MOOD, ASSOCIATIVE MEMORY, AND THE FORMATION AND DYNAMICS OF BELIEF

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Abstract. We provide a model of the effect of associative memory and mnemonic cues on agent beliefs and choices in generic stochastic decision problems. Our model is specialized to the study of affective state as a cue for information congruent with the affect of the agent, a phenomenon called mood congruent memory. We apply our model to demonstrate that the effect of employee morale on effort choice depends on the complementarity between morale and agent beliefs regarding marginal productivity of effort. We employ our model of decision making with mood-regulated associative memory to study financial asset pricing in a dynamic context, which allows us to provide novel explanations for phenomena such as excess volatility, short-run underreaction and long-run overreaction to news, and the influence of non-fundamental events.
"The existence of forgetting has never been proved: We only know that some things don’t come to mind when we want them." - Nietzsche

1. Introduction

Mood regulated memory is a psychological phenomenon wherein agent affective states serve as cues for information stored in a long-term associative memory system (Isen [35]). We will be focusing on the valence of the affective state\(^1\) associated with moods and emotions as a cue for information of the same valence. Appraisal theories of emotion predict that the valence of the emotion elicited by and associated with a piece of information, situation, or a cognitive process is determined by the implications for the individual’s well-being (Smith et al. [64]). This suggests that positive affective states of an agent are correlated with the satiation of an agent’s preferences, which makes affective valence a natural point of connection between the psychology of emotion and economic theory. Mood-regulated memory causes positive (negative) affective states of a decision maker to cue the recollection of positive (negative) information from memory, and the beliefs founded on the recalled data will be optimistically (pessimistically) biased. We provide a model of the effect of emotion on memory in order to generate conclusions about the influence of sentiment, which we interpret as experienced or anticipated utility, on belief formation and dynamics.

We model the effects of mnemonic cues, such as agent affective state, within a generic decision problem under uncertainty. Prior to the time of decision, the agent observes a history of informative signals and stores this complete history in long-term memory. At the time of decision, the agent recollects a sample of data from memory in order to update his prior beliefs and form a posterior. Given the posterior beliefs, the agent makes an optimal decision given his preferences and the feasible set he faces. Whereas in a traditional model of decision making the entire set of previously observed information is recalled and employed to form a posterior belief, in our model the information is recalled incompletely and the set of data recollected is biased by mnemonic cues. The mathematical formulation of the biased, incomplete recall is

\(^1\)The valence of an affective state refers to the subjectively experienced goodness of the state. For example, joy is a positively valanced affect, while anger and sadness are negatively valanced affective states.
inspired by robust stylized facts from the large body of research on the psychology of emotion and memory.

The benefits of our approach are twofold. First, because we use an abstract static decision problem as our benchmark model, the behavioral model and the comparative statics we derive can be incorporated into a variety of economic theories. Second, since our model is grounded in psychological primitives, we can employ the wealth of data on the determinants of affect and the sources of idiosyncratic sentiment to study how affective state influences the formation and dynamics of beliefs, identify situations in which agents will be optimistic or pessimistic relative to an unbiased Bayesian benchmark, and predict how these effects evolve dynamically within a population of agents.

In order to study the dynamics of beliefs generated by associative memory and the influence of these dynamics on the time series behavior of asset prices, we embed our static theory of associative memory in a model of intertemporal cue dynamics. We use agent affective state as the memory cue and assume a positively valenced affective state encourages the recall of positive information about future asset values from memory. Agent affective state is influenced by news in the present period with positive (negative) dividend shocks encouraging positive (negative) affective states. Since we assume the news is autocorrelated, good news that encourages a positive affective state this period implies that the news next period will also be good and further shift the distribution of affect in the market towards more positively valenced states. We prove that the affective dynamics cause changes in the asset traders’ judgements of net present asset value to be autocorrelated, which in turn induces autocorrelation in market prices. We use our model to provide novel explanations for the phenomena of asset price over- and underreaction to news, excess volatility, and price responses to irrelevant information.

The effect on asset prices has a unique time series signature. Suppose the agents observe a positive dividend announcement in the present period. Although agents incorporate the likelihood of positive news next period into their estimate of asset value by correctly assuming that the news is autocorrelated, the agents do not account for the fact that the high probability of good news next period will positively shift the affective states of the agents in the market. The repeated positive shocks
to affective state amplify the bias in the agents’ estimations of asset value and causes a short run upward trend in asset price, also known as underreaction of asset price to news. As these biased expectations are not met in the long run, affective states of the agents will converge over a sufficiently long time horizon towards the unbiased expectation of price, which appears as long-run overreaction to the news releases. Excess volatility and reaction to irrelevant news are the result of affective state or other cues inducing price movement without a corresponding cause rooted in the asset price fundamentals.

In a second application, we consider the debate in the organizational behavior literature between two schools of thought regarding the link between employee morale and productivity (see Staw and Barsade [65] for a recent study and literature review). The "Happier-and-Smarter" school of thought suggests that optimistic workers will prove more productive through increased effort choice and heightened creativity. The "Sadder-but-Wiser" hypothesis emphasizes that experimental subjects in a depressed mood tend to have more accurate beliefs regarding uncertain events than their happier peers. Ceteris paribus, more accurate beliefs lead to improved decision quality. If one interprets an employee’s morale as mood or affective disposition, our model of mood-regulated memory provides an analysis of the relationship between morale and productivity in an otherwise traditional principal agent model.

We focus on cases where an optimistic belief regarding the marginal productivity of effort is a complement or a substitute for agent affect and show that high morale can either enhance or reduce employee effort choices. Specifically, if the agent believes that a "good" state of the world has a high marginal productivity of effort, then a positive affective state generates an optimistic bias regarding marginal productivity of effort. High morale thus yields an increase in agent effort level. Conversely, if the "good" state of the world has a low marginal productivity of effort, exactly the opposite effect occurs and positive affective states reduce agent effort choices. By clarifying the interactions between morale, beliefs, and effort choice, our model sheds light on the conflicting empirical work in this area and points to an important factor for future studies to assess.

Section 2 presents a brief review of the economics and psychology literature salient for this study. Section 3 describes the model of memory that serves as a foundation
for the behavioral model, while section 4 builds upon these foundations to generate comparative statics results for generic decision problems. Section 5 analyzes the issue of mood and morale and uses our structure to study asset pricing anomalies. Section 6 concludes. Lengthy proofs are relegated to the appendix. Additional appendices describes extensions to and reinterpretations of our model as well as welfare implications of our model.

2. Related Literature - Emotions and Memory in Economics

Affective state has been identified as a cause of a variety of changes in the cognitive processes that drive agent decisions. A concise summary of these effects is given in Clore et al. [15] and Isen [34]. The review below focuses on the economics literature studying the impact of emotion and memory on decision making. Elster [26] and Loewenstein and Lerner [48] provide qualitative overviews of the role of emotion in economic decision making.

2.1. Emotion. Emotions experienced at the time of decision making, known as visceral states, have a profound impact on an agent’s decision process. Loewenstein [47] models the effect of visceral factors through state dependent utility functions. Agents have been shown experimentally to exhibit a failure to understand how visceral factors change over time and how these changes can influence preferences, a phenomenon known as the intrapersonal empathy gap. A classic example, studied in Read et al. [58], is that when subjects are asked what they would like to eat as an after lunch snack in a week’s time, agents in a hungry visceral state choose fattier snacks than agents in a satiated state. The subjects are unable to fully shed their present feeling of hunger to imagine how they will feel when the snack is consumed in the future. This failure of introspection results in agents being unable to predict future preferences and the appearance of dynamic inconsistency when revealed preference approaches are naively applied. Loewenstein et al. [49] call this phenomenon projection bias.

Hermalin and Isen [30] present a model of preferences over affective states and provide examples ranging from a model of savings to the effect of affect on inter-agent cooperation. The authors demonstrate the existence of multiple equilibria in problems for which effort and affect are complementary, providing a motivation
for firms and other principals to break agents out of inefficient equilibrium states through transient utility enhancements. Although several interesting phenomenon can be explained through the use of preferences over affect, the goal of our study is to assess the effect of beliefs biased by affective state. To this end, we have chosen not to incorporate preferences over affect into the model.

2.2. Memory. Mullainathan [54] provides a model of long-term associative memory in order to explain deviations from the consumption paths predicted by the permanent income hypothesis. Mullainathan develops his model in the context of an agent using Bayesian updating techniques to form estimates of the behavior of a random walk. The agent is assumed to observe a series of events with both an informative and an uninformative component. The associative aspect of memory is captured by allowing past events to have increased probability of recall if either the informative or the uninformative portion of the currently observed event are close to the values of any prior event. Unfortunately, Mullainathan’s analysis techniques cannot be generalized to generic stochastic processes or economic models. The model we derive allows for a general choice of stochastic process, which will allow us to apply the model to a wide array of decision problems.

Several models of memory of limited volume have appeared in the literature. Wilson [68], Miller and Rozen [53], and Hellman and Cover [31] derive optimal memory processes and decision strategies in the context of a decision problem with an infinite repetition of informative signals, but where memory is of a fixed, finite length. Dow [23] discusses optimal memory schemes in the short-run context of a two period decision problem where the agent stores information from period to period in an optimal, but limited, fashion. Benabou and Tirole ([6], [7], [8], [9]) have utilized models of malleable, imperfect memory in the context of agents with self-control problems in order to explain the use of intrapersonal rules for behavior and numerous other phenomena regarding self-regulation. Gottlieb [29] uses a model of agents with malleable memory and preferences over their own attributes to explain anomalies in the literature of choice under risk.

Models of limited memory volume could be interpreted as capturing the effects of short-term memory. Our model and Mullainathan’s work are models of a long-term memory system of infinite volume combined with an imperfect process for recalling
data from memory. For surveys of these topics, please see Cowan [18]. We take the view that long-term memory is the appropriate channel for examining the effects of memory on decisions due to the extremely short time scales on which short-term memory operates.

2.3. Emotion and Belief. Affective state has a clear and significant effect on histories recollected by agents in experimental settings (Isen [34]). Experimental subjects faced with a decision in a distinct affective state are prone to recollect histories that match their present affect. The effect of this bias in memory is that agents will skew the beliefs based on these recollected histories in a direction consistent with the present affective state.

Study of these phenomena was prompted by the associative network model of emotion and memory developed in Bower [12]. Nodes in the network (affective states, memories, cognition processes, etc.) are connected by edges representing association. Activation of one node stimulates the activation of nodes that are connected in the network. Affective states are hubs for the interconnection of memories and information with similar valence to the affective state. Therefore, emotions experienced at the time of decision will activate the corresponding node in the memory network, which in turn causes information of a similar valence to become active. This active information influences the belief formation process and causes biases in agent beliefs commensurate with the emotion experienced. This is known as mood congruent recall, a form of cued associative memory.

Kida et al. [38] provide a series of experiments involving subjects and tasks mimicking managerial activities that reveal that affective reactions to information are related to information in a straightforward fashion, encoded into memory, and have significant impact on the recall of the information at later times. The subjects, experienced managers, were required to remember and utilize numerical data, a task that was familiar from their job function. The Kida et al. experiments showed that when provided with a natural benchmark from which to establish the valence of

\footnote{The psychology literature strongly supports the notion that positive affect generates a mood-regulated recall process biased towards positive recollections, but the evidence regarding the effects of negative mood is more contentious (Isen [34]). The model we develop is agnostic to asymmetries between positive and negative emotions and their relative influence on memory.}
experimental data (ex: industry performance benchmarks), managers were markedly better able to recall the affective reactions to the data than the actual comparison to the benchmark. Further, the managers had difficulty recollecting data about a firm when the valence of the data did not conform to the affective impression the subjects had of the firm. For example, if a firm’s accounting data was typically below market benchmarks, subjects had difficulty recollecting that the firm had an exceptionally high cash flow relative to industry standards. In addition, subjects had greater difficulty identifying incorrect information as false when the valence of the incorrect information matched the subject’s overall affective impression of the firm. This suggests that trained managers, who are exposed to large volumes of numerical data on a regular basis, are subject to affective influences on their recollection processes.

2.4. A Canonical Experiment. Isen et al. [35] present an early study of the affective biases of memory recall processes. The authors employed a field study to demonstrate a large effect on consumer product evaluations through the use of low cost affect manipulations and a supplementary laboratory study to isolate the mood-regulated memory effect underlying the phenomenon. In the field study, shoppers in a suburban mall were assigned randomly to two treatments. In the first treatment, a confederate of the experimenter posing as a company representative distributed free samples of products valued at $0.29 in 1977 dollars to the subjects. The subjects in the control condition did not receive any gifts. The experimenter approached the subjects without knowing to which condition they had been assigned and administered a consumer satisfaction survey. The shoppers were asked to rate on a numeric scale the performance of a variety of consumer goods unrelated to the sample products offered by the confederate. The participants in the gift treatment registered significantly higher product satisfaction ratings than those in the control group.

The laboratory experiment involved inducing an affective state in the participants, requiring the subjects to memorize a list of words, inducing a (possibly different) affective state in the subjects, and then determining the number and affective valence of the words that could be recollected in the new affective state. The affective state was created by having the subject play a computer game developed by the experimenter. Winning the computer game was presumed to induce a positive affect,
while losing the computer game induced a neutral or negative affective state. Winning and losing were randomly determined by the experimenter for each repetition of the game. After playing the computer game once, the participants were provided a tape recording containing 36 words. 18 of the words were traits with 6 words each of positive, negative, and neutral valence, while 18 additional non-trait words were added as a control. After playing the computer game a second time, the participants were given 5 minutes to recall as many words as possible. Consistent with mood congruent recall, recollection was improved most when the valence of the material memorized matched the valence of the affective state at recollection.

Two factors regarding the experimental method are of particular interest to economists. First, mood induction is rapid and inexpensive. In the first experiment, the participants had a significant positive mood induced with a gift of such low value that it is plausible that economic actors would frequently use such techniques to manipulate agents. Second, the field experiment studied a judgement consumers are required to construct frequently, predictions of future utility based on the recollection of prior judgements of product quality. This experiment proved both how easily consumer affective state can be manipulated and that the effects of this manipulation can influence judgements of economic interest.

3. General Model

In our benchmark model of emotion and memory, we model the effects of mnemonic cues, such as agent affective state, within a generic decision problem under uncertainty. Prior to the time of decision, the agent is assumed to have observed a history of informative signals and stored this history in long-term memory. At the time of decision, the agent recollects a sample of data from memory in order to update his prior beliefs and form a posterior. Given his posterior beliefs, the agent makes an optimal decision given his preferences and the feasible set he faces. Our goal is to provide a formal model of these phenomena and provide monotone comparative statics for the influence of mood on memory.

