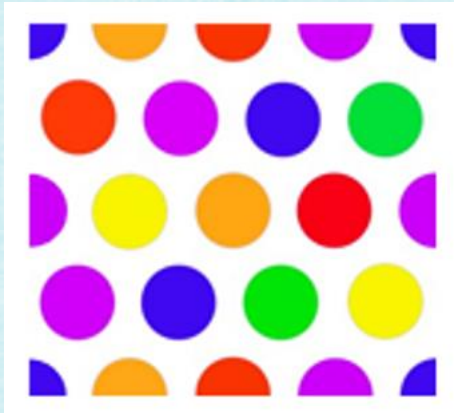
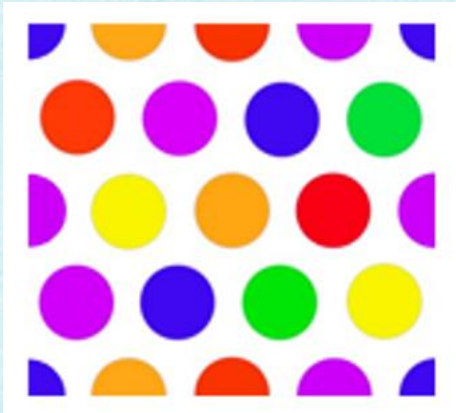


☒ Image 1 is better

☒ Image 2 is better

Don't Stretch Textures!

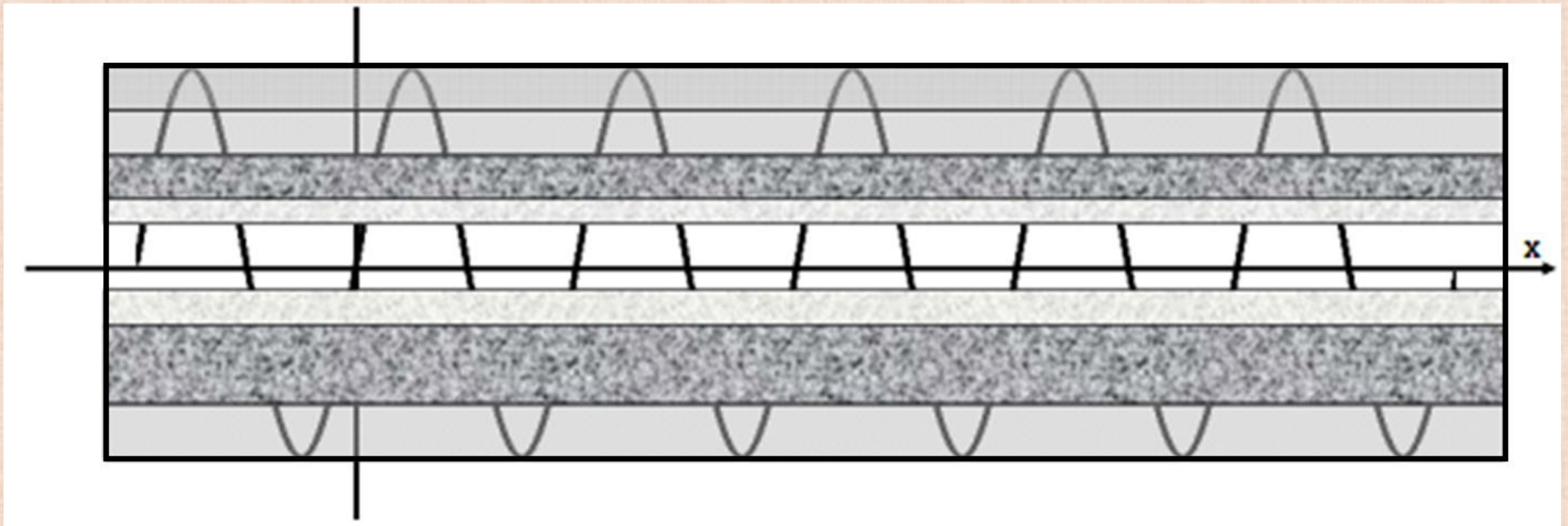
- Stretching out 10 bricks to cover an entire wall of a building is going to look unrealistic!
- Instead, **tile the texture** (requires periodic boundaries)



