DECISION UNDER SEVERAL OBJECTIVES

by

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ABSTRACT

In this paper the author formulates and develops solutions of problems encountered in endogenous treatment of environmental aspects in an economic model. These are:

(1) an aggregated environmental utility measure,

(2) a method for achieving a compromise between economic and environmental goals, and

(3) providing for feedback of abatement "industries" (activities) to the rest of the economy.

Problems (1) and (2) are related to decision making under several objectives.

The methodology is developed for an environmental utility function and demonstrated with an illustrative aggregate hazard (air pollution) index for each energy source of fossil power stations and nuclear power stations. The aggregate hazard index is the sum of the hazard indexes of various pollutants, computed from specific emissions and air quality norms for the pollutant.

A compromise between economic and environmental goals is achieved by adding a constraint that provides a Pareto optimum type solution which best fulfills both goals as it requires equal degrees of satisfaction of an economist and an environmentalist.

An environmental matrix and an environmental flow model for the PILOT energy/economic model are presented which demonstrate abatement activities and how the interaction of industries improves or weakens the environmental situation. Consumer pollution is attached to the industry where it is emitted. Abatement activities are subdivided according to

high and low abatement measures with respect to quota. Results depend on the level of environmental detail at which the environmental sector is modeled. However, major changes in a model's energy supply picture are expected to result with endogenous treatment of environmental aspects.

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1. The Problem

The energy crises has brought about much modeling work regarding the interrelationships of energy and the economy. One of these to which this paper will refer as an example is PILOT [1], a linear programming optimization model based on a multisector input-output structure. Up to now, covering eight, five year periods, it maximizes individual consumption subject to capacity growth constraints for single industries and technology substitution options in various energy fields. It mainly models the feedback of different energy scenarios to the economy, but also the energy demand is subject to endogenous considerations to some extent; only pure price effects on the specific energy needs of industrial processes or of the consumer still are not endogenous.

It is obvious that energy production and consumption influence not only the economy but also the environment, a field of recent public interest. So a model also should take into account the environmental situation. As a first step, a model like PILOT can pay regard to environment policy restrictions by incorporating those abatement measures within the technical coefficients of an input-output structure which meet the policy restrictions. The detailed measures could be derived from disaggregated site-specific models. Here, the treatment of environmental aspects remains exogenous. What is wanted, however, is an endogenous treatment of the environmental aspects in an economic model. To provide for this, the following problems arise:

to find a utility measure for the environmental situation, (1.1)

to provide for abatement industries regarding their feedback to the rest of the economy, (1.2)

and to establish rules, how to accomplish a good economic and a good environmental result in a model optimization. (1.3)

Problem (1.2) presents no difficulties which have not arisen already in the economic models. It will be addressed in Chapter 4. Problems (1.1) and (1.3) are the main problems which this paper will address. They will be treated in Chapters 2 and 3 and are both related to the question of decision making under several objectives. Though continuously referring to the energy model task, the stated decision problem shall be presented in a more generalized way.

Problem (1.1) as well as problem (1.3) addresses the problem of combining different aspects in a way which allows an overall assessment statement. In the case of problem (1.1), various environmental effects should be combined to an overall measure of the environmental situation. In the case of problem (1.3), the different goal functions should be combined to find the most preferred feasible solution of the model, which we will call a good solution. More generally stated, both problems can be expressed as follows:

Let X denote a set of feasible solutions of a model. We will call those feasible solutions with respect to our problem options. Let k be an assessment aspect and $u_k(x)$, $x \in X$, be a corresponding utility measure which produces a complete, transitive ordering on X regarding k, denoted as a k-ordering. There may exist several assessment aspects, e.g., the amount of imported oil or the probability for some kind of disease caused by pollutants. The utility functions of those different aspects may induce different k-orderings on X. If all assessment aspects which are relevant for a total assessment in X form a set K, then we get an assessment vector $(u_k(x) | k \in K)$ for each $x \in X$. The general statement of our problem now is:

to find a mapping z of these vectors to a strongly ordered set such that it provides a K-ordering on X (that is, one complete, transitive ordering on X regarding all $k \in K$ simultaneously). (1.4)