3.1. Basic Model and Notation. The agent is assumed to experience and (potentially) recollect up to \( N \) previously observed informative signals \( \{\omega_i\}, i \in \{1, 2, \ldots, N\} \) that are stored in long-term memory. The signals \( \omega_i \) are members of a space \( \Omega \) that
can be represented as a finite Cartesian product of totally ordered subspaces. For example, we could assume $\omega_i \in \mathbb{R}^d$ for $d < \infty$. The agent will use these signals to conduct Bayesian updating of a prior belief regarding the distribution of a parameter $\theta_0 \in \Theta$ with associated cumulative distribution function (CDF) $G(\theta)$ where $\Theta$ is a partially ordered parameter space. Assume the signals are identically and independently distributed according to probability density function (PDF) $f(\omega|\theta)$. We will focus on the case where $\Theta \subseteq \mathbb{R}$ and $\Omega \subseteq \mathbb{R}$, although the results below generalize readily.

**Assumption 1.** $f(\omega|\theta)$ is log supermodular (log-spm) in $(\omega, \theta)$.

**Assumption 2.** $f(\omega|\theta)$ has full support over $\Omega$.

If $\theta \geq \theta'$, assumption 1 is equivalent to $\frac{f(\omega|\theta)}{f(\omega|\theta')}$ being increasing in $\omega$, which implies that higher values of $\omega$ represent evidence of higher values of $\theta$ regardless of any other information observed. Formally, this property is sufficient for the distribution of $\theta$ given $\omega$ to first order stochastic dominate the distribution of $\theta$ given $\omega'$ where $\omega > \omega'$. Therefore higher values of $\omega$ are referred to as "good news" (see Milgrom [50] for examples). Let the order relation $\succ$ refer to the strong stochastic order, which in a one-dimensional setting is equivalent to first order stochastic dominance. Where the strong stochastic ordering is strict, we use the symbol $\succ$.

We will assume that there is a state variable, denoted $\varphi \in \mathbb{R}$, that represents the memory cues facing the agent during the decision problem. The parameter is meant to capture the intensity of cues present at the time of decision as well as the strength of association between the cues and the relevant data stored in memory. In this study we use the convention that positive values of $\varphi$ are associated with positive values of $\omega$ stored in memory. In the context of emotion, $\varphi$ indexes the valence of the agent’s affective state with higher values reflecting more positively valenced moods. We do not model the source of these associations, but the psychology literature has shown that cues become associated with data that have similar features such as valence through conscious processes of association as well as unconscious channels (Smith et al. [64]).

Consider a worker whose beliefs are complementary with effort, by which we mean that optimistic beliefs increase the perceived marginal value of effort. In this case her
employer (the firm) may attempt to bias her belief formation process by providing cues that promote the recollection of positive evidence from memory. For example, the firm might place reminders of past successes on display (i.e. commendations from clients or industry groups) to promote recollection of these positive outcomes. The firm could make costly improvements to the workplace or the employee contractual arrangements (i.e. flexible work hours) to engender positive moods in the employees with this mood serving as a cue for positive information about the firm and the employee’s job function. All of these efforts can be interpreted in the model as firm efforts to increase $\varphi$ in order to increase the probability of employees recollecting high values of $\omega_i$. This example will be analyzed formally when we discuss employee morale.

A history, $H_N = \{\omega_1, \omega_2, ..., \omega_N\}$, is a set of data observed by the agent prior to decision and stored in long term memory. A recollected history is a random variable that consists of those events that are recalled by the agent at the time of decision. Let the random variable $H^R(\varphi)$ represent the recollected history (where the random variable takes the cue state as a parameter) with a typical realization denoted $H^R \subseteq H_N$. The length of a recollected history is denoted by $|H^R| \in \{1, ..., N\}$. We will let $\mathbb{H}(H_N, n)$ denote the set of all possible length $n$ recollected histories given the true history $H_N$. We will assume naivety on the part of the agent in the sense that the agent does not use knowledge of the cue state to correct for the biases induced in the recollection and Bayesian updating process.\footnote{The agent is assumed to be either unaware of the cue’s effect on his memory process or of the cue’s existence. Alternately, the problem of removing the bias may suffer from a catastrophic identifiability problem. Although the author prefers the notion that the agent is unaware of the bias as most psychological subjects seem to be ignorant of their own mnemonic biases, either of these explanations suffices to support the behavior modeled herein.}

3.2. Recall Probability Functions. A simple model for associative memory would be to treat the recollection of each datum in memory as an independent random event with the recollection probability a function of both cue state and the value of the datum. However, this generates a model in which both the length and content of memory are influenced by the cue state.\footnote{One of the important assumptions of Mullainathan \cite{54} is the use of a random walk information process. For a random walk, a datum consists of an innovation that is unambiguously good or...} For general stochastic processes it is
impossible to stochastically order the posteriors generated by data series of different lengths and without such an ordering it is impossible to generate comparative statics results for generic decision problems.

Let the recall probability function, $\rho(H^R|\varphi, H_N, n)$, be the probability that a set of data $H^R$ is recollected at the time of decision given a complete history of data $H_N$, a cue state $\varphi$, and a restriction that the recollection contain precisely $n \leq |H_N|$ elements. Let the probability that the recollected history, $H^R$, is of length $n$ given the true history, $H_N$, is of length $N$ be denoted

$$k(n, N) = \Pr\{|H^R| = n \text{ given } |H_N| = N\}$$

We assume $k(n, N)$ is independent of the cue state and that $k(0, N) = 0$ to preclude the case of complete forgetfulness.\(^5\)

We assume that, contingent on recollecting $n$ events, the contents of the memory are generated by a process of independent sampling without replacement from the true history. In order to generate associative memory effects, the relative sampling probabilities are assumed to be functions of the cue state and the signals in memory. High cue states are assumed to encourage recollection of high signal values. Denote the relative sampling probability of recalling event $\omega$ in a recollected history of length $n$ from a true history $H_N$ as $\rho_\omega(\omega|\varphi, H_N, n)$. We then have

$$\rho(H^R|\varphi, H_N, n) = \frac{\prod_{\omega \in H^R} \rho_\omega(\omega|\varphi, H_N, n) \cdot \prod_{\omega' \in H_N \setminus H^R} (1 - \rho_\omega(\omega'|\varphi, H_N, n))}{\sum_{H^R \in \Pi(H_N, n)} \prod_{\omega \in H^R} \rho_\omega(\omega|\varphi, H_N, n) \cdot \prod_{\omega' \in H_N \setminus H^R} (1 - \rho_\omega(\omega'|\varphi, H_N, n))}$$

$\propto \prod_{\omega \in H^R} \rho_\omega(\omega|\varphi, H_N, n) \cdot \prod_{\omega' \in H_N \setminus H^R} (1 - \rho_\omega(\omega'|\varphi, H_N, n))$

\(^5\)Throughout this work $\Pr\{E\}$ denotes the probability of an event $E$ under the true probability distribution over the states of the world.

bad news, in the sense of Milgrom [50], regardless of the number or magnitude of prior innovations. Therefore, beliefs contingent on memories of different lengths can be compared in the strong stochastic order in the random walk setting. One of the contributions of our model is identifying restrictions on the model of memory sufficient for comparative statics predictions to be made outside of the random walk setting.
Assumption 3. $\rho_\omega(\omega|\varphi, H_N, n)$ is log supermodular in $(\omega, \varphi)$

Assumption 4. $\rho_\omega(\omega|\varphi, H_N, n) > 0$ if and only if $\omega \in H_N$

Assumption 3 states that high values of $\varphi$ increase the probability of recollecting good news events (high $\omega$) relative to bad news events (low $\omega$). Assumption 4 defines the support of $\rho_\omega(\omega|\varphi, H_N, n)$. Combining all of these elements, the probability that $H^R(\varphi) = H^R$ is then

$$
k(n, N) \frac{\prod_{\omega \in H^R} \rho_\omega(\omega|\varphi, H_N, N) * \prod_{\omega' \in H_N \setminus H^R} (1 - \rho_\omega(\omega'|\varphi, H_N, n))}{\sum_{H^R \in H(H_N, N)} \prod_{\omega \in H^R} \rho_\omega(\omega|\varphi, H_N, N) * \prod_{\omega' \in H_N \setminus H^R} (1 - \rho_\omega(\omega'|\varphi, H_N, n))}
$$

4. Effects on Estimation and Choice

In this section we use the framework outlined above to generate monotone comparative statics results relating a mnemonic cue, such as affective state, to posterior beliefs and choices. The first application is to the agent’s estimation of parameters where this estimate is derived from data recalled in the presence of cues. The second application is to comparative statics in stochastic decision problems wherein the agent is making a choice from an abstract choice set with the utility a joint function of the choice and a random variable whose distribution the agent infers from prior beliefs updated via recollected signals.

4.1. Memory Cues and Estimation. Of primary interest for the purposes of this study is not the memory process, which is not observable (perhaps even to the agent), but the observable consequences on the decision outcomes. In this section we are concerned with developing comparative statics for the relationship between an agent’s judgements regarding $\theta_0$ and the cue state. The impact of biases in these judgements can be seen in choices over lotteries under different cue states. We assume that the agent observes the true history prior to the decision point and engages in recollection at the time the judgement is formed. These imperfect recollections comprise the data on which the judgement is based.

In this section we study the effect of cue state on judgements about the expected values of functions of random variables. Let $q : \Theta \to \mathbb{R}$ denote an increasing function
of \( \theta \). For example, consider an estimate of the expected utility of a choice \( x \in X \) where the utility as a function of \( \theta \) is \( q(\theta) = U(x; \theta) \). Note that as the agent’s judgement is based on what she recollects, the judgement about the mean of \( q(\theta) \) is a random variable that is a function of the recollection. Denote this random variable as

\[
q(\varphi) = \int q(\theta)G(d\theta|H^R(\varphi))
\]

**Theorem 5.** \( \varphi_1 \geq \varphi_2 \) implies \( q(\varphi_1) \succ q(\varphi_2) \).

Given a higher cue state, \( \varphi_1 \geq \varphi_2 \), for any given length of memory \( n \) the log supermodularity of the recall process weights the recalled data towards more positive events under cue value \( \varphi_1 \) relative to \( \varphi_2 \). Since we have assumed that the signal distribution \( f(\omega|\theta) \) is log supermodular, high signal values are good news. This good news property implies that higher expectations will result when a more positive set of data is recollected. Hence, for each fixed \( n \), the distribution of expected values under cue \( \varphi_1 \) will first order stochastically dominate the distribution under \( \varphi_2 \). Since the distribution of \( n \) is independent of the cue state, these relations holds when we consider randomizations over \( n \).

The theorem above uses a relation in the strong stochastic order that is not strict, whereas in certain circumstances we may require a strict stochastic ordering. In order to achieve this, we will have to strengthen the assumptions on the distributions made above.\(^6\)

**Corollary 1.** Assume that:

1) \( f(\omega|\theta) \) is strictly log-spm in \( (\omega, \theta) \) and \( \rho(\omega|\varphi, H_N, n) \) is strictly log-spm in \( (\omega, \varphi) \)

3) There exist \( \omega_1, \omega_2 \in H_N \) such that \( \omega_1 \neq \omega_2 \)

4) \( q(\omega) \) is strictly increasing

Then \( \varphi_1 > \varphi_2 \) implies \( q(\varphi_1) \succ q(\varphi_2) \)

\(^6\)The proof of the corollary follows from a straightforward modification of theorem 5 and is omitted.
4.2. **Stochastic Optimization and Cues.** Suppose the agent faces a stochastic optimization problem of the form

\[(4.2) \quad x^*(H^R(\varphi)) \in \arg\max_{x \in X} \int u(x; \theta) G(d\theta|H^R(\varphi)) \]

where \(X\) is a lattice, the utility function \(u(x; \theta)\) is a function of the choice variable \(x\) and an unknown parameter \(\theta\), and the agent has a CDF of posterior beliefs \(G(\cdot|H^R(\varphi))\) after recollecting history \(H^R(\varphi)\). Note that as the recollection of the agents, \(H^R(\varphi)\), is a random variable parameterized by \(\varphi\), \(x^*(H^R(\varphi))\) is a random variable whose distribution is induced by \(H^R(\varphi)\). Assume sufficient regularity conditions that the maximizer is unique for all \(H^R\).\(^7\) The agent observes the true history prior to the decision point and engages in recollection at the time the choice is made. These imperfect recollections comprise the data on which the beliefs about \(\theta_0\) are based and these beliefs influence agent choices. We are interested in how the distribution of choices of the agent are related to changes in the cue state.

In order to prove our monotone comparative statics result relating cue state, \(\varphi\), to agent choices, \(x^*(H^R(\varphi))\), we need to place assumptions on how \(\theta\) influences the agent felicity function, \(u(x; \theta)\). We will require that the felicity function of the agent obeys a single cross condition with respect to \((x, \theta)\).

**Definition 1.** (from Athey [1]) The function \(u(x; \theta)\) satisfies the two-dimensional single crossing condition if for each \(x_H > x_L\) pair there exists \(\theta_0\) such that for all \(\theta < \theta_0\) we have \(u(x_H; \theta) - u(x_L; \theta) \leq 0\) and for all \(\theta > \theta_0\) we have \(u(x_H; \theta) - u(x_L; \theta) \geq 0\).

The two dimensional single crossing condition, when combined with our assumptions on the information and memory processes, allows us to show that increases in the agent affective state induce the agent to choose higher values from \(X\). This is captured by the following theorem.

\(^7\)If the maximizer is not unique, the results of Athey [1] reveal that \(|H^R| = |\hat{H}^R|\) and \(H^R \geq \hat{H}^R\) imply \(x^*(H^R) \geq x^*(\hat{H}^R)\) where \(x^*(H^R)\) is a set valued function and the order refers to the strong set order. In this case, the notation \(x^*(H^R(\varphi_1)) \succ x^*(H^R(\varphi_2))\) implies that for all measures \(f\) that are monotone increasing in the strong set order we have \(Ef(x^*(H^R(\varphi_1))) \geq Ef(x^*(H^R(\varphi_2)))\). Although interesting to note, it is not clear to the author that this yields useful economic insights. One possible resolution to this impasse is to allow a set of maximizers but insist that the agent always choose the highest or lowest element from the set.
Theorem 6. Assume that \( u(x; \theta) \) satisfies the two-dimensional single crossing condition. Then \( \varphi_1 \geq \varphi_2 \) implies \( x^*(H^R(\varphi_1)) \succ x^*(H^R(\varphi_2)) \)

Given a higher level of cues, \( \varphi_1 \geq \varphi_2 \), for any given length of memory \( n \) the log supermodularity of the recall process weights the recalled data towards more positive events under cue value \( \varphi_1 \). Since we have assumed that the signal distribution \( f(\omega|\theta) \) is also log supermodular, high signal values are good news. This good news property translates into posteriors that are increasing in the strong stochastic order with increasing values of the cue state for fixed \( n \). We show in the appendix that \( x^*(H^R) \) is weakly increasing in \( H^R \). Combined with the effect of cue on the distribution of recollections \( H^R \), we have that the distribution of choices of the agent has the same strong stochastic ordering with respect to cue state. Since the distribution of \( n \) is independent of the cue state, these orderings holds when we consider randomizations over \( n \).