Marble Texture

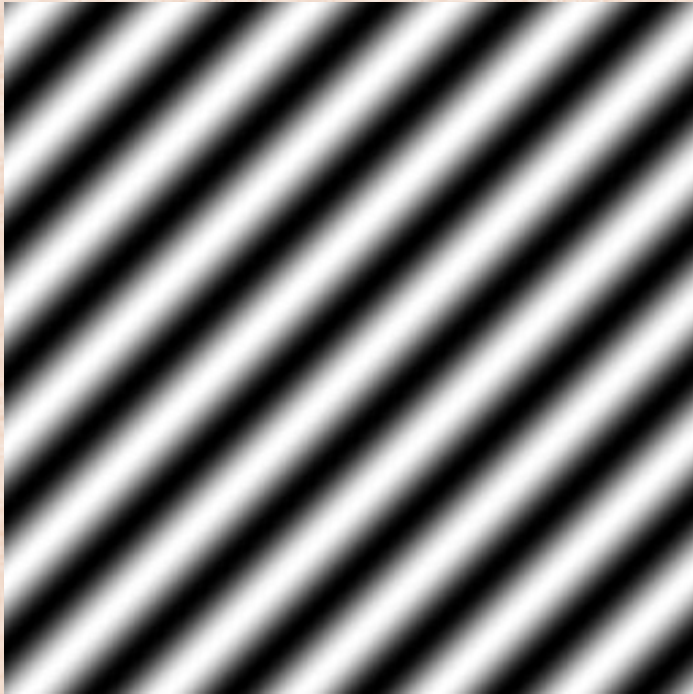
- Define layers of different colors
- Use a function to map layer colors to (u, v) texture locations
- For example:

$$\text{marbleColor}(u, v) = \text{LayerColor}(\sin(k_u u + k_v v))$$

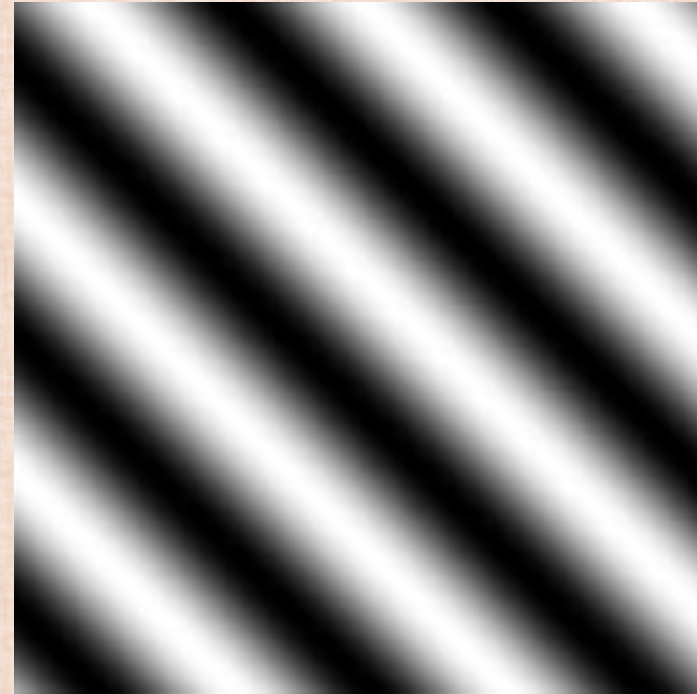


Marble Texture

- k_u and k_v are spatial frequencies
- (k_u, k_v) determines the direction, and $\frac{2\pi}{\sqrt{k_u^2 + k_v^2}}$ determines the periodicity
- Problem: too regular (still need to add noise/randomness)



higher frequency



lower frequency

Perlin Noise

- Noise should have both coherency and structure, in order to look more natural
- Ken Perlin proposed a specific (and amazing!) method for doing this