Let K_i be a partition of K, $i = 1,2,\ldots$, such that K_i denotes a more general goal, e.g., the economic prosperity or man made environmental pollution. For convenience let us gather the goals K_i in G and define an index set I which allows us to distinguish the goals in G. We have two similar problems on different aggregation levels of K if we break down z

into a mapping z_1^i of the K_i restricted vectors $(u_k(x)|k \in K_i)$, $x \in X$, to produce a K_i-ordering on X, $i \in I$, (1.4.1)

and, denoting $f_i(x) = z_1^i(u_k(x)|k \in K_i)$, a mapping z_2 of the vectors $(f_i(x)|i \in I)$, $x \in X$, to produce a K-ordering on X. (1.4.2)

Though problems (1.4.1) and (1.4.2) have the same structure, the concept of solving them will be quite different.

The reason is, first, that all $k \in K_i$ are noncontroversial for people supporting goal K_i; that is, with $K_i = \{k_1, k_2\}$ it should be possible for those people to find a goal function $f_i(u_{k_1}(x), u_{k_2}(x))$ which meets for each of them the assessment of the situation with respect to all $k \in K_i$. Let $i = 1$ represent the economic aspects. We know that PILOT assumes $f_1(x)$ to be the consumption. Let $i = 2$ represent the environmental aspects. To perform (1.4.1) for the environmental aspects is the essence of problem (1.1) and will be treated in Chapter 3.

On the other hand, let all $K_{i_1}, K_{i_2} \in G$, be chosen such that they represent controversial goals; that is, there is no i_1, $i_2 \in I$, $i_1 \neq i_2$, where the optimum of K_{i_1} and K_{i_2} can be met simultaneously. This may be evident if, e.g., K_1 represents the economic and K_2 the environmental goals in X. It will be shown in Chapter 2 for this case that it is not advisable to establish an overall goal function, but to treat the problem (1.4.2) differently. This treatment covers problem (1.3).

It should be noted that the breakdown of (1.4) into (1.4.1) and (1.4.2) is quite natural in the following sense. Problem (1.4) generally requires too many individual judgments to be processed analytically. On the other hand, it is too complex to be treated democratically. So we want (1.4.1) to be an analytical process with as few judgmental aspects as possible, with the goal to reduce the complexity of (1.4). Consequently (1.4.2) then should be a political process, with as few technocratic mechanisms as possible [2]. The author hopes for a move in the latter direction. However, as adequate, democratically legitimized structures to perform (1.4.2) along these lines still are missing, we searched for a method which the decision maker can apply to solve (1.4.2), using a second best approach, i.e., an analytical procedure. This is addressed in Chapter 2.

2. Tradeoffs Between Competing Goals

The first two sections of this chapter will be addressed to the restricted problem (1.3). In order to show the usefulness of the concept, the arguments will follow closely the conditions PILOT provides. It will be shown that a compromise between the economic and the environmental goals can be achieved by adding one constraint to the economic PILOT version. This constraint provides for a Pareto optimum type solution and best fulfills both goals. The new constraint requires equal degrees of satisfaction of an economist and an environmentalist.

The third section of this chapter will generalize the proposed methodology and gives a detailed mathematical treatment of problem (1.4.2).

2.1. The Decision Maker's Problem

Most planning models addressing the energy and economy question, including PILOT, are chiefly nationally aggregated models; they cannot conveniently include site-specific conditions. Thus it is not useful to include constraints regarding maximal acceptable burdens which would have to be compared with site-specific ones. Even if models could be developed which are capable of doing this, one should realize that the politically agreed upon environmental quality norms may not be thresholds with respect to undesirable environmental effects. That is to say, the acceptability of meeting environmental quality norms may vary with the utility of the related economic situation. So one might put up with the fact that a model disregards site-specific conditions if a way is found which enables one to compare the economic with the environmental situation generally.