Although stated in terms of individual decisions, this result implies that the distribution of judgements in an economy will be shifted by an increase in a public cue state. Consider a large economy where supply and demand are equated to form market prices. Assume that all of the agents are in the same cue state and that the law of large numbers holds so that the demand curve in the market is equal to the distribution of demands formed as a function of the random recollection process. The price is then formed by the activity of the marginal agent in the system, and the theorem above shows that the marginal agent’s choice will shift with changes in cue state regardless of the identity of the marginal agent in the economy.

5. Applications

In section 5.1, we use our theorems to study the role of employee morale in principal agent relationships with an emphasis on when positive and negative affect are complementary with effort. This analysis provides theoretical structure for evaluating studies on worker morale in the organizational behavior literature. We then model several phenomena in the behavioral finance literature by embedding the static model studied above within a model of intertemporal cue dynamics.
5.1. Mood, Motivation, and Morale. The organizational behavior literature has seen a long debate between two schools of thought regarding the link between employee morale and productivity (see Staw and Barsade [65] for a brief literature review). The "Happier-and-Smarter" school of thought suggests that optimistic workers will prove more productive through increased effort choice and heightened creativity. The "Sadder-but-Wiser" hypothesis emphasizes that experimental subjects in a depressed mood tend to have more accurate beliefs regarding uncertain events than their more optimistic, happier peers. Ceteris paribus, more accurate beliefs lead to improved decision quality. We interpret an employee's mood and disposition as morale and use our model to provide an analysis of the relationship between affective state and productivity in an otherwise traditional principal agent model.

We assume that morale and the cued-memory effects it causes influence opinions about forces that determine the marginal product of the employee's labor. Examples of marginal productivity enhancing forces include the effort level of other employees, the competence of firm management, and general market conditions in addition to beliefs about the agent's own competence. However, optimism regarding the outcome of an effort choice (optimism about levels) does not equate to optimism regarding the marginal effects of effort (optimism about margins). Our model allows optimistic beliefs about outcomes to be positively (the complements case) or negatively (the substitutes case) correlated with the agent's expectations regarding the marginal productivity of effort. We show that the presence of complementarities between effort choice and optimistic beliefs implies happy agents are more productive, but in the substitutes case agents in a positive affective state will be less productive. Therefore, even without a trade-off between effort and accuracy of beliefs, it is not clear that optimism will result in greater agent effort. By providing a theoretical framework for analyzing the effect of morale in principal-agent relationships, we have a tool for interpreting the existing empirical research and suggest novel predictions for empirical testing.

Koszegi ([40], [41], [42]) provides studies of ego utility in principal agent settings. Koszegi interprets mood and morale as preferences over ego or as anticipatory utility, whereas our model is a purely informational account of the effect of mood.
An alternative view of our model is that we are providing economic and psychological microfoundations for the confidence enhanced productivity model of Compte et al. [17]. The Compte et al. model has two crucial behavioral components. First, Compte et al. assume a reduced form model for agent memory wherein successes are more likely to be remembered than failure, which results in agents possessing optimistic views about their future performance. Our model of memory provides an avenue through which these optimistic beliefs could be formed and suggests when the influence of optimistic beliefs will be more (or less) prevalent. Second, the Compte et al model assumes that confidence increases the chance of successfully executing the task. Our model of morale provides a simple structure based on the complementarity of effort choice and optimism that founds these effects. In addition, we highlight the possibility that optimistic beliefs can lower the equilibrium probability of success given appropriate economic primitives, a phenomena called defensive pessimism. Finally, since we have provided microfoundations for these phenomena, we are able to identify environmental factors, such as mnemonic cues, that influence the direction and degree of bias.

We assume that the agent has a strictly increasing utility function for wealth, \( v(w) \), and chooses his effort given a strictly increasing wage function, \( w : \mathbb{R} \rightarrow \mathbb{R}_+ \). The wage function maps production outcome, \( y \), to income. As our focus is not on designing the wage contract per se but on the role of affect in motivating the worker, we will assume that the wage contract is given exogenously. Production is a random variable with a probability density, \( f(y|e, \theta) \), that is a function of both an unknown parameter, \( \theta \in \mathbb{R} \), and the agent’s choice of costly effort, \( e \), at associated cost \( c(e) \). \( \theta \) parametrizes the marginal productivity of agent effort. Assume the agent has prior \( G(\theta) \) over the parameter space and has observed a history of signals distributed as \( f(\overline{\omega} | \theta) \) prior to making his effort choice.

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9This is an example of a self-affirming bias, which is significantly different than the associative memory model we are using. Compte et al. suppose that failures are less likely to be recorded in long-term memory. We assume that all data is in long-term memory and that data is recollected in an imperfect fashion.
Given a recollection $H^R$ of the true history $\bar{w}$ the agent’s problem is then:

\begin{equation}
\max_{e \in \mathbb{R}^+} E[v(w(y))|e;H^R(\varphi)] - c(e) = \\
\max_{e \in \mathbb{R}^+} \int \int v(w(y)) \ast f(y|e, \theta) \ast dy \ast G(d\theta|H^R(\varphi)) - c(e)
\end{equation}

We assume that for any $\theta$ there is a unique $e$ that solves (5.1). Given an effort choice $e$ and production parameter $\theta$, the agent’s expected utility from wages is

\begin{equation}
V(e, \theta) = \int v(w(y)) \ast f(y|e, \theta) \ast dy
\end{equation}

Our proof requires that $V(e, \theta)$ have the two dimensional single crossing property (2-SCP) in $(e, \theta)$ where $e$ is the choice variable. In a deterministic problem,

\begin{equation}
\max_e V(e, \theta) - c(e)
\end{equation}

2-SCP implies that the agent’s choice of effort is monotone increasing in $\theta$ (Milgrom and Shannon [51]). For 2-SCP to hold it suffices that either:

1. $y \in \{0, 1\}$ and $f(y|e, \theta)$ is strictly supermodular in $(e, \theta)$\(^{10}\)
2. $f(y|e, \theta)$ is strictly log supermodular in $(y, e)$ and $(y, \theta)$

**Example 1.** Consider a two point distribution for $y$ such that $y \in \{0, 1\}$, states of the world $\theta \in \{0, 1\}$, and stochastic output function $f(y = 1|\theta, e) = \theta \ast g(e)$ where $g(e)$ is chosen so that $g(e) \in (0, 1)$, $g'(e) > 0$, $g''(e) < 0$. This example represents an extreme case of complementarities between the marginal productivity of effort and state wherein effort is only efficacious in state $\theta = 1$ with failure assured if $\theta = 0$.

We assume that states of the world with high equilibrium wage realizations induce and are associated with positive mood states on the part of the agent. Mood regulated memory effects imply that positive mood states encourage the recollection of data from memory that supports a belief of a high probability of the occurrence of these high wage states. Further assume that these high wage states are also states with high marginal productivity of effort, as in the example above. In this case, affective cues bias the agent towards optimistic beliefs regarding the productivity of

\(^{10}\)This condition requires that the marginal effect of effort on the probability of success ($y = 1$) be increasing in $\theta$. 

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MOOD AND ASSOCIATIVE MEMORY
Given his biased beliefs, the agent chooses a level of effort that is increasing in his affective state. This relationship is captured in the following theorem.

**Theorem 7.** If \( \varphi_1 \geq \varphi_2 \), then \( e(\varphi_1) \geq e(\varphi_2) \)

Therefore a firm can achieve higher effort levels from the agents through manipulations that induce a positive affect in the agents.\(^{11}\) For simplicity, assume the firm can deterministically choose the agent’s affect level \( \varphi \) by spending \( m(\varphi) \).\(^{12}\) Further suppose the firm has a reduced form expected revenue function \( \Pi(y, \kappa) \), where \( y \) denotes the employee productivity level and \( \kappa \in \mathbb{R} \) is a parameter that indexes the effort sensitivity of the firm’s revenues to employee productivity \( (\frac{\partial^2 \Pi(y, \kappa)}{\partial \kappa \partial y} > 0) \). Creative and service industries are relatively sensitive to the effort level of the employees,\(^{13}\) whereas capital intensive manufacturing firm output is relatively insensitive to employee effort.\(^{14}\) Therefore the firm’s problem is

\[
(5.5) \quad \max_{\varphi \in \mathbb{R}} E[\Pi(y(\varphi), \kappa) - w(y(\varphi))] - m(\varphi)
\]

where \( y(\varphi) \) refers to the distribution of productivity induced by the distribution of employee effort levels \( e(\varphi) \). The objective is supermodular in \((\varphi, \kappa)\) and so by Topkis’s theorem (Topkis [67]) we know

\[
(5.6) \quad \varphi(\kappa) \in \arg \max_{\varphi \in \mathbb{R}} E[\Pi(y(\varphi), \kappa) - w(y(\varphi))] - m(\varphi)
\]

is weakly increasing in \( \kappa \). Therefore, industries in which corporate revenues are relatively sensitive to employee effort have a higher level of affect inducing compensation

\(^{11}\)Since the effort choice is based on the stochastic recollection data, we mean the term higher to be interpreted as a greater distribution of effort choices in the strong stochastic order.

\(^{12}\)We think of \( m(\varphi) \) as including costly ancillary benefits, mood enhancing improvements to the workplace, and time spent by managers and other figures investing in employee morale and a mood enhancing work-culture.

\(^{13}\)An inspection of the Fortune "100 Best Place to Work" list has seven of the top ten workplaces within the high-tech or business services industries. These industries are very human capital intensive and firm profits are likely to be sensitive to the effort exerted by the firm’s staff.

\(^{14}\)Suppose the firm’s operate in a competitive market and output is of the Cobb-Douglas form

\[
(5.4) \quad y = A * K^\alpha L^{1-\alpha}
\]

Our comparative statics parameter \( \kappa \) is analogous to the parameter \( 1-\alpha \) of the production function. Our theory predicts a correlation between \( 1-\alpha \), the amount of money firm’s spend on employee morale, and proxies for the happiness and morale of firm employees.
ceteris paribus. To the author’s knowledge, a comparative static of this form has not been tested in the organizational behavior literature on employee morale.

The flip side to confidence enhanced performance is defensive pessimism, the mental habit of holding pessimistic moods (and hence beliefs) in order to induce greater effort. Consider a graduate student studying for qualifying exams. A student who is optimistic about the difficulty of the exam or the degree to which he has mastered the requisite material might believe that there is little marginal benefit to increased effort studying. However, by focusing attention on the difficulty of past exams and the consequences of failure, the student can induce a negative affective state that encourages the recall of negative information about his preparedness for the upcoming exam, and these negative beliefs motivate the student to study.

Modifying the structure above, suppose that \( f(y|e, \theta) \) is log supermodular in \( (y, e) \) and \( (y, -\theta) \). Therefore, states with a high marginal productivity of effort are associated with low levels of output.

**Example 2.** Consider a two point distribution of output, \( y \in \{0, 1\} \). Suppose there are two possible states of the world, \( \theta \in \{0, 1\} \), and stochastic output function \( f(y) = 1|\theta, e = \theta + (1 - \theta)g(e) \) where \( g(e) \) is concave and chosen so that \( g(e) \in (0, 1) \), \( g'(e) > 0, g''(e) < 0 \). In the good state, \( \theta = 1 \), success is assured. In the bad state, \( \theta = 0 \), success is possible, but the probability is increasing in effort.

The output function, \( f(y|e) \), in the example in supermodular in \( (y, e, -\theta) \) as required. Appealing to theorem 6, we have that \( \varphi_1 > \varphi_2 \) implies \( e(\varphi_2) \geq e(\varphi_1) \). Again assuming that \( \kappa \) indexes firm profit sensitivity to effort, we have from the logic above that \( \varphi(\kappa) \) is weakly decreasing in \( \kappa \). Firms facing such a production function can take actions to reduce employee affective state by, for example, cultivating a high-stress corporate-culture that induces the agents to hold pessimistic beliefs and therefore increase work effort. A high stress level, if applied selectively so as not to make employment at the firm worse than the available outside options, could induce high employee effort when required even if employee wages are inflexible.

The theory outlined above focuses on individual morale and effort choice, but mood and morale are social phenomena and can spread between members of a team.
Barsade [5] provides evidence of emotional contagion from a confederate of the experimenter exhibiting either positive or negative emotions to experimental participants. Barsade surveys evidence that negative emotions spread more effectively than positive emotional states, which suggests that firms ought to take care to limit the interactions between unmotivated, demoralized workers and the remainder of the work force in situations where positive affect increases employee effort. Conversely, if a pessimistic outlook encourages effort, the easy contagion of negative affect might allow for limited firm effort at affective suggestion to have significant effects on employee output.

5.2. Financial Products. In this section we will trace out linkages between affective state, beliefs, and market prices for financial assets. Two complementary interpretations of our model are provided. In the first interpretation, asset dividend announcements\(^\text{15}\) activate agent affective states directly, and these activated affective states bias the recall of information from memory.\(^\text{16}\) Under this interpretation good (bad) news about an asset signals high (low) future utility, which induces a positive (negative) affect in the market participants. Mood regulated memory processes imply that the positive affect state biases recollection from long-term memory and the beliefs formed from the recollection.

In the second interpretation, good (bad) news about a firm reinforces links in associative memory between a firm and the positive (negative) valence affect node. As noted in Smith et al. [64], the reinforcement is driven by unconscious associations between memories of positive (negative) news and positively (negatively) valenced affective states. When the agent considers the firm in the future, the affect node is activated in memory and this contributes to a similarly valenced mood in the agent. For example, good news about Google strengthens the mnemonic association between Google and positive affective states, and Google will then serve as a stronger cue for the induction of positive affect in the agent. The valenced mood evoked by

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\(^{15}\)We assume that all firm earnings are immediately returned to investors in the form of dividends. Although earnings can be retained by firms for reinvestment, we could readily incorporate this feature of dividend policy into our model of net present value at the expense of increased notation.

\(^{16}\)Alternatively, we could assume that recent positive outcomes serve as cues for positive news in memory directly and reach qualitatively similar outcomes. To the extent that present positive news announcements cue good news from memory, the results of this section will be amplified.
an agent’s explicit consideration of the firm causes biases when the agent recalls data from memory and forms beliefs about the net present value (NPV) of assets.

We assume that positive information regarding asset dividends has a positive valence, which fits the attribution theory of valence (Smith et al. [64]). We show that the effects of dividend distributions on mood can explain a variety of time-series phenomena such as over- and underreaction of securities prices to the release of news. We will close our analysis by showing how cuing effects can also explain excess volatility of asset prices and price responses to non-fundamental information releases. The cue dynamics framework developed below is of more general interest as it represents one method for applying the associative memory model developed above to dynamic problems.

5.2.1. Prior Literature. The efficient markets hypothesis is conventionally taken to entail that all information is immediately incorporated into asset prices. One consequence is that current information should have no predictive power for future asset returns once the current price is taken into account. However, studies in behavioral finance have found a variety of regularities within the securities price data that suggest that present information has significant predictive power for future returns even after controlling for price. For example, Campbell and Shiller [13] have found that a recent series of high returns is indicative of low future returns, which suggests that financial assets become overpriced after the revelation of a series of positive news events. Empirical research on other regularities, such as short-run underreaction to news and excess price volatility, are surveyed in Barberis, Shleifer and Vishny [3]. We show that the effect of positive news on the affect of investors can provide a concise explanation all of these pricing regularities.

