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Shubnikov–de Haas oscillations in a two-dimensional electron gas in a spatially random magnetic field

F. B. Mancoff,* L. J. Zielinski, and C. M. Marcus
Department of Physics, Stanford University, Stanford, California 94305-4060

K. Campman and A. C. Gossard
Materials Department, University of California at Santa Barbara, Santa Barbara, California 93106
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We report measurements of transport in a two-dimensional electron gas in a spatially random magnetic field in which the average magnetic field extends from the classical regime $\langle \omega_c \rangle \tau < 1$ into the quantum Hall regime. Experiments make use of a rough Nd-Fe-B permanent magnet on the surface of a GaAs heterostructure. Effective mass, transport and total scattering times, and g-factor-enhancement values (all measured from Shubnikov–de Haas oscillations) are comparable to those found for potential scattering in a uniform magnetic field.

Magnetotransport in a two-dimensional (2D) electron system with a spatially random magnetic field has generated recent interest for a number of reasons. First, several recent experimental realizations using high-mobility heterostructure materials and overlayers of superconductors or ferromagnets[1–4] have yielded interesting classical transport results. Second, a theoretical debate has arisen concerning the existence of extended states analogous to quantum Hall edge states in a two-dimensional electron gas (2DEG) in a spatially random field with zero average ($\langle B \rangle = 0$, $\langle B^2 \rangle \neq 0$).[5] Third, there are indirect connections between the random-field problem and high-T_c superconductivity.[6] Finally, and perhaps most importantly, there is a fascinating connection via a Chern-Simons gauge transformation between electrons in a random magnetic field around $\langle B \rangle = 0$ and composite fermions (CF's)—electrons coupled to an even number of magnetic-flux quanta—near even-denominator fractional quantum Hall states, particularly Landau-level filling factor $\nu = \frac{1}{2}$.[7,8]

In the CF picture, the vicinity around $\nu = \frac{1}{2}$ for electrons can be mapped (via the "law of corresponding states"[9]) to the region near zero effective field for CF's. The principle sequence of odd-denominator fractional quantum Hall states at $\nu = p/(2p \pm 1)$ for integer p corresponds to the integer quantum Hall states $\nu = p$ of CF's.[10–15] Fractional states at larger values of p, which lie close to $\nu = \frac{1}{2}$, correspond to (and resemble experimentally) familiar Shubnikov–de Haas (SdH) oscillations around $B = 0$. Indeed, analysis of the magnetoresistance oscillations near $\nu = \frac{1}{2}$ in terms of the standard Ando formula for SdH oscillations[16,17] has been used to extract the CF effective mass and total scattering time.

An important ingredient of the Chern-Simons transformation is that electron density fluctuations due to static potential disorder as well as due to phonons lead to fluctuations in the effective magnetic field for CF's.[8] As a result, the CF's see not only a uniform field B^* equal to the deviation from half filling, $B^* = B - B_{1/2}$, but also a fluctuating field that is correlated with both static and dynamic density fluctuations. This fact connects transport of CF's to the problem of electron transport in a random magnetic field. While the relatively simple problem of 2D transport in a static random field is a subset of the issues raised by the CF picture, surprisingly, even this problem has been addressed only recently. In this paper, we extend the experimental investigation of 2D electron transport in a random B field into the SdH regime, allowing a direct comparison to recent work equating fractional quantum Hall states to SdH oscillations of CF's.[10–15]

Experimental study of 2D transport in a random magnetic field in the SdH regime using a superconducting film or grains on a heterostructure surface is problematic because a large applied B field either drives the superconductor normal[2] or causes the high density of overlapping vortices to wash out the fluctuating field, $\delta B \equiv (\langle B^2 \rangle - \langle B \rangle^2)^{1/2}$.[3] As an alternative approach, we use a neodymium-iron-boron (Nd-Fe-B) permanent magnet attached to the surface of a high-mobility GaAs/Al_xGa_{1-x}As heterostructure [Fig. 1(a), inset].[4] An optical micrograph of the Nd-Fe-B material [Fig. 1(b), inset] shows a large surface roughness with a characteristic length scale $\xi_B \sim 20$ μm. The sintered Nd-Fe-B is fixed using polymethyl methacrylate (PMMA) to the surface of a 200 μm by 3 mm Hall bar[4] ($n = 3 \times 10^{15}$ m^{-2}, $\mu \sim 100$ m^2/V s, transport mean free path $l \sim 9$ μm) with the easy axis of magnetization perpendicular to the surface. The samples are cooled in a ^{3}He refrigerator or ^{3}He/^{4}He dilution refrigerator. Simultaneous longitudinal and Hall resistance measurements both on the end of the Hall bar under the Nd-Fe-B magnet ("covered") and on the end of the Hall bar without the magnet ("uncovered") [see Fig. 1(a), inset] were made using standard ac lock-in techniques at 317 Hz with a current bias of 0.5 μA.