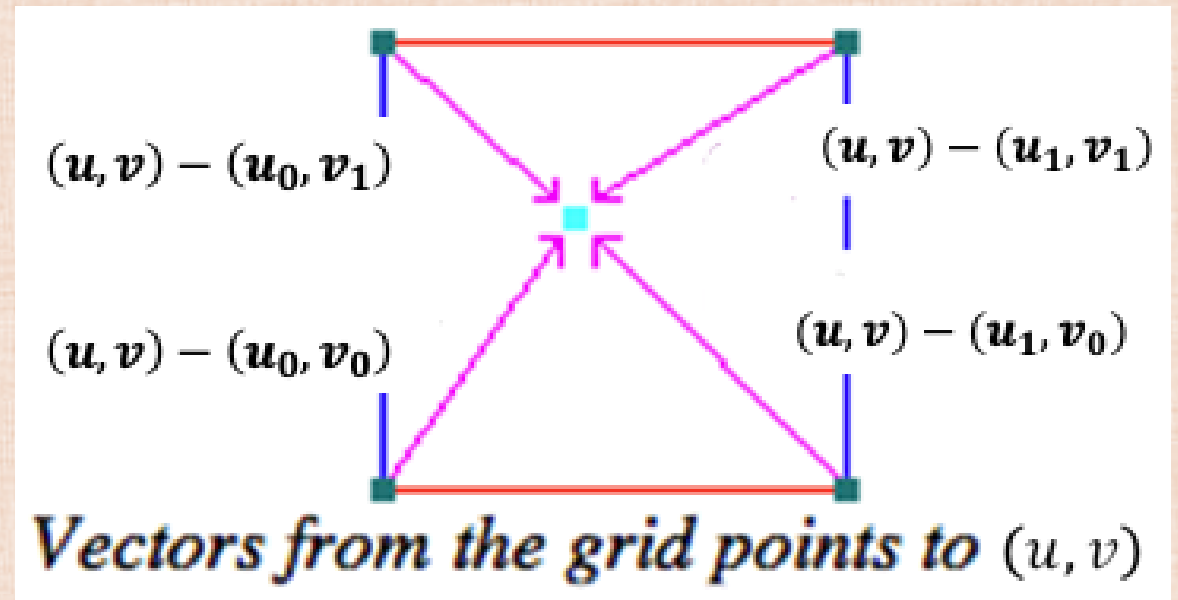
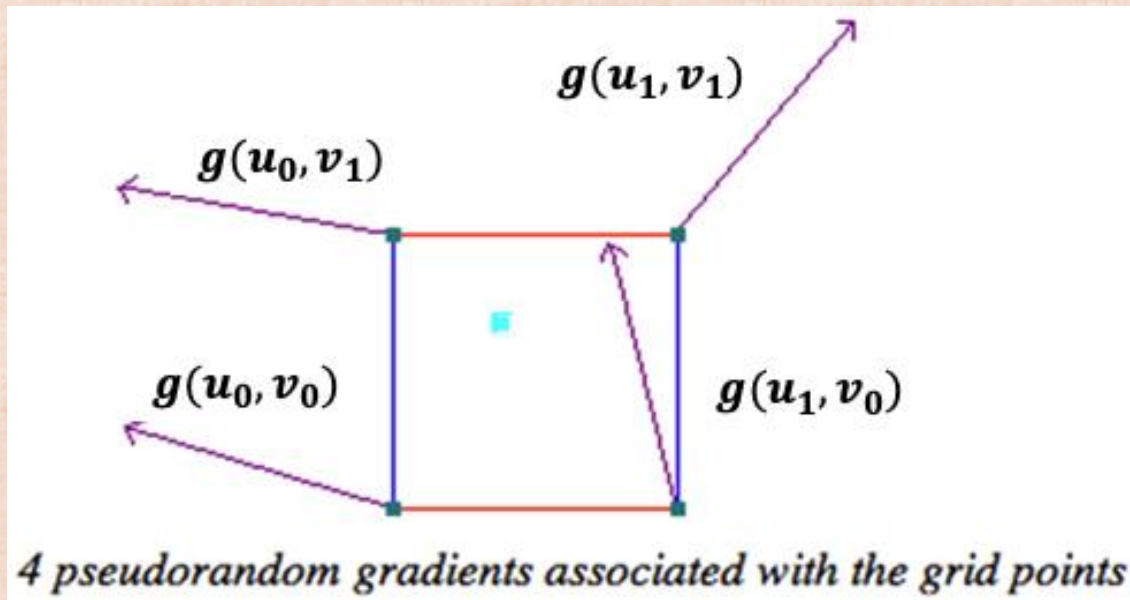


Perlin Noise

- Place a 2D grid over the texture image, and assign a random (unit) gradient $g(u_i, v_j)$ to each grid point
- For each pixel, compute the dot-products between vectors from the grid corners and the corresponding gradients
- Take a weighted average of the result:

$$\text{noise}(u, v) = \sum_{i=0,1; j=0,1} w\left(\frac{u - u_i}{\Delta u}\right) w\left(\frac{v - v_j}{\Delta v}\right) g(u_i, v_j) \cdot (u - u_i, v - v_j)$$

- Cubic weighting (for smoothness): $w(t) = 2|t|^3 - 3|t|^2 + 1$ for $t \in (-1, 1)$



Multiple Scales

- Natural textures tend to contain a variety of feature sizes
- Mimic this by adding together noises with different frequencies and amplitudes:

$$\textit{perlin}(u, v) = \sum_k \textit{noise}(\textit{frequency}(k) * (u, v)) * \textit{amplitude}(k)$$

- Each successive noise function is twice the frequency of the previous one:

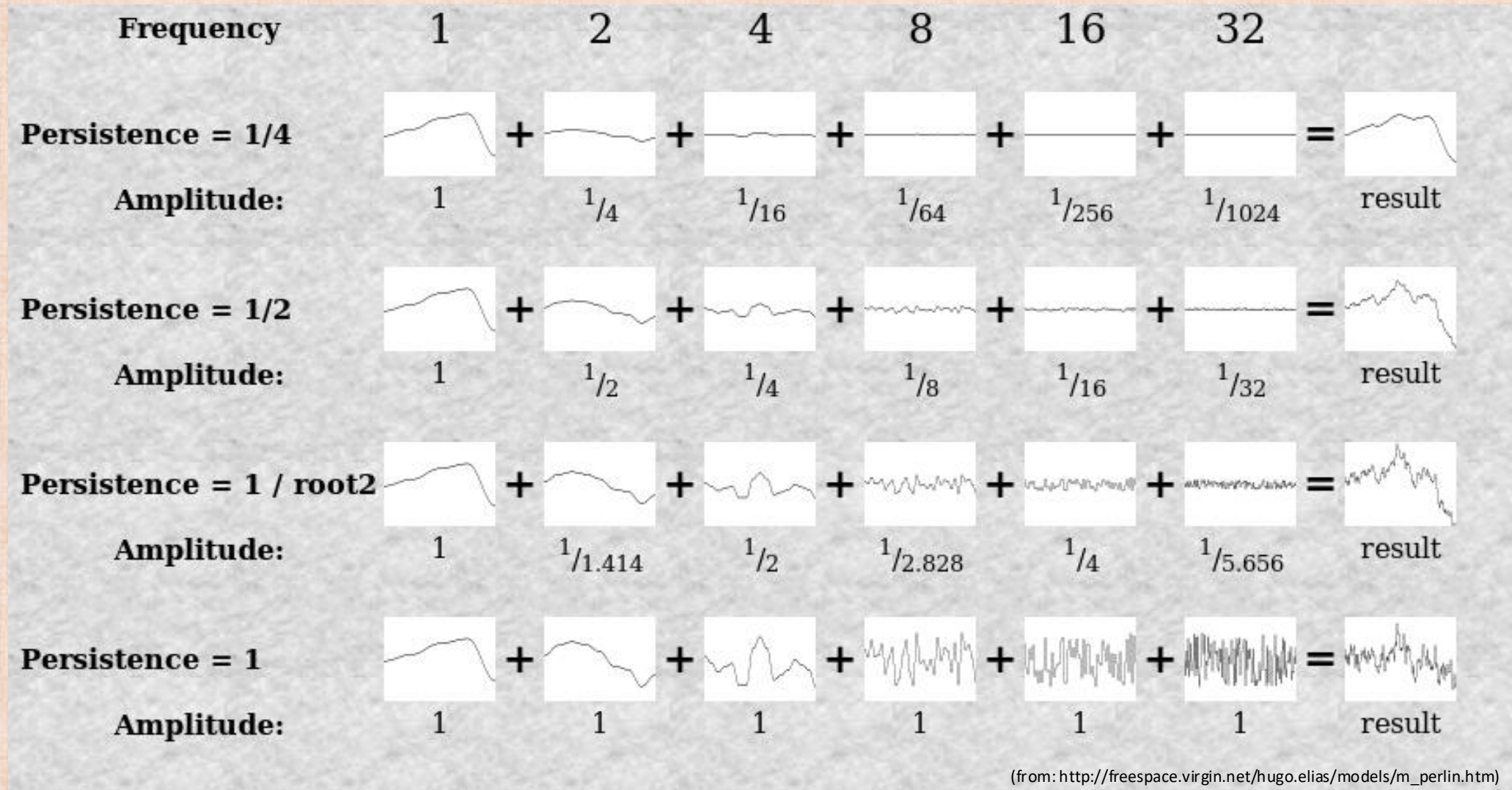
$$\textit{frequency}(k) = 2^k$$

- The amplitude of higher frequencies is measured by a persistence parameter (≤ 1)
- Higher frequencies have a diminished contribution:

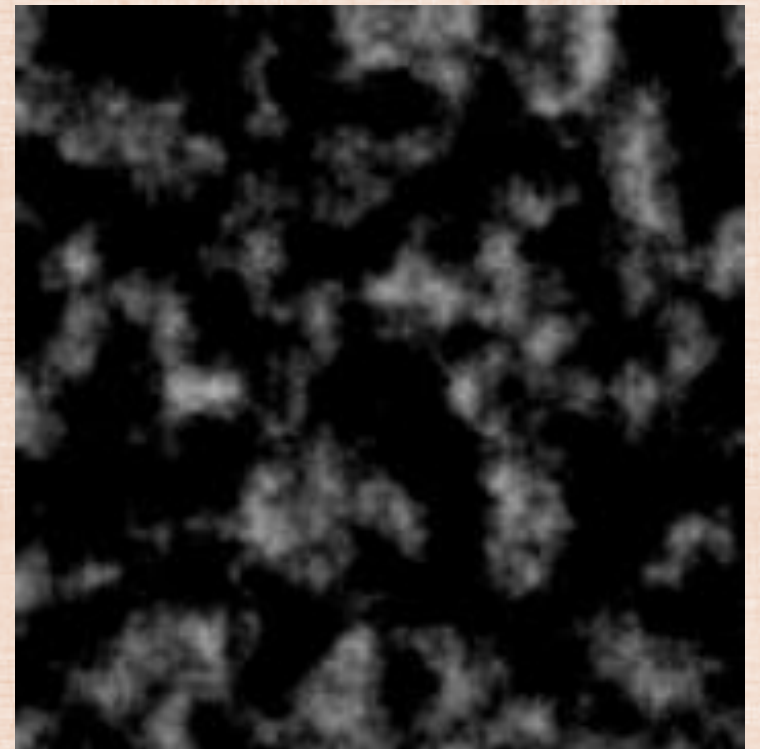
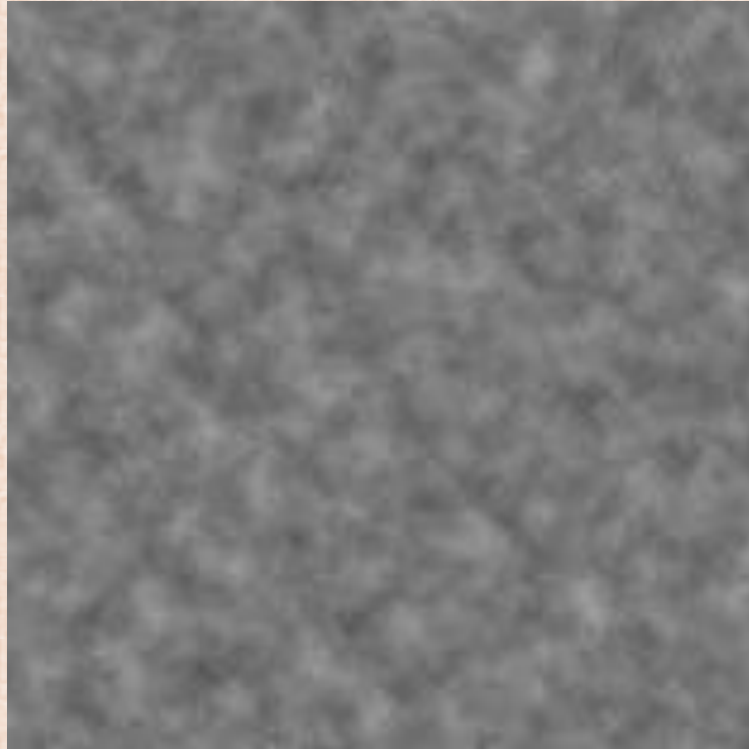
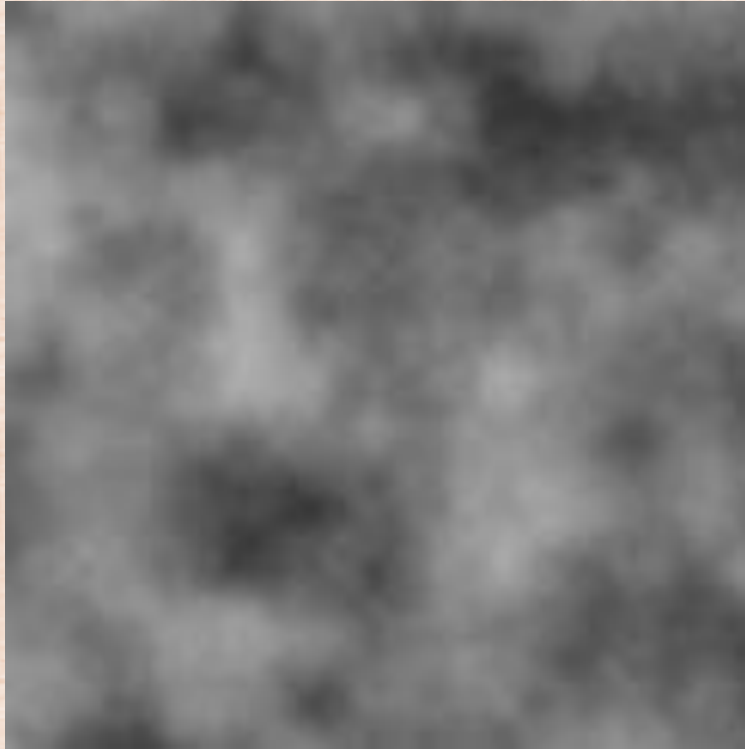
$$\textit{amplitude}(k) = \textit{persistence}^k$$

1D Examples

- Smaller persistence gives less higher frequency noise and thus a smoother result