The concept of affect has appeared in the behavioral finance literature in the form of sentimental noise traders that serve as a source of liquidity in markets. In an early model, DeLong et al. [21] assume that noise traders are subject to exogenous sentiment shocks and study how these shocks can create risk that prevents arbitrageurs from eliminating mispricings and allows the noise traders to earn supernormal profits. Barberis et al. [3] provide a model of investor sentiment wherein traders alternate between two incorrect asset pricing models based on the representativeness heuristic to explain over- and underreaction of asset prices to news. Daniel et al. [20] provide
a model of investor sentiment driven by overconfidence and positively biased self-attribution in order to explain price over- and underreactions. All of these studies interpret sentiment as one of a variety of biases in belief or other behavioral anomalies, whereas we use the term sentiment in the conventional sense of an affective state.

Another branch of the behavioral finance literature of particular interest to this paper is the effort to find regularities between asset prices and weather within the geographical region of a trading market. In an early contribution, Saunders [59] finds that the cloud cover in Manhattan has a significantly negative effect on the prices of stocks traded through the New York Stock Exchange. Hirshleifer and Shumway [32] extend this analysis to cover markets across the globe and find the same negative correlation between cloud-cover and asset prices. Kamstra et al. [36] provide an analysis of seasonal effects around the world under the assumption that depressive effects caused by short days during the winter in turn depress stock prices. Edmans et al. [24] show stock market valuations decline following a loss in important soccer matches and the magnitude of the loss is greater for smaller stocks and more important events. Edmans et al. interpret this as a mood-related effect after controlling for plausible material explanations. Our model supplies foundations for these asset price anomalies in a Bayesian framework with the optimism formally encoded in the biased recollection of information salient for estimating market values. The change in prices is due to the fact that weather causes the moods and the corresponding biases in belief of traders within the market to become correlated.

Ljungqvist et al. [46] and Derrien [22] provide models wherein IPO prices reflect the optimism of individual traders in the market. Of principle interest to the authors is why IPOs tend to appear underpriced in the short run and overpriced in the long run. Crucial to their models is that the IPO is conducted when investor sentiment is optimistic regarding the fair IPO price and that this sentiment could become less optimistic in the future. Our model provides microfoundations for the belief formation process that leads the initial beliefs of investors to be either bearish or bullish and what factors cause these beliefs to change over time.

Cook et al. [16] study the role of sentiment in IPO pricing by examining whether investment bankers are compensated for generating excitement over a future IPO. If
the marketing efforts of investment bankers in charge of an IPO can have the effect of increasing the bullishness of investor sentiment, then the optimism of investors will increase the IPO profits reaped by the offering firm. To incentivize bankers’ efforts, the degree of marketing should be reflected in the investment bankers’ compensation. Indeed, Cook et al. find that investment banker compensation is significantly influenced by factors such as the number of headlines in the LexisNexis database with the company’s name in the six months preceding the IPO. This suggests that investment bankers are well aware both of the effect of sentiment on IPO pricing as well as their ability to manipulate the degree of sentiment.

Tetlock [66] studies the influence of the Wall Street Journal’s (WSJ) "Abreast of the Market" section on market sentiment. Included in this WSJ section are a summary of market events, explanations of the market behavior from third parties, and predictions about future market behavior. Tetlock conducts time series analysis of the relationship between the number of negatively valenced words in the WSJ section and the performance of the Dow Jones Industrial Average (DJIA) as well as a variety of other financial market statistics. Unsurprisingly, Tetlock finds that low DJIA performance causes negatively valenced words to appear in the WSJ section. More surprisingly, and salient for our study, negatively valenced words in the WSJ predict depressed performance of the DJIA in the following week. Tetlock argues that the widely read WSJ influences investor sentiment and the induced mood causes market prices to become depressed. Tetlock’s work suggests a feedback system is at work wherein depressed market performance leads to negatively valenced news coverage, which in turn leads to depressed market performance in the future. Our model provides a description of the psychological foundations leading to this relationship and allows us to make predictions regarding the magnitude and direction of these effects across different investors and asset classes.

5.2.2. Overview of Analysis. The model outlined in section 3 was developed in a static setting, whereas the majority of behavioral finance phenomenon involve time series behavior. Therefore, we will incorporate a model of affective dynamics into our analysis. We allow the agents within the market to have heterogeneous affective states, which implies that the agents have heterogeneous beliefs about the net present value of the assets future dividends. A price setting mechanism maps from the
distributions of beliefs about net present value into market price. The price will
generically not conform to the mean beliefs in the population, but will be determined
by the beliefs of the agent on the margin between buying and selling the security.

Our analysis assumes that each agent in each period makes inferences about the
net present asset value of the asset by recalling past dividends and updating her
beliefs through Bayes’ rule. A firm’s dividend payouts consist of hard information,
and it might strike the reader as implausible that this information could be forgot-
ten. However, dividend reports and earnings announcements are accompanied by
soft information about the firm’s future that is more difficult to locate within readily
available data sources. For example, the firm’s CEO might provide qualitative infor-
mation about the future of the firm and the industry within which it operates, equity
analysts could ask questions that hint at relevant information about competitors, or
the trader could have heard news through professional connections about the future
prospects for the firm. All of these data sources would be difficult to recover except
through memory and each can be relevant for assessing the future performance of the
asset. A reinterpretation of the model below is that hard information is perfectly
recalled and that the biases are induced by the imperfect recall of relevant soft infor-
mation. The qualitative effects of such a model would be identical to those found
below although the notation would be significantly more complicated.

5.2.3. Dynamics of Over- and Underreaction. As a positive datum is added to the
time series on which the NPV estimate is based, an unbiased estimate of the NPV of
the stock is increased. In addition, suppose that after a positive dividend announce-
ment the agent’s affect is shifted to a more positively valenced mood. Similarly,
after a negative announcement the agent’s affect is moved to a more negatively va-
lenced affective state. The memory biases induced by the changing affective state
will amplify the reactions of the agents to the news release.

The agents in our model fully and correctly incorporate recollected information
into beliefs about asset valuation, but they do not foresee how shifts in the affective
states of agents in the market will alter future prices. Our first goal will be to capture
two phenomena known as underreaction and overreaction of prices to information.\footnote{See Shleifer [63] for a discussion of the empirical evidence.
In the formulation below, \( \eta_t \) refers to a dividend (or earnings) innovation announced in period \( t \). The event \( \{ \eta_t > 0 \} \) is referred to as good news and \( \{ \eta_t < 0 \} \) is referred to as bad news. Let \( E_t^* \) refer to the expectations of an outside observer (i.e., an econometrician examining market price data) regarding \( P_t^M \), the market price of the asset at time \( t \).

**Definition 2.** Underreaction is a short run phenomenon wherein stocks for which good news has been recently reported have a tendency to have positive price correlation over short time horizons. Formally

\[
E_t^*[P_{t+1}^M - P_t^M | \eta_t \geq 0] > E_t^*[P_{t+1}^M - P_t^M | \eta_t < 0]
\]

**Definition 3.** Overreaction is a long run phenomenon wherein stocks for which good news has been repeatedly reported have a tendency to have negative price movements in the future. Formally for \( j \) sufficiently large

\[
E_t^*[P_{t+1}^M - P_t^M | \eta_t \geq 0, ..., \eta_{t-j} \geq 0] < E_t^*[P_{t+1}^M - P_t^M | \eta_t \leq 0, ..., \eta_{t-j} \leq 0]
\]

In the case of underreaction, a short series of positive news induces the market price to drift higher in the next period. The agents incorporate the news into their evaluation of future asset value and do not expect the price to drift upwards since they are naive regarding their mnemonic biases. A long series of positive news announcements pushes agent affective state to a high level, causing market participants to have optimistically biased expectations that cannot be met in the long run. Once this occurs, there is a high likelihood of asset performance below market expectations in the following period, and the worse than expected performance pushes the agents’ affective states towards a neutral valence and less biased beliefs.

5.2.4. Pricing and Affective State. Suppose an agent’s belief about the net present value of an asset given he recollects \( n \) signals is denoted \( P^n(\omega_1, ..., \omega_n) \), which defines a family of functions \( \{ P^n(\omega_1, ..., \omega_n) \}_{n=1}^{\infty} \). In an abuse of notation, we will refer to this set of functions collectively as \( P(H^R) \). Assume from hereon that \( P(H^R) \) is strictly increasing in each argument. Although we could choose any strictly increasing function, we formally define a family of belief functions since the estimator will be different for recollections of different lengths. For example, the mean of a data set \( \{x_1, ..., x_N\} \) is \( \frac{1}{N} \sum_{i=1}^{N} x_i \), which is a formally distinct function for each value of \( N \).
increasing pricing function for our model, to focus ideas we will model an agent’s assessment of the fair valuation of an asset as the risk neutral net present value of future dividend payouts

\[ P(\varphi) = \mathbb{E}_t[\mathbb{P}(\varphi)] = \mathbb{E}_t\left[\sum_{\tau=1}^{\infty} \frac{d_{t+\tau}}{(1+r)^\tau} \mid \varphi \right] \]

where \(d_t\) denotes the time \(t\) dividend, \(r\) is the market discount factor, and \(\mathbb{E}_t[\cdot \mid \varphi]\) represents the time \(t\) expectations of a naive agent in cue state \(\varphi \in [0,1]\). We will assume that dividends are produced by a random walk of the form \(d_{t+1} = d_t + \eta_{t+1}\) where \(\eta_{t+1} = \varepsilon_{t+1} + \gamma \varepsilon_t\), \(\gamma \in [0,1]\), and \(\varepsilon_t\) is symmetrically, identically, and independently distributed with a mean of 0.\(^{19}\) The exact form of dividend innovation autocorrelation is not crucial to any of the results below.

The price equation can be written

\[
(5.7) \quad P(\varphi) = \sum_{\tau=1}^{\infty} \frac{1}{(1+r)^\tau} \mathbb{E}_t[d_{t+\tau} \mid \varphi]
= \frac{d_t + \gamma \varepsilon_t}{r} + \sum_{\tau=1}^{\infty} \frac{1}{(1+r)^\tau} \frac{1 + \gamma + r}{r} \mathbb{E}_t[\varepsilon_{t+\tau} \mid \varphi]
= d_t + \gamma \varepsilon_t + \frac{1 + \gamma + r}{r^2} \mathbb{E}_t[\varepsilon_{t+1} \mid \varphi]
\]

Denote the expectation of future dividend innovations based on the recollected history \(H^R(\varphi)\) as

\[
(5.8) \quad \delta(\varphi) = \frac{1 + \gamma + r}{r^2} \mathbb{E}_t[\varepsilon_{t+1} \mid H^R(\varphi)]
\]

Theorem 6 shows \(\varphi_1 > \varphi_2\) implies \(\delta(\varphi_1) \succ \delta(\varphi_2)\). Although the agents’ estimation of the mean of the expected dividend shocks may be biased by their affective states, the agents are utilizing the correct model for the dividend process since the persistence inherent in the dividend process has been fully incorporated into beliefs regarding net present asset value.

\(^{19}\) The persistence of dividend innovations is required for underreaction to news in general. However, underreaction will occur with no persistence of dividends (\(\gamma = 0\)) if \(T_1^B T_2^B \succ T_1^C T_2^B\) (See Appendix A.5 for notation). Numerical tests reveal that most intuitive choices of \(f_G\) do not have this property. However, if \(T_2^B T_1^C \succ T_1^C T_2^B\) nearly holds, then the persistence can be made small (\(\gamma\) arbitrarily close to 0) and short-run underreaction will occur. Overreaction occurs for all \(\gamma \in [0,1]\).
Suppose the strength of the associations between a security and positive affective state is distributed as either $\psi_1$ or $\psi_2$ in the market with $\psi_1 \succ \psi_2$.\textsuperscript{20} Our analysis above implies that shifts of the distribution of the affective cue state (in the strong stochastic order) yield corresponding shifts in the distribution of beliefs about the asset values. In a model of supply and demand where the agents have heterogeneous beliefs about the asset value, it is the marginal buyer that determines the price of the asset in the market. Denote the time $t$ market price of the asset in the market given a distribution of affective states $\psi$ as $P_t^M(\psi) : \Delta([0,1]) \to \mathbb{R}_+$ where $\Delta([0,1])$ represents the set of probability measures over $[0,1]$. In accordance with the NPV model above, we make the following assumption about the price formation process:

**Assumption 8.** $P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t) = \frac{(1+\gamma+r)}{\rho} \ast \varepsilon_{t+1} + \Lambda(\psi_{t+1}) - \Lambda(\psi_t)$ where

1. $\psi_{t+1} \succ \psi_t$ implies $\Lambda(\psi_{t+1}) - \Lambda(\psi_t) > 0$
2. $\psi_{t+1} \prec \psi_t$ implies $\Lambda(\psi_{t+1}) - \Lambda(\psi_t) < 0$
3. $\Lambda(\cdot)$ is continuous in the total variation norm or any coarser topology on the space of distributions over affective states.

This assumption states that if the distribution of market beliefs about the fair value of the asset is shifted in the strong stochastic order the price changes correspondingly. Any reasonable model of market price formation will have the market price increase if the distribution of beliefs about the assets fair price execute a shift in the strong stochastic order.\textsuperscript{21}

For an example of such a market mechanism at work, suppose we hypothesize that the market consists of identical agents that do not have unlimited ability to act as arbitrageurs regarding perceived price irregularities.\textsuperscript{22} Therefore, even if some

\textsuperscript{20} The cue state of a single agent is denoted by $\varphi_t$. The distribution of these cue states in the market is denoted $\psi_t$.

\textsuperscript{21} We note that our pricing structure is a partial equilibrium in that we do not explicitly model both a buy and a sell side of the market. However, we conjecture that most plausible models would exhibit a price response of the form we are positing. A previous version of this paper developed a general equilibrium asset pricing model using a simplified version of the Kyle [43] noise trader model.

\textsuperscript{22} The agents in the analysis above possess beliefs that are, at turns, either too optimistic or pessimistic about the price of the asset in future periods. It seems plausible than an arbitrageur could profitably enter the market. However, there is a large and growing literature on the limits of arbitrage. See Shleifer [63] for a survey on the limits of arbitrage.
agents believe the market price is lower than the net present value, the agent with the highest assessment of net present value either cannot or will not purchase the entire supply. Suppose that there is sufficient supply of the asset to provide the top $p\%$ of the buyers with the asset. Then the market price will be set by the beliefs of the agent at the $(1 - p)$ percentile of the distribution.$^{23}$

Let the affective state of the market be defined by the PDF $\psi_t$ over the set $[0, 1]$ of possible affective states. We assume that after good news the associations between a security and positive affective states are strengthened and the cue state increases, and after an event of bad news these associations are weakened. If the agents in the market attribute positive news to better predictions of their future well-being, then attribution theory predicts the associations between affect and the asset will be altered in this qualitative way (Smith et al. [64]).$^{24}$ In the event of good news, $\{\eta_{t+1} \geq 0\}$, affective dynamics are modeled using a transition function

$$f_G(u, v) = \Pr\{\varphi_{t+1} = v | \varphi_t = u, \eta_t \geq 0\}$$ (5.9)

$f_G$ is a PDF dictating the distribution over affective states in period $t + 1$ of an agent in affective state $v$ at time $t$ contingent on having received good news in period $t$. In the event of bad news, $\{\eta_{t+1} < 0\}$, affective dynamics are modeled using a transition function

$$f_B(u, v) = \Pr\{\varphi_{t+1} = v | \varphi_t = u, \eta_t < 0\}$$ (5.10)

$^{23}$In the original working paper version of Mullainathan [54], an application to asset pricing was developed. Mullainathan’s model required the market price to be monotone in the average beliefs about net present value amongst market traders, which is not required when one leverages the more powerful monotone comparative statics results we have provided. Without strong assumptions regarding the price formation process in the market and the distribution of cues, no predictions regarding the influence of mnemonic cues on the time series of asset prices can be made using Mullainathan’s results directly.