Assuming one could minimize a disutility with respect to the environment, this would lower consumption. Let us assume there is an economist who wants to maximize consumption and an environmentalist who wants to minimize pollution, and the two resulting optimal solutions are different. The question is how a decision maker resolves this situation when the decision maker knows no authority which could silence the economist as well as the environmentalist and then impose his own goal function. Therefore, all the common methods with which one tries to elaborate on the hidden values of persons fail to be applicable. The economist and the environmentalist each represents a pressure group and the decision maker expects to be blamed by at least one of them. The decision maker assumes that a pressure group will not blame him if its optimum is met, but it will blame him maximally if the optimum of the competing pressure group is met. Let us assume further that the decision maker has established between those limits a measure for the degree of dissatisfaction of each pressure group, depending on the chosen compromise between economic and environmental goals, and that he expects the degree of blame by each pressure group will be proportional to the degree of dissatisfaction of that pressure group, i.e., the decision maker regards his blame to be the degree of dissatisfaction of any of those pressure groups that he wanted to satisfy.

There are two ways now to approach a solution of the decision maker's problem:

One way is for the decision maker to regard it as his duty not simply to minimize his blame but to search for a solution which minimizes the dissatisfaction for each of the pressure groups; that is, to choose a realization of the economy and the environment such that the larger of the two pressure groups' degree of dissatisfaction is as low as possible. (2.1.1)

The other way to approach the problem is for the decision maker to try to minimize the blame he expects to receive but assumes there is a third party who may play off the dissatisfaction of the economy group and the environment group, i.e., the third party could be a holding company of both pressure groups, which supports its members in their attempts to persuade the decision maker. The third party does this by manipulating his influence so as to maximize the blame. The decision maker does not know the weightings of the goal functions, or the power behind the individual groups. (2.1.2)

In the first approach, it is relatively easy to set down extra conditions for a model that would yield a solution. The second approach can be attacked using the theory of games. It will be shown that both approaches yield the same solution and guarantee a Pareto optimum with respect to the degrees of dissatisfaction, i.e., there exists no other realization of the economic and the environmental goals which would lower the degree of dissatisfaction for one of the pressure groups without increasing the degree of dissatisfaction of the other. Any realization of the economic and environmental goals as established along the above lines, when there is no information about the relative importance of either goal, will be called a compromise.

The best insight in either approach is obtained by treating now a simplified game situation.

2.2. A Game with Linear Payoff

With respect to a model like PILOT, let us assume now (2.2.1)
that feasible solutions of one scenario, represented by constant technological conditions and constant consumption patterns, are described by an n-dimensional (closed, bounded) convex polyhedron, say, X. Let f_1 and f_2 be utility functions corresponding to the economic and environmental goal, respectively. To avoid future confusion, we will substitute f_i by g_i if we want to represent disutilities. So let f_1 be a measure of the economic situation, representing a linear utility function, and let g_2 be a measure of the environmental situation, representing a linear disutility function. (The case of nonlinear functions f and g will be treated in Section 2.3.) Where no confusion can occur, we will omit the subscripts and take f for f_1 and g for g_2.

Assuming $f^* = f(x^*)$ and $g_* = g(x^*)$ if x^* maximizes (2.2.2)
$f(x)$ subject to $x \in X$, correspondingly $f_* = f(x_*)$ and $g^* = g(x_*)$ if x_* minimizes $g(x)$ subject to $x \in X$, then

$$r_f(x) = \frac{f(x) - f^*}{f_* - f^*} \quad , \qquad\qquad r_g(x) = \frac{g(x) - g^*}{g_* - g^*}$$

give measures of the previously discussed degrees of dissatisfaction in comparable figures. Now $r_i(x)$, $i \in \{f,g\}$, represents the dissatisfaction degree with respect to goal K_i if the feasible solution x, $x \in X$, is chosen.

To obtain a compromise solution with respect to the problem of minimizing in some sense the vector $(r_f(x), r_g(x))$ subject to $x \in X$ and to show how to deal with the problem within the PILOT linear program structure, a game is defined as follows:

Let X' represent the set of extreme points of X and J the corresponding indexset. Note that for any $x \in X$ with $x \notin X'$ there exists a $\eta = (\eta_j | j \in J)$ with $\eta_j \geq 0$ and $\sum_{j \in J} \eta_j = 1$ such that $x = \sum_{j \in J} \eta_j \cdot x^j$ with $x^j \in X'$. All those η, covering all $x \in X$, we unite in $\mathcal{H}$. Now the decision maker is treated as a column player controlling η. So in fact the decision maker is able to choose any $x \in X$. (2.2.3)

The third party or holding company of the economist and the environmentalist acts as the row player and controls $\xi = (\xi_f, \xi_g)$ with ξ_f, $\xi_g \geq 0$ and $\xi_f + \xi_g = 1$. All those ξ we unite in Ξ. ξ_f and ξ_g can be interpreted to be a weighting of the goal K_i. But some caution must be applied to this interpretation, as explained in Section 2.3.1.