Prior to the transport measurements, the Nd-Fe-B is permanently magnetized by ramping an external, perpendicular magnetic field B_{ext} to several tesla. Because of the surface roughness, the remnant magnetization of the Nd-Fe-B produces a spatially disordered field in the 2DEG plane that is essentially insensitive to B_{ext} on the return towards $B_{ext} \sim 0$, during which magnetoresistance data are taken. The magnitude of the random magnetic-field fluctuations is $\delta B \sim 0.1$ T, as determined by the low-field magnetoresistance and by

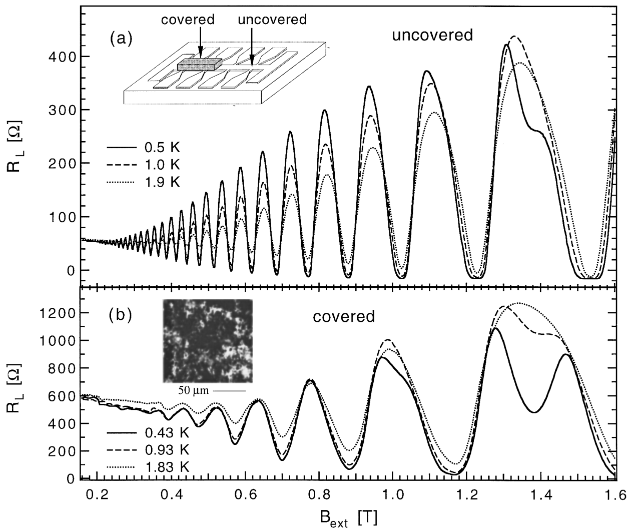

FIG. 1. Longitudinal resistance R_L versus applied magnetic field B_{ext} shows Shubnikov–de Haas oscillations in (a) uniform magnetic field and (b) random magnetic field. Solid, dashed, and dotted curves give R_L for three temperatures in each case. (a) Inset shows schematic side view of the Nd-Fe-B random magnet covering half of a GaAs Hall bar. (b) Inset shows optical micrograph of Nd-Fe-B surface roughness on a ~ 20 μm length scale.

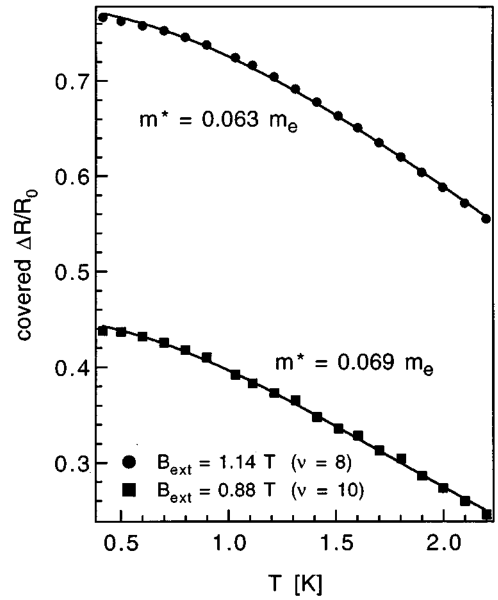

FIG. 2. Normalized Shubnikov–de Haas oscillation amplitude $\Delta R/R_0$ versus T for resistance minima at $\nu=8$ (circles) and $\nu=10$ (squares) in the random magnetic field. Solid lines are fits to T dependence of Eq. (1) and yield effective mass for electrons in a random magnetic field that is consistent with the usual value for electrons in GaAs.

modeling the Nd-Fe-B surface in terms of displaced spherical dipoles.[4] The magnitude δB is significant compared to the average field at which the SdH oscillations appear, so that the classical orbits are significantly affected by δB. The frozen-in remnant field from the Nd-Fe-B also has a nonzero average, $\langle B_{\text{Nd-Fe-B}}\rangle \sim 0.25$ T, which can be measured directly from the Hall resistance [i.e., $R_H(-\langle B_{\text{Nd-Fe-B}}\rangle)=0$] and accounted for in the external field ($B_{tot}=B_{ext}+\langle B_{\text{Nd-Fe-B}}\rangle$).