$^{24}$The attribution of valence for the future can be indirect. For example, even if the agent does not hold the security, the associations would be strengthened if the good news was a signal of future good news regarding elements of his portfolio or of the market generally. On the other hand, agents holding short positions in an asset for whom a good news event has occurred would not have positive affective reactions to good news for that asset, so our analysis assumes the negative change in associations that occur for the fraction of traders who hold short positions in any given asset is overwhelmed by the positive associations formed by the remainder of the market participants.
Since each transition function is asymptotically used in 50% of the time periods, the long-run transition function is

\begin{equation}
(5.11) \quad f(u, v) = 0.5 \ast f_B(u, v) + 0.5 \ast f_G(u, v)
\end{equation}

We assume that $f_G, f_B$ are Lebesgue measurable and continuous except at points in the set \{(x, x) : x \in [0, 1]\}.

5.2.5. Analysis of Market Effects. Empirical tests of the underreaction and overreaction phenomena examine panels of security prices. We study these phenomena by examining the autocorrelation in expected prices observed by an unbiased econometrician following a short or a long series of valenced news events. In both cases, we assume that agent affect is distributed according to the ergodic distribution prior to the news events.

**Theorem 9.** Assume \( \rho = \Pr\{\eta_{t+1} \geq 0|\eta_t \geq 0\} = \Pr\{\eta_{t+1} \leq 0|\eta_t \leq 0\} \) is sufficiently large and market affective state begins at the stationary distribution \( \psi_{t-1} = \tilde{\psi} \). Then asset prices exhibit underreaction in the short run

\begin{equation}
(5.12) \quad E_t^* [P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t)|\eta_t \geq 0] > E_t^* [P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t)|\eta_t < 0]
\end{equation}

The short-run underreaction is due to the fact that while traders realize that good news this period implies likely good news for next period and incorporate this into their calculations of the asset value, good news next period will further shift the affective state of the market participants and increase next period’s price further than the asset traders’ estimates predict. If agent memory were unbiased, then market prices would form a random walk as suggested by the efficient markets hypothesis.

We now show that a sufficiently long series of good (or bad news) causes overreaction. Intuitively, a long series of good news will push the market participants into positive affective states so that they hold, in general, optimistic (positive) beliefs about future dividend payouts that are unlikely to be met in reality. This causes future asset prices to revert to the market price given unbiased beliefs as successive bad news events depress the distribution of affective states in the market.\(^{25}\)

\(^{25}\)This result does not require persistence in the dividend process, although we will maintain the assumption for continuity.
Theorem 10. Asset prices exhibit overreaction in the long run

\[
E_t^*[P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t)]|\eta_t \geq 0, \ldots, \eta_{t-j} \geq 0 < \\
E_t^*[P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t)]|\eta_t \leq 0, \ldots, \eta_{t-j} \leq 0
\]

for sufficiently large \( j \).

When the market is exposed to a long series of positive news items, the distribution of affective states becomes concentrated around the most positive affective state, \( \varphi = 1 \). However, since the distribution of affective states is near its upper bound, the distribution of affective states next period will with high probability shift towards a less positive distribution in the strong stochastic order. Therefore, an unbiased econometrician would expect market price to shift downwards next period (under \( E_t^* \)) given a long series of positive news has been observed. Symmetrically, if a long series of negative information has been observed, then the expectations under the true dividend process is that prices will rise. The net expected effect is that asset market prices will exhibit long run overreaction.

Using a stylized model for simulation purposes, figure 1 describes the autocorrelation of changes in the mean beliefs about net present value of an asset.\(^{27}\) Without associative memory effects, there would be no autocorrelation of the changes in beliefs. However, we see short-run underreaction in that changes in beliefs about net present value are positively correlated over a single period. Long run overreaction is captured by the negative correlation of changes in mean beliefs over longer horizons. In cases where multiple successive news events of the same valence are realized, these correlations could extend over longer time horizons.

Long run overreaction to a series of news releases caused by the dynamics of sentiment is one explanation for the existence of bubbles in the stock market driven

\(^{26}\)It is trivial to show our argument holds for initial distributions away from the ergodic distribution as Harris chain dynamics push any initial distribution monotonically towards the unique ergodic measure in the total variation norm as \( T \to \infty \).

\(^{27}\)Without a market model, we cannot translate a distribution of beliefs about net present value into a market price. We plot the mean beliefs about net present value as a form of convenient summary statistic without claiming that there is a direct mapping from the average expected net present value of an asset to market price. See Footnote 22.
by "animal spirits." Positive announcements by firms that are considered to be industry leaders are informative for the industry at large and, due to their visibility, cause positive shifts in affective state across the market. The affective shift causes a movement of the asset prices that is purely due to changing sentiment. For example, an investor might note that Yahoo! and Microsoft have released positive performance reports, associate a positive affect with the industry broadly, and more readily recollect other evidence of positive future performance for internet stocks. The bias towards recollection of positive information about the industry will lead to higher valuations for internet related stocks. Conversely, if Pets.com and other prominent internet stocks release negative reports and cause the industry to be associated with negatively valenced outcomes and affects, the agents’ mood-regulated memory would more readily evoke evidence that would lead the agent to be skeptical of the industry’s future prospects, leading to a shift to lower stock prices.

5.2.6. Heterogeneous Market Beliefs. If agents share common beliefs about the future returns of equity ownership, one would expect little trade in the stock market. Although the literature on uncommon priors is too large to discuss in depth, we note

Figure 1. Autocorrelation of Increments of Mean Expectation of NPV
the work of Miller [52] as an early example that makes predictions about the relationship between heterogeneity of opinion, market prices, and trading volume. Hong and Stein [33] provide a recent overview of the literature and develop a simple model relating trading volume and price momentum effects with heterogeneous beliefs caused by the gradual release of information in the market. The stochastic nature of the model we have developed and the presence of heterogeneous cues and mood shifters amongst the agents in the market provides a novel source for heterogeneity of beliefs. The advantage of the model we have presented is its firm grounding in psychological foundations rather than assumptions of uncommon priors or other behavioral effects.

Some models assume that myopic noise traders with idiosyncratic demand shocks provide the observed liquidity in the stock market (for example, DeLong et al. [21]). Our model can provide microfoundations for the determinants of the sentiment shocks that drives noise trader demand levels. If we assume that noise traders are subject to idiosyncratic cues and mood influencing effects, then our model provides a rationalization for changes in noise trader demand. By analyzing the cues facing the traders, we can predict the direction and volatility of noise trader demand. For example, if the market receives positive news about consumer confidence, this would provide a correlated positive shift of noise trade affective state that would cause asset market prices to shift correspondingly. As our prediction has not survived an empirical test, these claims should be treated as preliminary.

5.2.7. Excess Volatility and Non-Fundamental News. In our model changes in market prices for assets are driven by two complementary effects. Information about future asset dividends is revealed by dividends in the current period. This information is incorporated into models of the net present value of the asset, and the movements of price caused by this updating are referred to as fundamental volatility. Empirical studies find that asset prices exhibit volatility greater than can be explained by the fundamental contribution alone (see, for example, Shiller [61], [62]), and our model of asset pricing suggests that affective volatility can partially explain this added volatility. Mood-regulated memory effects on agent beliefs cause affective state to change based on the release of dividend information. The change in affective state will also cause prices to shift in the market, an effect we refer to as affective volatility.
In our model the two effects can be separated allowing us to generate predictions about the relative magnitude of the affective and fundamental contributions to volatility in different market conditions. From period $t$ to period $t+1$ the price increment is determined by

$$P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t) = \frac{(1 + \gamma + r)}{r^2} \cdot \varepsilon_{t+1} + \Lambda(\psi_{t+1}) - \Lambda(\psi_t)$$

We interpret the variance of this price increment as volatility. We can identify the fundamental contribution to volatility as

$$Var \left[ \frac{(1 + \gamma + r)}{r^2} \cdot \varepsilon_{t+1} \right]$$

As the variance of dividends increases, the fundamental contribution to volatility increases. The non-fundamental contribution is

$$Var \left[ \Lambda(\psi_{t+1}) - \Lambda(\psi_t) \right]$$

The more mercurial affective state is with respect to news revelations, the greater the excess volatility will be.

5.2.8. **Empirical Testing.** The advantage of rooting our model in the psychological primitives of affective state and memory is that we can utilize the deep literature on the determinants of mood as the basis of tests of our model. Mood can be driven by a number of phenomenon, such as cloud cover, that have little if anything to do with stock market fundamentals. Other interesting examples include the release of survey results regarding consumer opinion, negative reports in industries or markets unrelated to the asset under consideration, political events, or almost any other valenced event that would cause correlated moods in the market participants. Our theory predicts volatility and price movement in response to these cues, especially in cases where the affective cues are salient to the agents and highly variable. We predict these effects will occur even when no fundamental news has been released and classical asset pricing models would predict no price response.

We can further test our model using panel data of asset portfolio holdings in combination with data regarding cues the agents are facing. For example, we would expect asset holders in cloudy cities to divest from risky assets as their view of the
net present value of the risky dividend flows in the future decreases relative to the beliefs of asset holders in sunnier regions. This provides a cross-sectional test based on the cues facing investors in different geographic regions. Furthermore, if an agent observes an affective cue regarding one element of his portfolio, his affective state could bias his beliefs about unrelated segments of his portfolio. For notational ease, consider an agent invested in industries A and B that have no fundamental connection economically. If positive news is released about firms in market A, the agent’s affective state will increase and his beliefs about the securities of firms in both markets will become more optimistic. This will induce the agent to invest more in the risky asset of markets A and B (relative to his holdings of riskless securities), even though the news announcement held no information about firms in industry B. One could test for the presence of a mood-regulated memory bias by examining the cross-sectional holdings of agents and determine how the market price and agent purchase decisions are altered by news about the individual securities the agents hold.\textsuperscript{28}

Another test of the mood-regulated memory model would be to examine the difference in security holdings and price responses of agents that exhibit differing degrees of mood-regulated memory bias. Presumably professional traders, who are not risking their own assets, suffer fewer changes in affective state when news is revealed. Under this hypothesis, one metric for the volatility of affect amongst traders in the market for a security would be the fraction of the asset supply in the hand’s of semi-professional traders (i.e. day traders). A natural test then would be to compare the fraction of the asset held by semi-professional traders with a measure of excess volatility or response to non-fundamental news. This provides a test of our model of mood as a cue for associative memory in financial markets that, to the best of the author’s knowledge, is not contained within the literature.

\textsuperscript{28}This test is complicated by a variety of factors. For example, if an agent’s holding of securities in market A increases in value, a rational agent should rebalance his portfolio for diversification (absent transaction costs) and so we would expect purchases of securities unrelated to market A even without the presence of mood-regulated memory effects. One can show in a parametric model that agent’s subject to mood regulated memory will increase the riskiness of their holdings of assets in market B in response to good news about assets in market A. A positive affective state causes the agents to hold optimistic beliefs about the returns of the risky assets in market B that result from their positive affective state. The optimistic views about risky assets in market B causes agents to rebalance their portfolios of assets in market B in favor of the risky securities.
6. Conclusion

The principle goal of our paper is to provide a framework for modeling the effect of mood on associative memory in generic decision problems under uncertainty. Our model allows us to derive robust comparative statics that can be employed in a variety of economic settings. We chose to explicitly model the psychological primitives of our associative memory process and its relation to affective state. By incorporating the psychological primitives in our model, we can use the wealth of psychological data on cued memory and affect to determine market conditions under which the effects of our model will be prevalent. These insights from the psychology literature, once incorporated into our theoretical framework, allow us to make novel predictions regarding long standing problems interpreting the effect of employee morale on firm productivity and puzzling time-series behaviors of asset prices in security prices.

The application to morale provides a framework for interpreting the conflicting results in the empirical literature on mood and employee productivity. One conclusion of our work is that the influence of affect on employee effort and resulting firm profitability depends on whether optimistic employee beliefs about productivity levels increase or decrease the employees beliefs about the marginal value of effort. By identifying this relationship between mood, biased beliefs, and the marginal value of effort, we are able to generate novel predictions regarding correlations between firm industry, job function, and the amount of firm resources expended on morale enhancing activities. Additional care must be taken when interpreting the prior literature since, as a rule, these studies do not present evidence sufficient to answer the question as to whether beliefs and effort are complementary. Testing our predictions in either a laboratory or field setting remains a subject for future research.

We apply our model to securities markets in order to explain a variety of phenomenon observed within the behavioral finance literature. We are able to predict the short-run underreaction and long-run overreaction of price to news, excess price volatility, and the response of prices to nonfundamental events. We also provide microfoundations for the notion of sentiment that is used in noise-trader models. In addition, our model supplies a source for heterogenous market beliefs, both due to the fundamentally stochastic nature of our theory and the possibility that agents are influenced by idiosyncratic cues that distort their beliefs. Although the ability of
our model to explain observations in the finance literature is significant, of greater consequence for tests of the theory is the novel predictions regarding how cues affect the magnitude and direction of these phenomena. Novel predictions can be generated by studying the influence of nonfundamental affective cues such as economic conditions local to the trader, the quality of the weather, or the performance of local sports teams. In addition our theory can be extended to make predictions about how portfolio choice problems are affected by mnemonic cues and how trading behavior varies as a function of responsiveness to these cues.

There are a number of additional applications such as advertising and persuasive speech where affect and cued memory are important influences on agent behavior. One example includes the choice of firms to employ either affective or informative advertising strategies. By incorporating our theory as to how consumers respond to affective cues into a model of advertising and market competition, we can derive predictions as to which types of firms use each form of advertising and how these advertising strategies segment the customer base. Persuasive speech often contains emotional cues with little novel information. This form of speech is important within the political sphere where politicians face a trade-off between revealing policy information to voters or spending their valuable media exposure opportunities attempting to influence the voters with affective appeals. It would be of interest to study both the determinants of the politicians’ choice of the nature of their public speeches and how this influences the decision processes of the voting public.