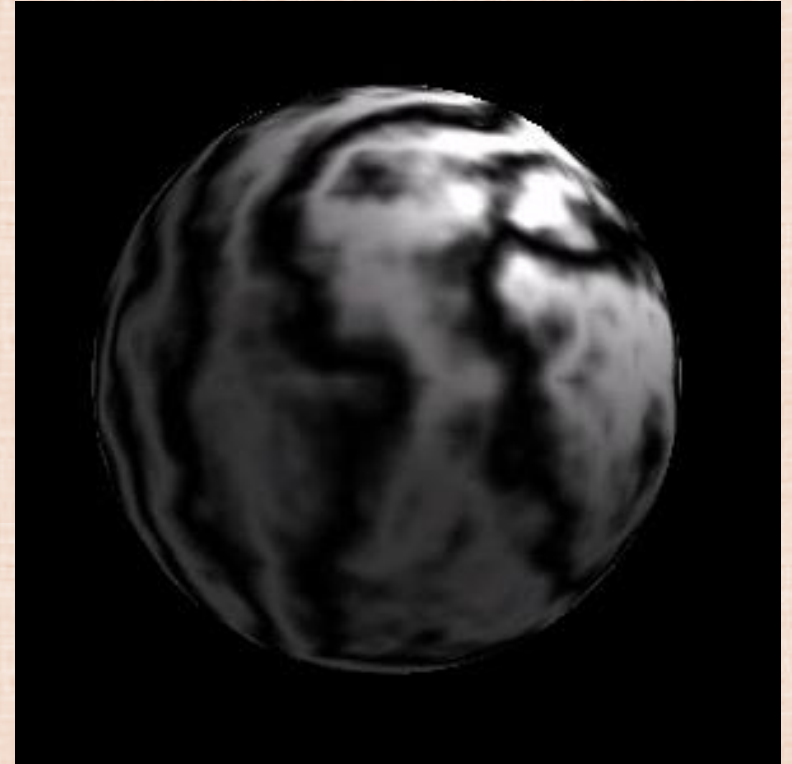
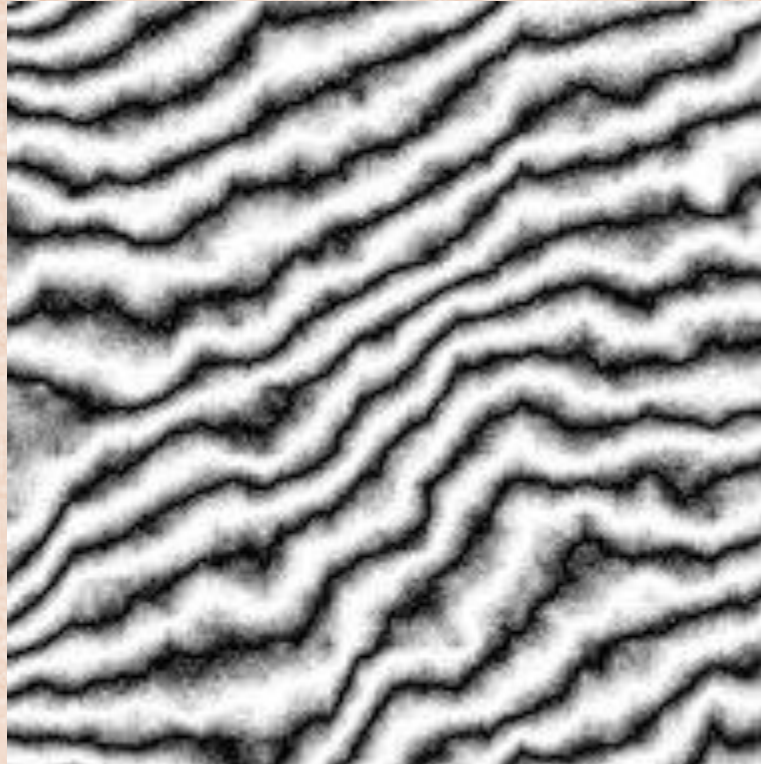
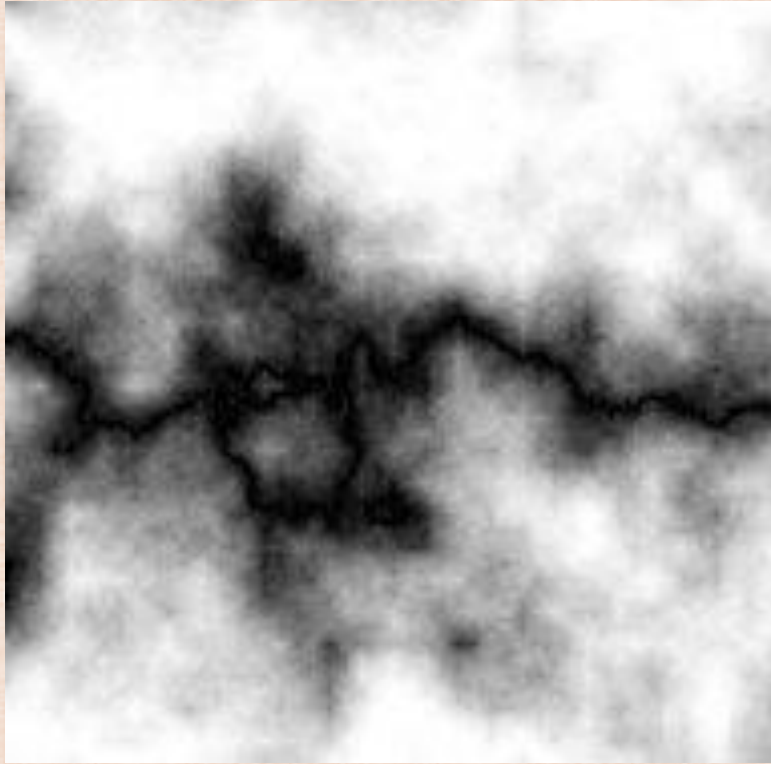
2D Examples