Figure 1 shows a typical longitudinal resistance ($R_L = V_L/I_{bias}$) versus B_{ext} for a representative temperature range both underneath the Nd-Fe-B [Fig. 1(b)] and in the uncovered half [Fig. 1(a)]. The Hall slopes indicate that the electron sheet density is unaffected by the presence of the Nd-Fe-B,[4] but the mobility is greatly reduced under the Nd-Fe-B, changing from $\mu_{\text{uncovered}} \sim 100$ m^2/V s to $\mu_{\text{covered}} \sim 10$ m^2/V s. We have confirmed that the decrease is not due to surface damage of the GaAs by removing the Nd-Fe-B and observing that the two ends of the Hall bar behave identically again. The reduced mobility can be understood in terms of a classical model of a nonuniform resistivity tensor in the plane of the 2DEG.[4] The envelope function of the oscillations in R_L was found to be well described by the conventional Ando formula:[16,17]

$$\frac{\Delta R}{R_0} = \frac{A_T}{\sinh(A_T)}\, 4e^{-\pi/\omega_c \tau_{tot}}, \qquad (1)$$

where $A_T = 2\pi^2 kT/\hbar\omega_c$ accounts for thermal smearing at temperature T, $\omega_c = eB_{tot}/m^*$ is the cyclotron frequency (m^* is the electron effective mass), and τ_{tot} is the total scattering time. R_0 represents the classical resistance in zero applied magnetic field, with ΔR the difference between R_0 and R_L at the resistance oscillation extrema. Since R_0 varies slightly with B_{ext}, we determine $R_0(B)$ at each extremum by averaging the value of R_L at that extremum with a linear interpolation in B of R_L at the two neighboring opposite extrema.[15]

Figure 2 shows a plot of $\Delta R/R_0$ versus T in the covered case for two different Shubnikov–de Haas minima, at $B_{ext}=0.88$ T ($\nu=10$) and 1.14 T ($\nu=8$). Fits to Eq. (1) give $m^* \sim 0.06 m_e$ in the covered region (m_e is the bare electron mass), as well as $\sim 0.06 m_e$ in the uncovered region, consistent with the standard value $0.067 m_e$ for GaAs. Apparently, a static random magnetic field does not affect m^* values. This result is of interest since in experiments around $\nu=\frac{1}{2}$, the use of Eq. (1) to determine the CF effective mass gives an enhanced value,[11–15] which is observed to increase[12,14] or decrease[13] upon approaching $\nu=\frac{1}{2}$. While perhaps not surprising, this result suggests that the enhancement of m^* as well as its dependence on ν is not a feature of fermions in a random B field and presumably is due to interactions specific to CF's.

From Eq. (1), the slope of a Dingle plot,[17] i.e., $\ln[\Delta R/4R_0 \times \sinh(A_T)/A_T]$ versus $1/B_{tot}$, for the Shubnikov–de Haas envelope is $\sim -\pi m^*/e\tau_{tot}$. From this slope, we extract the total scattering time τ_{tot}, assuming a constant $m^* = 0.067 m_e$. Typical plots (Fig. 3) for the R_L oscillation amplitudes, measured in a single magnetic-field sweep at 30 mK for both the covered and uncovered cases, are well described by this linear form at lower magnetic fields. We attribute the deviation from linearity at $\omega_c\tau_{tot} \gtrsim 1$ to the formation of quantum Hall states. The linearity of $\ln(\Delta R)$ versus $1/B_{tot}$ suggests that the usual Ando expression with $\Delta R \propto e^{-\pi/\omega_c\tau_{tot}}$ accurately characterizes the SdH oscillations in a strong static random field. This result is in conflict with recent theoretical treatments, which give $\Delta R \propto e^{(-\pi/\omega_c\tau_{tot})^4}$ when the magnetic-field fluctuation correlation