References


Appendix A. Proofs

A.1. Notes on Stochastic Orders. The model and subsequent analysis will make use of multivariate stochastic orders. Of principal interest are distributions that obey the condition of multivariate total positivity of order 2 (MTP-2). A probability density function (PDF) $f(x), x \in \mathbb{R}^N$, is MTP-2 if for any $x, y \in \mathbb{R}^N$ we have

\begin{equation}
 f(x)f(y) \leq f(x \land y)f(x \lor y)
\end{equation}

where $x \lor y = (\max(x_1, y_1), \ldots, \max(x_N, y_N))$ and $x \land y = (\min(x_1, y_1), \ldots, \min(x_N, y_N))$ are lattice operations. This condition is identical to the notions of affiliation and log supermodularity (log-spm) studied in the mechanism design and monotone comparative statics literatures. Random variable $X$ is larger than $Y$ in the strong likelihood ratio order or tp-2 order (denoted $X \succ_{tp_2} Y$) if the respective PDFs obey for all $x, y \in \mathbb{R}^N$,

\begin{equation}
 f_X(x)f_Y(y) \leq f_Y(x \land y)f_X(x \lor y)
\end{equation}
Random variable \( X \) is larger than \( Y \) in the *strong stochastic order* (denoted \( X \succ Y \)) if for all increasing functions \( u : \mathbb{R}^N \to \mathbb{R} \), \( Eu(X) \geq Eu(Y) \). The strong stochastic order is a multivariate generalization of the one dimensional first order stochastic dominance. Mueller and Stoyan [55] show that \( X \succ_{tp2} Y \) implies \( X \succ Y \).

**A.2. Proofs from Section Three.** Our first theorem uses assumption three and four to show that improved affect represses the recollection of bad news from memory.

**Lemma 1.** If \( \rho_\omega (\omega | \varphi, H_N, n) \) is log-spm in \((\omega, \varphi)\), then \( 1 - \rho_\omega (\omega | \varphi, H_N, n) \) is log-spm in \((-\omega, \varphi)\).

**Proof.** Note that \( 1 - \rho_\omega (\omega | \varphi, H_N, n) \) is log-spm in \((-\omega, \varphi)\) is equivalent by definition to the claim that for all \( \omega \geq \omega' \), \( \frac{1}{\rho_\omega (\omega | \varphi, H_N, n)} \) is decreasing in \( \varphi \). Let \( g(\omega'; \varphi) = \frac{1}{\rho_\omega (\omega | \varphi, H_N, n)} \) and \( L(\omega, \omega'; \varphi) = \frac{\rho_\omega (\omega | \varphi, H_N, n)}{\rho_\omega (\omega | \varphi, H_N, n)} \). Then \( \frac{1-\rho_\omega (\omega | \varphi, H_N, n)}{1-\rho_\omega (\omega | \varphi, H_N, n)} \) is decreasing in \( \varphi \).

We will now use our assumptions and the lemma above to show that the recall of sequences of signals inherits the log-spm properties assumed for the functions \( \rho_\omega \) defining the relative recall probabilities of the individual signals.

**Lemma 2.** Assumptions 3 and 4 imply that \( \rho (H^R | \varphi, H_N, n) \) is log-spm in \((H^R, \varphi)\).

**Proof.** (of Lemma 2) We will proceed by showing the log-spm relation holds between any pair of variables in \((H^R, \varphi)\) (See Karlin and Rinott [37] for a proof of the sufficiency of this argument). It is well known that this condition then implies log-spm amongst all of the variables.

In a slight abuse of notation, we will denote the set of recollected items (as opposed to the majorized vector) as \( H^R \). Then consider the two sets of recollected histories \( H \cup \{ \omega_i, \omega_j \} \) and \( H \cup \{ \tilde{\omega}_i, \tilde{\omega}_j \} \) where \( \omega_i \geq \tilde{\omega}_i \), \( \omega_j \geq \tilde{\omega}_j \) where \( H \cup \{ \omega_i, \omega_j, \tilde{\omega}_i, \tilde{\omega}_j \} \subseteq H_N \). To show log-spm of the recall probability function holds between pairs of signal values, note

\[
(A.3) \quad \frac{\rho(H \cup \{\omega_i, \omega_j\} | \varphi, H_N, n)}{\rho(H \cup \{\tilde{\omega}_i, \tilde{\omega}_j\} | \varphi, H_N, n)} = \frac{\rho(\omega_j | \varphi, H_N, n) * [1 - \rho(\tilde{\omega}_j | \varphi, H_N, n)]}{\rho(\tilde{\omega}_j | \varphi, H_N, n) * [1 - \rho(\omega_j | \varphi, H_N, n)]} \]

\[
= \frac{\rho(H \cup \{\tilde{\omega}_i, \omega_j\} | \varphi, H_N, n)}{\rho(H \cup \{\tilde{\omega}_i, \tilde{\omega}_j\} | \varphi, H_N, n)}
\]
Therefore we have log-spm in pairs \(\{\omega_1, \omega_2\}\).

Suppose that \(\omega \geq \omega', \varphi \geq \varphi',\) and \(H^R_1, H^R_2 \in \mathbb{H}(H_N, n)\) differ only in that \(\omega \in H^R_1, \omega' \notin H^R_1\) \text{ and } \(\omega' \in H^R_2, \omega \notin H^R_2\). Then we have \(H^R_1 \geq H^R_2\) (where this inequality refers to the majorizations of both recollected histories). Therefore

\[
\frac{\rho(H^R_1|\varphi_1, H_N, n)}{\rho(H^R_2|\varphi_1, H_N, n)} = \frac{\rho(\omega|\varphi_1, H_N, n) * (1 - \rho(\omega'|\varphi_1, H_N, n))}{\rho(\omega'|\varphi_1, H_N, n) * (1 - \rho(\omega|\varphi_1, H_N, n))}
\]

From Assumptions 3 and 4 it follows that

\[
\frac{\rho(H^R_1|\varphi_1, H_N, n)}{\rho(H^R_2|\varphi_1, H_N, n)} = \frac{\rho(\omega|\varphi_1, H_N, n) * (1 - \rho(\omega'|\varphi_1, H_N, n))}{\rho(\omega'|\varphi_2, H_N, n) * (1 - \rho(\omega|\varphi_2, H_N, n))}
\]

Therefore the log-spm holds in \((\omega_1, \varphi)\) pairwise. It follows then that \(\rho(H^R|\varphi, H_N, n)\) is log-spm in \((H^R, \varphi)\). \(\square\)

Log-supermodularity is preserved by multiplication, which is convenient in our context since the probability of observing a history \(H^R\) is the product of the relative sampling probabilities for each recollected datum, \(\rho_\omega(\omega|\varphi, H_N, N)\), and each forgotten datum, \(1 - \rho_\omega(\omega'|\varphi, H_N, N)\). Our assumptions above insure that both \(\rho_\omega(\omega|\varphi, H_N, N)\) and \(1 - \rho_\omega(\omega'|\varphi, H_N, N)\) have the requisite log supermodularity properties. Therefore, the product of these functions, \(\rho(H^R|\varphi, H_N, n)\), inherits these properties. Note that since log supermodularity is a property of the ratio of probabilities, the complex denominator of equation (3.5), which is common across all of the histories of length \(n\), plays no role in our proof.

**Lemma 3.** ([37] or theorem 3.11.4, [55]) If \(\rho(H^R|\varphi, H_N, n)\) is log-spm in \((H^R, \varphi)\), then for \(\varphi_1 \geq \varphi_2\), \(H^R(\varphi_1, n)\) dominates \(H^R(\varphi_2, n)\) in the strong multivariate stochastic order.

The above lemma is a restatement of the fact that if two random variables are ordered in the strong likelihood ratio order, then they are also ordered in the strong multivariate stochastic order. Two technical conditions are required in addition to
the log supermodularity of \( \rho(\mathbf{H}^R|\varphi, H_N, n) \). First, \( \mathbb{H}(H_N, n) \) must form a lattice, which is obviously true. Second, the distributions \( \rho(\mathbf{H}^R|\varphi, H_N, n) \) (for different values of \( \varphi \)) must be absolutely continuous with respect to some \( \sigma \)-finite measure. In this case, this measure can be taken to be the counting measure over the elements of \( H_N \). Finally, we note that although we have established that the \( \rho(\mathbf{H}^R|\varphi, H_N, n) \) are log supermodular in \( (\mathbf{H}^R, \varphi) \) for a given value of \( n \), this is a sufficient but not necessary condition for \( \mathbf{H}^R(\varphi, n) \) to be ordered in the strong stochastic order with respect to parameter \( \varphi \).

A.3. Proofs from Section Four. We begin with a theorem that extends Milgrom’s [50] representation result to the case of multiple signals. Assume that the agent has recollected a set of signals \( H^R = (\omega_1, \ldots, \omega_n) \). Given a prior belief about the distribution of the parameter \( \theta \), denoted \( G(\theta) \) with density \( g(\theta) \), the agent forms a Bayesian posterior equal to \( G(\theta|H^R) \).

**Theorem 11.** If the condition density of signals, \( f(\omega|\theta) \), is log-spm in \( (\omega, \theta) \), then

\[
H^R_1 = \overline{\omega}_1 = (\omega_1, \ldots, \omega_n) \geq H^R_2 = \overline{\omega}_2 = (\overline{\omega}_1, \ldots, \overline{\omega}_n)
\]

implies \( G(\theta|\overline{\omega}_1) \) first order stochastic dominates \( G(\theta|\overline{\omega}_2) \).

**Proof.** Choose \( \theta^* \) such that \( 0 < G(\theta^*) < 1 \). Our assumption of the log-spm of \( f(\omega|\theta) \) implies that \( f(\omega_1, \ldots, \omega_n|\theta) = \prod_{i=1}^{n} f(\omega_i|\theta) \) has the log supermodularity property in \( (\omega_1, \ldots, \omega_n; \theta) \) as products of log-spm functions are log-spm. This then implies

\[
(A.6) \quad \frac{\int_{\gamma \geq \theta^*} f(\omega_1, \ldots, \omega_n|\theta) G(d\theta)}{f(\omega_1, \ldots, \omega_n|\theta)} \geq \frac{\int_{\gamma \geq \theta^*} f(\overline{\omega}_1, \ldots, \overline{\omega}_n|\theta) G(d\theta)}{f(\overline{\omega}_1, \ldots, \overline{\omega}_n|\theta)}
\]

\[
(A.7) \quad \Rightarrow \quad \frac{f(\omega_1, \ldots, \omega_n|\theta)}{\int_{\gamma \geq \theta^*} f(\omega_1, \ldots, \omega_n|\theta) G(d\theta)} \leq \frac{f(\overline{\omega}_1, \ldots, \overline{\omega}_n|\theta)}{\int_{\gamma \geq \theta^*} f(\overline{\omega}_1, \ldots, \overline{\omega}_n|\theta) G(d\theta)}
\]

\[
\Rightarrow \quad \frac{\int_{\gamma \geq \theta^*} f(\omega_1, \ldots, \omega_n|\theta) G(d\theta)}{\int_{\gamma \geq \theta^*} f(\omega_1, \ldots, \omega_n|\theta) G(d\theta)} \leq \frac{\int_{\gamma \leq \theta^*} f(\omega_1, \ldots, \omega_n|\theta) G(d\theta)}{\int_{\gamma \leq \theta^*} f(\omega_1, \ldots, \omega_n|\theta) G(d\theta)}
\]

\[
\Rightarrow \quad \frac{G(\theta^*|\omega_1, \ldots, \omega_n)}{1 - G(\theta^*|\omega_1, \ldots, \omega_n)} \leq \frac{G(\theta^*|\overline{\omega}_1, \ldots, \overline{\omega}_n)}{1 - G(\theta^*|\overline{\omega}_1, \ldots, \overline{\omega}_n)}
\]

\[
\Rightarrow G(\theta^*|\omega_1, \ldots, \omega_n) \leq G(\theta^*|\overline{\omega}_1, \ldots, \overline{\omega}_n)
\]
as required.

The above proposition captures the notion of a multi-dimensional signal as either good or bad news. Since $\mathbb{R}^N$ is only partially ordered, there are many multi-dimensional signal comparisons for which this result remains silent. In addition, the result only holds for comparisons between signals of the same length. However, when both of these conditions are satisfied, we have established that $\omega_1 \geq \omega_2$ implies $G(\theta | \omega_1)$ first order stochastic dominates $G(\theta | \omega_2)$. The interpretation of the first order stochastic dominance relation as optimism, and hence $\omega_1 \geq \omega_2$ implying $\omega_1$ is good news relative to $\omega_2$, follows if the agent prefers high realizations of $\theta$.

The condition that $|H_1^R| = |H_2^R|$ is an important requirement of the result. As noted earlier, if the recollected histories are of different lengths, no clear stochastic ordering may be possible. To see this, consider the following simple example of updating a normal distributed prior with normally distributed signals.

**Example 3.** Consider prior beliefs $g(\theta) = N(0,1)$ and signals distributed as $\omega_i \sim N(\theta_0,1)$. Consider the two signal histories $H_1^R = (-1,1)$ and $H_2^R = (0.5)$ drawn from true history $H = (-1,0.5,1)$. The posterior beliefs are then $g(\theta | H_1^R) = N(0, \frac{1}{3})$ and $g(\theta | H_2^R) = N(\frac{1}{4}, \frac{1}{2})$. Although $g(\theta | H_2^R)$ has a mean than $g(\theta | H_1^R)$, the lower variance of $g(\theta | H_2^R)$ implies that neither a first nor a second order stochastic dominance relation can be established between these posteriors.

We now present the theorems for the theorems of section four

**Proof.** (Proof of Theorem 5) By proposition 11, we have that for $\omega_1, \omega_2 \in \mathbb{H}(H_N, n)$, $\omega_1 \geq \omega_2$ implies $G(\phi | \omega_1) \succ G(\phi | \omega_2)$. Therefore, $\int q(\theta)G(d\theta | \omega_1) \geq \int q(\theta)G(d\theta | \omega_2)$ as $q(\theta)$ is increasing in $\theta$. Since the distribution $\rho(\omega = H^R|\phi, H_N, n)$ is log-spms in $(\omega, \phi)$ and $\int q(\theta)G(d\theta | \omega)$ is increasing in each element of $\omega$, we have that the distribution of $q(\phi)$ contingent on $n$ is stochastically ordered by $\phi$ (Follows from Theorem 3.3.11 of [55]). Since the distribution of $n$ is independent of cue state, the unconditional distribution of $q(\phi)$ is ordered by $\phi$ in the strong stochastic order (so $q(\phi_1) \succ q(\phi_2)$).

Prior to providing a proof of Theorem 6, we will need to review some properties from the theory of monotone comparative statics. In the context of decision problems
under uncertainty, the ability to use monotone comparative statics theorems turns on showing that the distributions of the salient random variables obey the requisite log-spm properties. Let \( g(\theta|H^R) \) denote the probability density function of \( G(\theta|H^R) \). Now we will show that \( g(\theta|H^R) \) possesses the log-spm property required to apply Athey’s monotone comparative statics results.