Marble Texture + Perlin Noise

- Set the value of A to scale the amount of noise:

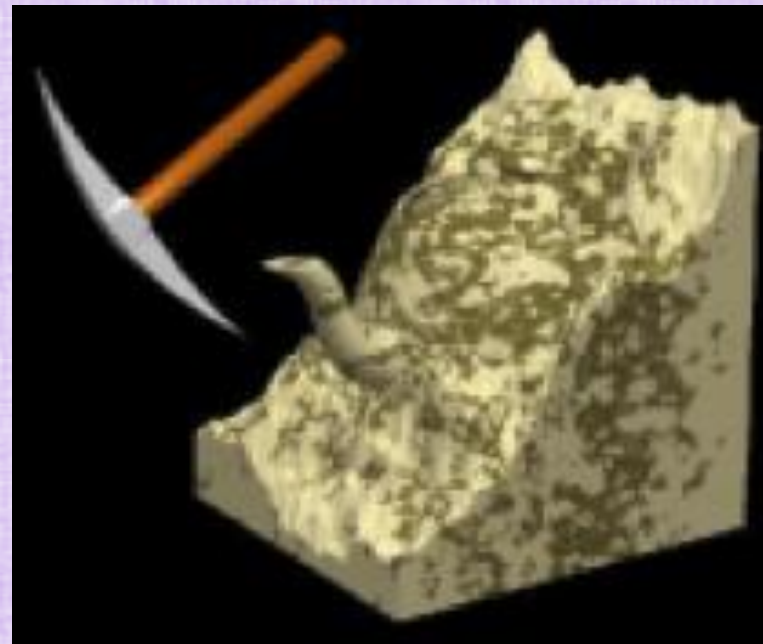
$$\text{marbleColor}(u, v) = \text{LayerColor}\left(\sin(k_u u + k_v v + A * \text{perlin}(u, v))\right)$$



3D Marble Texture

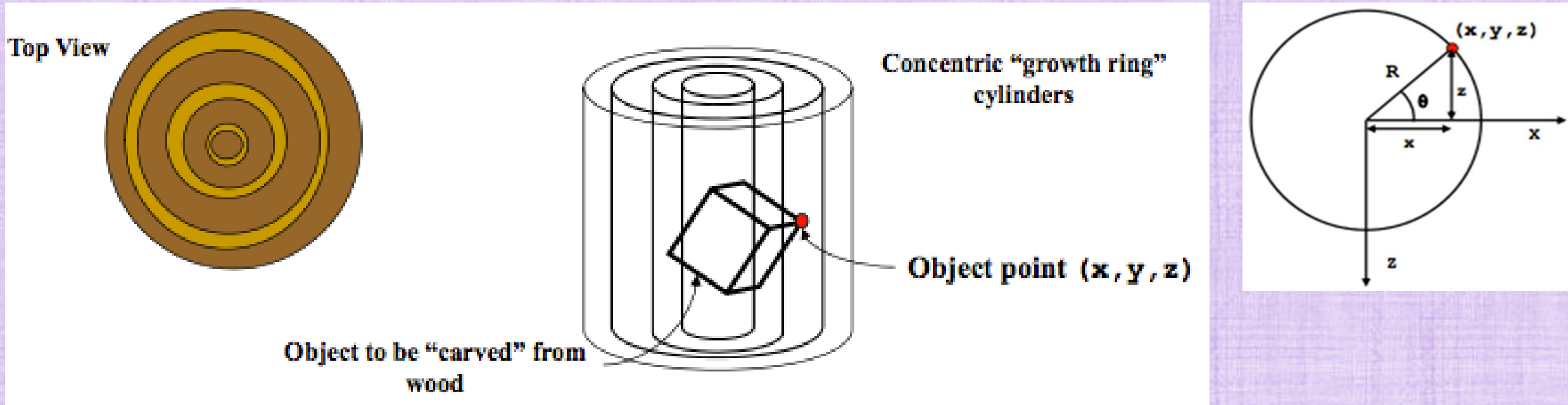
- “Carve” an object out of a 3D texture
- Marble texture function w/Perlin noise (for 3D):

$$\text{marbleColor}(u, v, w) = \text{LayerColor} \left(\sin(k_u u + k_v v + k_w w + A * \text{perlin}(u, v, w)) \right)$$