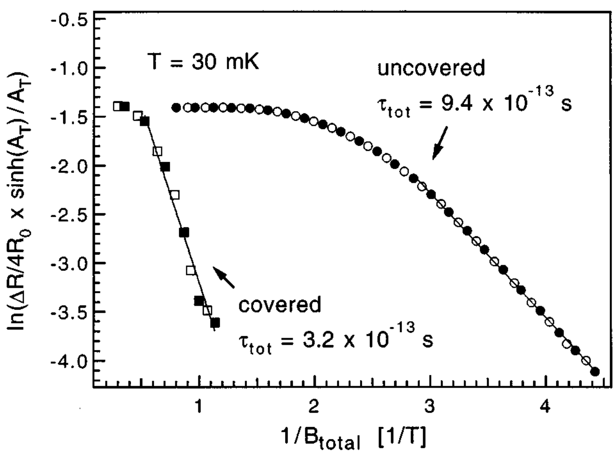

FIG. 3. Dingle plot of magnetoresistance oscillations for a 2DEG at 30 mK in uniform magnetic field (circles) and random magnetic field (squares). Solid curves are fits of Eq. (1), with total scattering time $\tau_{\rm tot}$ determined by the slope. Solid (open) symbols are resistance minima (maxima). $B_{\rm tot}=B_{\rm ext}$ for uncovered region and $B_{\rm tot}=B_{\rm ext}+\sim 0.25$ T for covered region since $\langle B_{\rm Nd\text{-}Fe\text{-}B}\rangle \sim 0.25$ T.

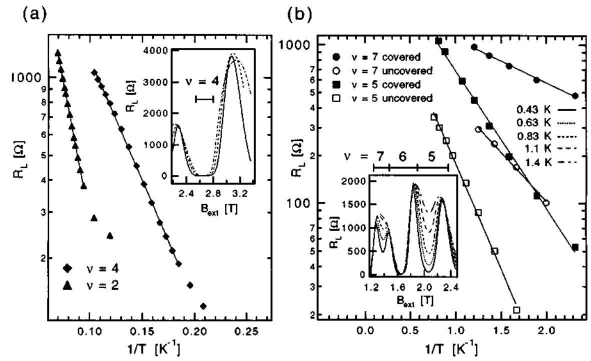

FIG. 4. (a) Longitudinal resistance R_L versus $1/T$ in the random magnetic field for Landau split minima at ν=4 (diamonds) and ν=2 (triangles). Slopes of solid line fits to Eq. (2) yield an additional measurement of $m^*\sim 0.09m_e$ for electrons in a random magnetic field. Inset: Temperature dependence of R_L versus $B_{\rm ext}$ in the random magnetic field around ν=4. (b) R_L versus $1/T$ in the random field for spin-split minima at ν=7 (solid circles) and ν=5 (solid squares), and at corresponding ν in uniform field (open symbols). Slopes of solid line fits to Eq. (2) determine g^*. Inset: Temperature dependence of R_L versus $B_{\rm ext}$ in the random magnetic field around ν=7 and ν=5.

length ξ_B is much greater than the cyclotron radius at $B_{\rm tot}$ ($\xi_B \gg R_c$) (Refs. 14 and 18) and $\Delta R \propto e^{(-\pi/\omega_c \tau_{\rm tot})^2}$ for a short correlation length ($\xi_B \ll R_c$).[18] The $1/B^2$ (or $1/B^4$) dependence of the Dingle plot for short (long) -range correlated random field assumes that the random field is sufficiently weak that circular orbits persist, and that the damping of the SdH oscillations at low field is due to differences in the phase accumulated within each circle.[18] In the present experiment, however, the random-field fluctuations are quite strong ($\delta B\sim 0.1$ T)—sufficient to reduce μ by a factor of 10—which leads to considerable disruption of orbits.

The ratio between the transport scattering time $\tau_{\rm tr} = m^*\mu/e$ and the total scattering time $\tau_{\rm tot}$ describes the degree of large-angle versus small-angle scattering in the 2DEG, with smaller $\tau_{\rm tr}/\tau_{\rm tot}$ indicating a greater proportion of large-angle scattering.[17] For the covered case, typically $\mu\sim 10$ m^2/V s and the ratio $\tau_{\rm tr}/\tau_{\rm tot}\sim 11$, while for the uncovered case, $\mu\sim 100$ m^2/V s and the ratio $\tau_{\rm tr}/\tau_{\rm tot}\sim 25$. As a possible mechanism for enhanced large-angle scattering in the random magnetic field, we suggest abrupt changes in the cyclotron radius for classical electron orbits crossing between regions of differing local magnetic field. However, a comparable trend towards lower ratios for lower μ was also reported by Coleridge and co-workers[17] for regular potential scattering in a variety of GaAs/Al$_x$Ga$_{1-x}$As 2DEG samples. Thus, the reduced ratio $\tau_{\rm tr}/\tau_{\rm tot}$ in the covered region indicates that scattering due to the random field is strong, with proportionately more large-angle events than in the high-mobility uncovered region.