**Lemma 4.** \( g(\theta|H^R, n) \) is log supermodular in \((\theta, H^R)\) where \( n \) is fixed.

**Proof.** Note that

\[
(A.8) \quad g(\theta|H^R, n) = g(\theta) \frac{1}{f(H^R)} \cdot f(H^R|\theta) = \frac{g(\theta)}{f(H^R)} \prod_{\omega \in H^R} f(\omega|\theta)
\]

where \( f(H^R) \) and \( f(H^R|\theta) \) denote the unconditional and conditional distributions of signals in a length \(|H^R| = n\) series. As log supermodularity is preserved by multiplication, this implies that \( \prod_{\omega \in H^R} f(\omega|\theta) \) is log supermodular in \((\theta, H^R)\). Also, \( g(\theta) \) and \( \frac{1}{f(H^R)} \) are separately and trivially log supermodular in \((\theta, H^R)\). Therefore, \( \frac{g(\theta)}{f(H^R)} \prod_{\omega \in H^R} f(\omega|\theta) \) is log supermodular in \((\theta, H^R)\). From the above equalities, we then have that \( g(\theta|H^R) \) is log supermodular in \((\theta, H^R)\).

As we have assumed the two-dimensional single crossing condition holds and have proven that \( g(\theta|H^R, n) \) is log supermodular in \((\theta, H^R)\) where \( n \) is fixed, it is straightforward to show using Athey’s monotone comparative statics results that for histories of fixed length, higher values recollected from memory lead to increasing choices.

**Lemma 5.** If \(|H^R| = |\widehat{H}^R|\) and \( H^R \geq \widehat{H}^R \), then we have \( x^*(H^R) \geq x^*(\widehat{H}^R) \)

**Proof.** This follows directly from [1] since we have, by assumption, \( u(x; \theta) \) obeys the single crossing property and shown in Lemma 4 that \( g(\theta|H^R) \) is log supermodular in \((\theta, H^R)\). Therefore \( x^*(H^R) \in \arg\max_{x} \int u(x; \theta)G(d\theta|H^R) \) is increasing in \( H^R \) element by element.

We can now provide a proof of Theorem 6
Proof. (Proof of Theorem 6) From Lemma 5 we have that $x^*(H^R)$ is an increasing function of $H^R$. Denote the random variable $H^R(\varphi)$ contingent on $|H^R| = n$ as $H^R(\varphi; n)$. Since $\rho(\overline{\varphi} = H^R|\varphi, H_N, n)$ is log-spm in $(\overline{\varphi}, \varphi)$, we have that $H^R(\varphi_1; n) \succ H^R(\varphi_2; n)$. Therefore, from Theorem 3.3.11 of [55], we have that $x^*(H^R(\varphi_1; n)) \succ x^*(H^R(\varphi_2; n))$. Since the length of recollected history is independent of $N$, we have that the same result holds for the unconditional distribution of $x^*(H^R(\varphi_1; n))$, so $x^*(H^R(\varphi_1)) \succ x^*(H^R(\varphi_2))$. \hfill \Box

A.4. Proofs from Section 5.1.

Proof. (Proof of Theorem 7) Note that since $f(y|e, \theta)$ is log-supermodular in $(y, e)$ and $(y, \theta)$, if $(e_1, \theta_1) \succeq (e_2, \theta_2)$, then the distribution of $y$ given $(e_1, \theta_1)$ strictly FOSD $y$ given $(e_2, \theta_2)$. Therefore, since $v(w(y))$ is a nondecreasing function, we have for $(e_1, \theta_1) \succ (e_2, \theta_2)$ that both

(i) $V(e_1, \theta_2) > V(e_2, \theta_2)$ and $V(e_1, \theta_1) > V(e_2, \theta_1)$

(ii) $V(e_2, \theta_1) > V(e_2, \theta_2)$ and $V(e_1, \theta_1) > V(e_1, \theta_2)$.

(i) implies the single crossing property in $(e, \theta)$ while (ii) implies the single crossing property in $(\theta, e)$. Therefore $V(e, \theta)$ has the two-dimensional single crossing property, which implies $V(e, \theta)$ is quasi-supermodular in $(e, \theta)$. From Milgrom and Shannon [51] this suffices to show that $e(\theta)$ is increasing in $\theta$. We then have from Theorem 6 that the distribution of efforts obeys $e(a_1) \succeq e(a_2)$. \hfill \Box

A.5. Proofs from Section 5.2.

A.5.1. Affective Dynamics. Our analysis begins by studying the intertemporal dynamics of affect as determined by the transition functions for a single agent’s affect. Each transition function, $f_G$ and $f_B$, generates a Harris chain over the space $([0, 1], \mathcal{B}([0, 1]))$ where $\mathcal{B}([0, 1])$ refers to the Borel sets on $[0, 1]$. The transition probabilities in the respective chains are then

\begin{align*}
F_G(u, A) & = \int_A f_G(u, v) dv \\
F_B(u, A) & = \int_A f_B(u, v) dv
\end{align*}

(A.9)
Note that we have implicitly assumed that \( f_G(u,v) \) and \( f_B(u,v) \) are Lebesgue measurable. We will use the notation \( F_x(u,A) \) with \( x \in \{ G, B \} \) in discussing the properties of these transition probability functions. We will denote the asymptotic transition probability function as \( F(u,A) \) with
\[
F(u,A) = 0.5 \ast F_G(u,A) + 0.5 \ast F_B(u,A)
\]
by definition.\(^{29}\) The following assumptions make the analysis more tractable, but are not crucial for any of the results below.

**Assumption 12.** \( f_G(u,v) \in (0,1] \) if and only if \( u \leq v \)

**Assumption 13.** \( f_G(u,v) = f_B(1-u,1-v) \)

The transition probability function \( F_x(u,A) \) defines a Markov operator \( T_x \) over the space \( B([0,1]) \) of bounded, Lebesgue measurable functions on \([0,1]\) of the form
\[
(T_x h)(u) = \int h(v) f_x(u,v) dv
\]
The adjoints of these operators, denoted
\[
(T_x^*) : \Delta([0,1]) \to \Delta([0,1])
\]
where \( \Delta(S) \) denotes the set of probability distributions on \( S \), is defined by
\[
(T_x^* \lambda)(A) = \int F_x(u,A) \lambda(du) = \int [ \int A f_x(u,v) dv ] \lambda(du)
\]
Note that our assumptions regarding the continuity of \( f_G, f_B \) imply that \( T_G^*, T_B^* \) are continuous in the Total Variation Norm. The long-run transition probability function generates the Markov operator
\[
(T^* \lambda)(A) = 0.5 \ast (T_B^* \lambda)(A) + 0.5 \ast (T_G^* \lambda)(A)
\]
We will now show that the above operator is stable and obeys Doeblin’s condition.

\(^{29}\)In order to properly define the invariant transition operator, we need to use the space \((\{0,1\} \times \{-1,+1\}, B([0,1] \times \{B,G\}))\) where the set \( \{B,G\} \) reflects the states that determine the autocorrelation of the dividend process generating the use of transition function \( F_X \). This has a proper invariant distribution, and throughout this work we refer to the marginal invariant distribution over \([0,1]\) as the ergodic distribution of \( F \).
Definition 4. The operator $T$ is stable if $Th$ is continuous and bounded whenever $h$ is bounded and continuous.

It is obvious that the operator $T$ induced by our transition probability functions is stable.

Definition 5. The operator $T$ induced by transition probability function $F(u, A)$ satisfies Doeblin’s condition if there exists some probability measure $\lambda$, an integer $n$, and an $\varepsilon \in (0, 1)$ such that if $A \subseteq [0, 1]$ and $\lambda(A) \leq \varepsilon$, then $F^n(u, A) \leq 1 - \varepsilon$ for all $u \in [0, 1]$ where $F^n_x(u, A)$ refers to the $n$-fold iterator on the transition probability function.

Lemma 6. $F(u, A)$ obeys Doeblin’s condition for each $x$

Proof. Noting that $f(u, v) = 0.5 * f_B(u, v) + 0.5 * f_G(u, v) > 0$ for all $(u, v)$, let $m = \max_{(u,v)\in[0,1]^2} f(u, v)$. As the space $[0, 1]^2$ is compact, the maximum is well defined. Let $\lambda$ be the usual Lebesgue measure, $\varepsilon \in (0, \frac{1}{m+1})$, and $n = 1$. Then we have for all $u \in [0, 1]$ and $A$ such that $\lambda(A) \leq \varepsilon$ that $F(u, A) \leq m \cdot \lambda(A) \leq m \cdot \varepsilon$. Noting that $m < \frac{1}{\varepsilon}$ yields $F(u, A) \leq 1 - \varepsilon$ as required. \hfill \qed

Futia ([28], theorems 4.9, 3.3, and 2.12) proves that if $T$ is stable and obeys Doeblin’s condition, then $T$ is quasi-compact and equicontinuous. This implies that there exists a unique invariant distribution $\tilde{\psi}$ such that $T^* \tilde{\psi} = \tilde{\psi}$. Note that by inspection the invariant distributions of $T^*_G$ and $T^*_B$ are $\delta_1$ and $\delta_0$ where $\delta_x$ refers to the measure that places a unit mass at $X \in [0, 1]$. In order to obtain our results, we will require the following monotonicity assumption on the adjoint operators induced by the Harris Chain. Assumptions 12 and 13 are sufficient for this property to hold.

Assumption 14. Suppose $\psi \succ \lambda$. Then we have $T^*_G \psi \succ T^*_G \lambda$ and $T^*_B \psi \succ T^*_B \lambda$

A.5.2. Proofs of Price Time Series Effects.

Proof. (of Theorem 9) In order to prove this theorem, we must compute the distribution $P^M_{t+1}(\psi_{t+1}) - P^M_t(\psi_t)$ conditional on $\{\eta_t \geq 0\}$ and $\{\eta_t < 0\}$. The theorem assumes we start at the steady state distribution, so $\psi_{t-1} = \tilde{\psi}$. In the case $\{\eta_t \geq 0\}$
we have $\psi_t^G = T_G^*\tilde{\psi}$ where $\psi_t^G(A) = \int_0^1 [\int_A f_G(u,v)dv]\tilde{\psi}(du)$. We then have the expected distribution over affective states at $t+1$ equal to

$$\psi_{t+1}^G = \rho * T_G^*\psi_t^G + (1-\rho) * T_B^*\psi_t^G = \rho * T_G^*T_G^*\tilde{\psi} + (1-\rho) * T_B^*T_B^*\tilde{\psi}$$

(A.15)

Note also that

$$T_G^*T_G^*\tilde{\psi} = T_B^*T_B^*\tilde{\psi} = T_G^*T_G^*\tilde{\psi} + \frac{1}{2}T_B^*T_B^*\tilde{\psi}$$

(A.16)

Therefore we have

$$\psi_t^G = T_G^*\tilde{\psi} = T_G^*T_G^*\tilde{\psi} = \frac{1}{2}T_G^*T_G^*\tilde{\psi} + \frac{1}{2}T_B^*T_B^*\tilde{\psi}$$

(A.17)

It is obvious that $T_G^*T_G^*\tilde{\psi} \succ T_B^*T_B^*\tilde{\psi}$. If $T_B^*T_G^*\tilde{\psi} \succ T_G^*T_B^*\tilde{\psi}$, then the above relation holds for any $\rho \geq \frac{1}{2}$. However, generically this will not be the case. But note that if $\rho = 1$ this equation reduces to

$$E_t^*\left[P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t)|\eta_t \geq 0\right] = E_t^*\left[\frac{(1 + \gamma + r)}{2} \varepsilon_{t+1} + \Lambda(\psi_{t+1}) - \Lambda(\psi_t)|\eta_t \geq 0\right]$$

$$= \rho \Lambda(T_G^*T_G^*\tilde{\psi}) + (1-\rho) \Lambda(T_B^*T_B^*\tilde{\psi}) - \frac{1}{2} \Lambda(T_G^*T_G^*\tilde{\psi}) - \frac{1}{2} \Lambda(T_B^*T_B^*\tilde{\psi})$$

(A.18)

Therefore, by the continuity of $E_t^*\left[P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t)|\eta_t \geq 0\right]$ in terms of $\rho$, there exists $\bar{\rho} \in [\frac{1}{2}, 1)$ such that for any $\rho \in (\bar{\rho}, 1]$ we have $E_t^*\left[P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t)|\eta_t \geq 0\right] < 0$.

Symmetric algebra can show that for sufficiently large $\rho$, we have $E_t^*\left[P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t)|\eta_t \leq 0\right] < 0$. Therefore

$$E_t^*\left[P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t)|\eta_t \geq 0\right] > E_t^*\left[P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t)|\eta_t < 0\right]$$

(A.19)

\[\square\]

30In the analysis below, we implicitly use the fact that $E_t^*[\varepsilon_{t+1}] = 0.$
Proof. (of Theorem 10) As in the proof of Theorem 15 above, we need to derive the distribution of

\[(A.20) \quad P_{t+1}(\psi_{t+1}) = \frac{(1 + \gamma + r)}{r^2} \cdot \varepsilon_{t+1} + A(\psi_{t+1}) - A(\psi_t)\]

contingent on \(\{\eta_t \geq 0, \ldots, \eta_{t-N} \geq 0\}\) and \(\{\eta_t \leq 0, \ldots, \eta_{t-N} \leq 0\}\). We will show the following stochastic ordering holds

\[E_t[P_{t+1}(\psi_{t+1})|\{\eta_t \geq 0, \ldots, \eta_{t-N} \geq 0\}] < P_{t+1}(\psi_t) \quad \text{(Overreaction)}\]

\[< E_t[P_{t+1}(\psi_{t+1})|\{\eta_t \leq 0, \ldots, \eta_{t-N} \leq 0\}]\]

Let \(\tilde{\psi}\) denote the invariant distribution of affective state. Note that

\[(A.21) \quad E_t[\psi_{t+1}|\{\eta_t \geq 0, \ldots, \eta_{t-N} \geq 0\}] = [\rho(T_G)^{N+1} + (1 - \rho)T_B(T_G)^N]\tilde{\psi}\]

We can then rewrite the first of our Overreaction formulas

\[(A.22) \quad E_t[P_{t+1}(\psi_{t+1})|\{\eta_t \geq 0, \ldots, \eta_{t-N} \geq 0\}] < P_{t+1}(\psi_t)\]

in the following form

\[\rho \cdot \{P_{t+1}((T_G)^{N+1}\tilde{\psi}) - P_{t+1}((T_G)^N\tilde{\psi})\} < \]

\[\quad (1 - \rho) \cdot \{P_{t+1}((T_G)^N\tilde{\psi}) - P_{t+1}(T_B(T_G)^N\tilde{\psi})\}\]

Note that \((T_G)^N\tilde{\psi} \rightharpoonup \delta_1\) in the Total Variation Norm, \(d_{TV,N}\), as \(N \to \infty\). From our assumption of the continuity of \(P_{t+1}\), this implies that for any \(\varepsilon > 0\) we can find \(N^* > 0\) such that for all \(N > N^*\) we have

\[(A.23) \quad P_{t+1}((T_G)^{N+1}\tilde{\psi}) - P_{t+1}((T_G)^N\tilde{\psi}) \in [0, \varepsilon)\]

Note that the full support conditions of Assumption 12 and 13 implies that there exists \(\gamma > 0\) such that \(d_{TV,N}(T_B\delta_1, \delta_1), d_{TV,N}(T_G\delta_0, \delta_0) > \gamma\). When combined with the continuity of \(T_B\) and \(P_{t+1}\) are continuous, we can conclude that there exists \(N^*\) such that for \(N > N^*\) we have both

\[P_{t+1}((T_G)^{N+1}\tilde{\psi}) - P_{t+1}((T_G)^N\tilde{\psi}) \in [0, \varepsilon \cdot \frac{1 - \rho}{\rho})\]

\[P_{t+1}((T_G)^N\tilde{\psi}) - P_{t+1}(T_B(T_G)^N\tilde{\psi}) > \varepsilon\]
Together these imply that relation (*) holds.