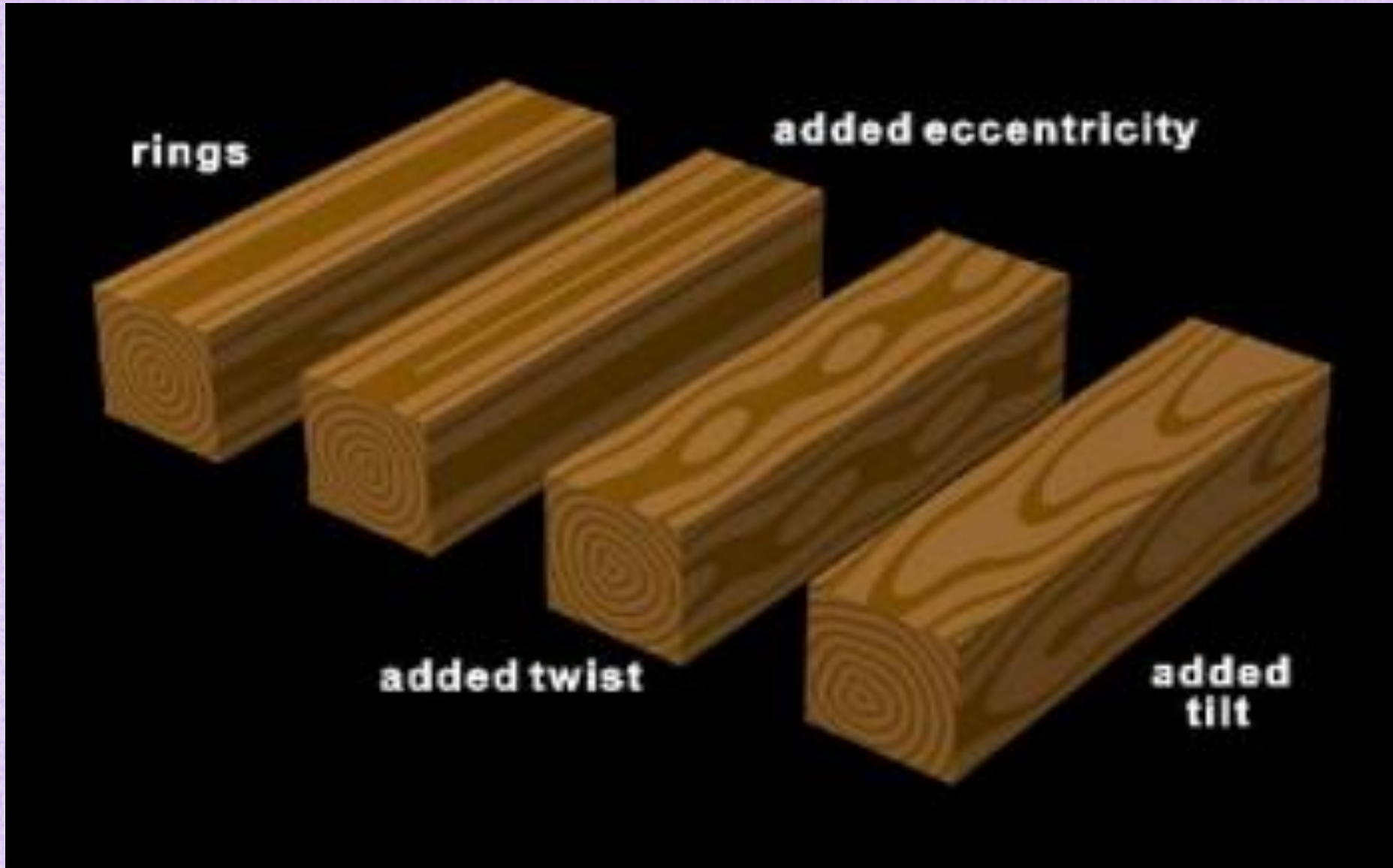


3D Wood Texture

- Procedurally generate tree rings (and cut the object out of the 3D texture)
- Cylindrical coordinates for (x, y, z) object points: $H = y, R = \sqrt{x^2 + z^2}, \theta = \tan^{-1} \left(\frac{z}{x} \right)$



3D Wood Texture



Machine Learning: Deep Fakes



Machine Learning: Text to Video



elon musk in a space suit

3d animation