In an effort to controllably add potential scattering to the 2DEG to allow a comparison of potential versus magnetic scattering *at the same* μ, we fabricated a nominally identical Hall bar from the same wafer with a metallic depletion gate over the uncovered half. Applying a negative voltage to the gate reduces the carrier density in the uncovered half ($n = 3.9\times 10^{15}$ m^{-2} at $V_g=0$ compared to 1.1×10^{15} m^{-2} at $V_g=-0.58$ V). As a result, the mobility in the uncovered half is reduced to 12.7 m^2/V s comparable to that in the covered half, and the scattering time ratio is reduced to $\tau_{\rm tr}/\tau_{\rm tot}\sim 5.5$. However, at less negative gate voltages, such that $\mu\sim 80$ m^2/V s on the uncovered half, the ratio becomes extremely large, $\tau_{\rm tr}/\tau_{\rm tot}\sim 60$, indicating that the gate alters the impurity screening in the 2DEG or changes the occupancy of surface states. This difference complicates any quantitative comparison between gated samples and samples with random magnetic-field scattering.

At higher fields where integer quantum Hall states are well defined, we can measure both m^* and the g factor for electrons from the activated transport at longitudinal resistance minima between both spin-split levels at odd ν [Fig. 4(b)] and Landau split minima at even ν [Fig. 4(a)]. The temperature dependence of R_L in the covered half versus $B_{\rm ext}$ is shown around ν=4 [Fig. 4(a), inset] and at ν=7 and ν=5 [Fig. 4(b), inset]. For a uniform magnetic field, one expects an activated longitudinal resistance R_L:

$$R_L \propto \exp(-\Delta/2kT), \qquad (2)$$

where $\Delta=\hbar\omega_c=\hbar e B_{\rm tot}/m^*$ for Landau split even-ν minima (neglecting the spin-splitting energy) and $\Delta=g^*\mu_B B_{\rm tot}$ for spin-split odd-ν minima.[19] Plots of R_L versus $1/T$ [Figs. 4(a) and 4(b)] for the quantum Hall minima at ν=4 and ν=2 and at ν=7 and ν=5 exhibit the activated form, Eq. (2), before saturating at lower T. For the minima at even ν [Fig. 4(a)], the slope of a linear fit of $\ln(R_L)$ versus $1/T$ gives $m^* \sim 0.09m_e$ for electrons in a random magnetic field, roughly consistent with our measurement of m^* using Eq. (1) to fit the Shubnikov–de Haas oscillations. For the minima at odd ν [Fig. 4(b)], a fit to the activated form, Eq. (2), gives $g^* = \Delta/\mu_B B_{\rm tot}\sim 0.9\pm 0.3$ at ν=7 and $g^*\sim 2.1\pm 0.5$ at ν=5, each roughly a factor of 2 less than g^* measured at corresponding values of ν in the uncovered region [Fig. 4(b)]. A possible explanation for the reduced g^* in the random field is that the population difference between spin-up and spin-down when the Fermi level lies between spin-split Landau levels (which leads to strong exchange enhancement of g^* in a uniform field) is reduced by the nonuniform B.[19]

In summary, we have created a strong static random magnetic field ($\delta B \sim 0.1$ T) that can coexist with a controllable background magnetic field. Using this method, we have analyzed Shubnikov–de Haas oscillations of a GaAs 2DEG in a random magnetic field and found good agreement with the Ando formula [Eq. (1)]. The effective mass m^* for electrons in a random magnetic field is unchanged from m^* in a uniform field. The ratio τ_{tr}/τ_{tot} of the transport to the total scattering time for electrons in our random magnetic-field system is roughly half that of a 2DEG in a uniform field, indicating enhanced large angle scattering off of the random field. The exponential temperature dependence of the quantum Hall minima provides an additional and consistent measure of m^* as well as indicating a reduction of g^* by a factor of ~ 2 in the random field. Inconsistencies between our observations and recent results for magnetoresistance oscillations around $\nu = 1/2$, particularly in the effective mass and the shape of the SdH envelope, suggest that the physics around $\nu = \frac{1}{2}$ may differ substantially from simple 2D transport in a strong static random magnetic field.

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*Present address: Department of Materials Science and Engineering, Stanford University, Stanford, California 94305-2205.

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