Now consider the second half of the overreaction relation
\[(A.26) \quad P_{t+1}^M(\psi_t) > E_t^*[P_{t+1}^M(\psi_{t+1}) | \{\eta_t \leq 0, \ldots, \eta_{t-N} \leq 0\}]\]

Note that
\[(A.27) \quad E_t^*[\psi_{t+1} | \{\eta_t \leq 0, \ldots, \eta_{t-N} \leq 0\}] = [\rho(T_B)^{N+1} + (1 - \rho)T_G(T_B)^N] \tilde{\psi}\]

As before \((T_B)^N \tilde{\psi} \to \delta_0\) as \(N \to \infty\). From symmetric logic to the argument above, we can show that the following relation holds
\[(**) \quad \rho \ast \{P_{t+1}^M[(T_B)^{N+1} \tilde{\psi}] - P_{t+1}^M[(T_B)^N \tilde{\psi}]\} > (1 - \rho) \ast \{P_{t+1}^M[(T_B)^N \tilde{\psi}] - P_{t+1}^M[T_G(T_B)^N] \tilde{\psi}\}\]

which in turn implies that the second overreaction inequality holds. Putting together the overreaction inequalities, we have
\[(A.28) \quad E_t^*[P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t) | \eta_t \geq 0, \ldots, \eta_{t-N} \geq 0] < E_t^*[P_{t+1}^M(\psi_{t+1}) - P_t^M(\psi_t) | \eta_t \leq 0, \ldots, \eta_{t-N} \leq 0] \]

as required.

\[\Box\]

**Appendix B. Extensions**

**B.1. Cues.** Although we have emphasized the use of affect and disposition as cues in the applications above, the model allows for general cues to be studied. One interesting extension is to treat the valence of the information contained in public signals as a cue in addition to or instead of the indirect cueing through influence on agent affect. In our asset pricing example, positive or negative public signals of stock performance might serve as cues for similarly valenced performance announcements in the past. The cues would have the effect not only of influencing the rational updates of agents with perfect recall, but of causing correlated recollections amongst the agents in the market. Good news can become self-enforcing as the asset holders not only use the announcement to positively update their assessment of the asset’s future performance but are prone to recollect positive information already announced, further increasing their assessment of the security’s potential future performance.
B.2. **Dynamics.** Suppose that cues are determined by a Markov process with transition probabilities taken as a function of the current cue and decision outcomes. In this case we could study the average long-run bias in the population and the durability of these biases over time under various market structures. The finance application above is an example of such a project. Other promising areas for future research include studies of the time series behavior of consumption streams, household savings patterns, and prices for consumption goods.

B.3. **Confirmation Bias and Attention.** An alternative interpretation of our model is that the agent is presented with a menu of information $H_N$ and stochastically attends to a subset of the information $H^R$. If the selection of attended information is biased, then the subsequent judgements will be biased as well. By studying the direction of these biases, we can use our framework to make predictions regarding agent beliefs and choices.

An example of attention cueing is confirmatory bias (see Rabin and Schrag [57] for a review of the psychology literature). Suppose the agent entertains a theory about the relation between variables $X$ and $Y$. The agent’s theory cues attention towards phenomena predicted by the theory relative to other data. By focusing attention on data that is supportive of the theory, the attention bias amplifies agent belief in the theory’s veracity. Not only would an agent become more confident in his or her theory when given mixed evidence, but agents holding different theories would become more certain in their different theories as evidence accumulated.\(^{31}\)

The Rabin and Schrag model treats confirmation bias as a fault in the interpretation of signals that are informative about the state of the world. Rabin et al. analyze a benchmark case of a sophisticated Bayesian observer who attempts to infer the state of the world from the signals interpreted and reported by a naive, confirmation biased agent. A two state, two signal structure is assumed to render the analysis of the unbiased Bayesian observer’s beliefs tractable. Our model has the benefit of applying to generic decision problems and having a readily analyzed

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To formalize this argument, we would need to describe a memory process wherein agents recollect an asymptotically infinite set of data as the true history of signals becomes infinite in size. Otherwise, agent beliefs could not converge to absolute confidence in the asymptotic limit. This represents an interesting extension for future work.
benchmark of an agent that has unbiased, stochastic attention.\textsuperscript{32} The Rabin et al. results, while proven in their specialized context, do not have obvious generalizations to other contexts such as larger hypothesis sets or different signal structures.

B.4. **Priming.** Priming refers to the elicitation of behavioral schema by the presence of cues in the environment. If one assumes that cues can selectively elicit information and that this information has an effect on the decisions of the agents acting within the environment, then our cued recall model can integrate these priming effects into traditional economic models. For example, Bargh et al. \cite{4} are able to prime aggressive behavior by presenting experimental subjects with subconscious displays of African-American faces. Although the authors believe this cue to be unwarranted and maladaptive, it is possible that subjects associate African-Americans with dangerous environments through a process of unconscious acculturation, and these cues for a dangerous environs in turn elicits information from memory supportive of aggressive behavior. Our model predicts that primes influence beliefs that in turn influence behaviors. The psychology literature typically hypothesizes a direct, automatic link between priming and behavior. Our information theory of priming can be distinguished from the automatic channel by using score functions designed to truthfully elicit agent beliefs and testing whether the primes influence beliefs as well as actions.

**Appendix C. Welfare Issues**

The author does not take a stand on the origin or adaptivity of the relationship between affective state and belief. From the perspective of an agent with imperfect memory and preferences only over choice outcomes, failing to recollect information is welfare decreasing. Therefore, a strict welfare benchmark is the choices made by an agent with perfect recollection. However, this standard conflates two effects of our model. First, given that the agent recalls a subset of the information observed prior to his decision, even recall unbiased by cues will be suboptimal relative to an agent with perfect recall. Second, given that the content of the recollected information is

\textsuperscript{32}Our benchmark is an unbiased, naive Bayesian observer. While the comparison with a sophisticated Bayesian observer is of theoretical interest, from a positive perspective it is not clear whom the sophisticated observer is meant to describe.
biased by the environmental cues, this bias can have a further welfare reducing effect. In order to isolate the welfare impact of the cued associations, our benchmark is an agent with imperfect, unbiased recall, which can be specified in our model by letting \( \rho_\omega(\omega_i | \varphi, H_N, n) = 1 \).

The psychology literature is energetically studying the potential functions of affective state in the decision process and how neurophysiological factors link the affective and decisive aspects of cognition. Our work has taken the bias in memory as a stylized fact and maps out the economic consequences of it. However, an economic model may provide an adaptive justification for the bias in terms of economic efficiency. We will briefly sketch two possible approaches below, leaving the details to be completed in later work.

C.1. **Effortful Recollection.** One adaptive explanation for an associative memory bias is that cued memory is a response to a bounded rationality constraint. If forming beliefs from recollections requires effort or time, it would be in the agent’s interest to have associative links in memory that selectively prompts the recollection of relevant information. In many cases, cues from the decision environment may generate only minimally biased recollections. However, if the cues generate significant biases, the memory effects may have a correspondingly significant negative welfare impact.

As discussed in our application to employee morale, the potential for an actor to influence the cues faced by another agent suggests that although associative-memory may have once been adaptive, strategic manipulations of the memory process could render this feature of memory maladaptive in some modern settings.

A natural extension of our model would make the recollection process a function of both environmental cues and an effort choice by the agent. In this expanded model, the welfare status of these biases becomes ambiguous. If agents may choose to enhance recall by exerting costly effort, then it may be optimal to trade-off between the cost of effort and the benefit of further debiasing the recollection process. In order to validate any model of effortful recollection, one would have to conduct a careful study of the determinants of mental effort in memory recall problems.

Models of bounded rationality that incorporate a metadecision problem of the agent regarding how much effort to exert in making a decision suffer from an infinite
regression of decision problems - deciding how to decide how to decide ad infinitum (see Lipman [45]). A more useful exercise would be to make the pragmatic assumption that agents are able to solve these second order problems perfectly and then determine comparative statics between environmental factors that would make recollection more costly, the impact of these costs on the biases induced, and the resulting effects on decision behavior. This would allow us to further analyze how actors manipulate each other’s recollections in order to amplify or alleviate these biases and suggest welfare enhancing policy interventions.

C.2. Preferences over Affect. Consider the following stylized facts:

1. Agents prefer positive affective states over negative states 
2. Biased beliefs occur primarily in positive affective states

Bower [12] has suggested, using a network model of mood and memory, that recollecting positive memories encourages activation of positive mood states. Therefore, thinking about positive memories encourages maintenance of a positive mood, whereas recollecting negative memories encourages the evocation of a negative mood. This suggests that the agents face a trade-off between biasing memory towards the recollection of positive events in order to support a preferred positive mood and the bias this will induce in the agent’s beliefs. Fully studying this explanation requires taking a stronger stance on agent preferences over affective states and is beyond the scope of this work.

C.3. Partial Cueing (Originally Section 4.3). In this section we develop a model wherein agents are heterogeneous in the degree of influence that associative memory cues have on their beliefs and resulting behavior. We assume that agent beliefs are a convex combination of a belief generated by the cued memory process and a benchmark belief. A natural benchmark is the beliefs generated by an unbiased stochastic recollection process, \( \rho(\omega_i|\varphi = 0, H_N, n) = 1 \), that samples all data in memory with equal probability.

Let \( \alpha \in [0,1] \) denote the degree of cue influence on recollection, where \( \alpha = 0 \) indicates beliefs determined by the benchmark, unbiased beliefs and \( \alpha = 1 \) denotes beliefs determined by the cued memory process. Define the recall probability recall
functions given cue $\varphi$, length of memory $n$, and degree of cue influence $\alpha$ as

$$
\rho_\omega(\omega|\varphi, \alpha, H_N, n) = \rho_\omega(\omega|\varphi, H_N, n)^{\alpha} \ast \rho_\omega(\omega|\varphi = 0, H_N, n)^{1-\alpha}
$$

$$
\rho(H^R|\varphi, \alpha, H_N, n) \propto \prod_{\omega \in H^R} \rho_\omega(\omega|\varphi, H_N, n)^{\alpha} \ast \prod_{Y \in H_N \backslash H^R} 1 - \rho_\omega(Y|\varphi, H_N, n)^{\alpha}
$$

Contingent on $\varphi > 0$, higher values of $\alpha$ increase the bias of the recalled data towards sampling more positive data from memory. Conversely, if $\varphi < 0$, the relative sampling probability functions of an agent with cued memory are increasingly biased towards recalling negative information from memory as $\alpha$ increases.

Mathematically, the log supermodularity of $\rho_\omega(\omega|\varphi, \alpha, H_N, n)$ in $(\omega, \alpha)$ contingent on $\varphi > 0$ has the same lattice properties as the log supermodularity of $\rho_\omega(\omega|\varphi, \alpha, H_N, n)$ with respect to $(\omega, \varphi)$. Therefore, the comparative statics results proven earlier relating the orderings of beliefs and choices to changes in $\varphi$ hold with respect to $\alpha$ contingent on $\varphi > 0$. Conversely, the lattice properties of $\rho_\omega(\omega|\varphi, \alpha, H_N, n)$ in $(\omega, -\alpha)$ contingent on $\varphi \leq 0$ are formally identical to the lattice properties of $\rho_\omega(\omega|\varphi, \alpha, H_N, n)$ with respect to $(\omega, \varphi)$. Hence, contingent on $\varphi \leq 0$, the orderings of beliefs and choices in the strong stochastic order with respect to $\varphi$ can be replicated for $-\alpha$.

The analog of our comparative statics results for beliefs (Theorem 5) and choices (Theorem 6) to the degree of cue influence on memory are presented in the two theorems below.\textsuperscript{33} Let $H^R_\alpha(\varphi)$ be a random variable denoting the history of data recollected under cue state $\varphi$ and degree of cue influence $\alpha$.

**Theorem 15.** Let $q^\alpha(\varphi) = \int q(\theta)G(d\theta|H^R_\alpha(\varphi))$. If $\varphi \geq 0$ and $1 \geq \alpha \geq \beta \geq 0$, then $q^\alpha(\varphi) \geq q^\beta(\varphi)$. Conversely, if $\varphi \leq 0$ and $1 \geq \alpha \geq \beta \geq 0$, then $q^\alpha(\varphi) \leq q^\beta(\varphi)$.

**Theorem 16.** Assume that $u(x; \theta)$ satisfies the two-dimensional single crossing condition. If $\varphi \geq 0$ and $1 \geq \alpha \geq \beta \geq 0$, then $x^*(H^R_\alpha(\varphi)) \geq x^*(H^R_\beta(\varphi))$. Conversely, if $\varphi \leq 0$ and $1 \geq \alpha \geq \beta \geq 0$, then $x^*(H^R_\alpha(\varphi)) \leq x^*(H^R_\beta(\varphi))$.

\textsuperscript{33}Given the similarity of the proof techniques to earlier results in this study, the proofs are omitted for brevity. Proofs of these results can be obtained from the author.
By increasing $\alpha$ we monotonically move the model predictions regarding the beliefs and actions of an agent with imperfect, unbiased memory towards the predictions for a biased agent. In cases where the biased agent holds optimistic beliefs about $\theta$ relative to an agent with an unbiased recall process, an agent with a lower value of $\alpha$ will hold beliefs that are less optimistic (but still more optimistic than an agent with unbiased beliefs). Conversely, if the biased agents are pessimistic in their beliefs about $\theta$ relative to an agent with an unbiased recall process, then an agent with a lower $\alpha$ will hold less pessimistic beliefs about $\theta$.

In practical applications, agents with differing degrees of bias could be sorted by firms using signaling or screening techniques. For example, advertisers may employ different marketing techniques depending on the bias level of the target audience. Principals may want to screen agents for low levels of bias if they rely on the agents to make informed task choices. On the other hand, principals in our employee morale application may want to employ agents who suffer from severe memory biases if this allows the principals to provide morale based incentives to the agent more effectively. Finally, demographic factors such as market experience may be correlated with the degree of cue influence on recall. If experience reduces the cued memory biases of the agent, one could model experts as having lower values of $\alpha$ than the general population. If experienced agents can be identified from market data and reliable cues can be found in the environment, this provides an opportunity to test the predictions of our theory.