$R = [r_i(x^j)]$, $i \in \{f,g\}$, $j \in J$, is the payoff matrix for the decision maker. He receives a blame $r_i(x^j)$ if he chooses $\eta_j = 1$ and goal K_i applies. A zero sum situation is assumed.

As will be shown in the generalized concept of Section 2.3, R provides for a mixed good strategy for the row player with $\bar{\xi}_f, \bar{\xi}_g > 0$. ("Good" means that the strategy is optimal, taking into account the possible reactions of the combatant.) Referring to [3, p. 286] and the previous Section 2.1, we say the players to act "conservatively" and find as a decision maker's problem

$$\min_{\eta \in \mathcal{N}} v \quad \text{subject to} \quad \sum_{j \in J} \eta_j \, r_i(x^j) \leq v \quad \text{for} \quad i \in \{f,g\}. \tag{2.2.4}$$

(v, corresponding to the payoff of the game, here represents a disutility for the decision maker.)

With the linearity of r_i and (2.2.3), the program can be restated as

$$\min_{x \in X} v \quad \text{subject to} \quad r_i(x) \leq v \quad \text{for} \quad i \in \{f,g\} .$$

From duality theory it is known that $\bar{\xi}_f, \bar{\xi}_g > 0$ provides for (2.2.5) $\sum_{j \in J} \bar{\eta}_j \, r_i(x^j) = \bar{v}$ for $i \in \{f,g\}$, where the bar denotes good strategies and the value of the game, respectively. It can be shown (see Section 2.3.1) that then the program (2.2.4) finally reads

$$\max_{x \in X} f(x) \quad \text{subject to} \quad r_g(x) \leq r_f(x) . \tag{2.2.6}$$

Note that this program corresponds to what the economist would do, except for the additional restriction that his dissatisfaction may not be smaller than (and will, in fact, be equal to) the one of the environmentalist. So (2.2.6) also can be stated intuitively to be a reasonable decision maker's choice concept without stressing game theory. Using game theory provides, however, a kind of proof of the soundness of the intuition.

Let $\bar{x}$ be the solution set of (2.2.6). Then, because of (2.2.5), $r_g(x') = r_f(x')$ for each $x' \in \bar{x}$. Therefore, if $\text{card}(\bar{x}) = 1$, the decision maker can choose one $x' \in \bar{x}$ randomly. The theory of linear programming provides well known methods to determine a unique solution, say $\hat{x}$, directly in case of degeneracy. Clearly $\hat{x} \in \bar{x}$ and the resulting payoffs are as good as those from a random choice. $\hat{x}$ corresponds to a realization of the economy and the environment such that the larger of both goal related dissatisfaction degrees is minimal. So it is necessary to add to a model like PILOT only the single restriction

$$r_g(x) \leq r_f(x) \ , \qquad (2.2.7)$$

as well as the parts which provide for $g(x)$ to be calculated, in order to get the solution $\hat{x}$ of the problem (1.3). $\hat{x}$ can be called a compromise between the economic and the environmental goals. (A similar concept was treated, e.g., in [4].)

2.3 A Generalized Concept

Note that only an informal reasoning will be given below to present a brief idea of the assumptions underlying the concept to be presented; with the aid of references, formal proofs easily can be made.

Let $I = \{1,\dots,m\}$ be a finite indexset of goals; the goal corresponding to $i \in I$ is K_i. Let X be a set of vectors, representing options; we understand an option $x \in X$ to be a feasible solution of a model, representing all facts which influence the assessment of the option with respect to all goals. (Though finiteness or infiniteness of X is not to be decided now, as that constitutes the subsequent different chapters, for convenience it now may be regarded to be finite.)

Let f_i be a mapping $f_i:X \to \mathbb{R}$ for each $i \in I$ with (2.3)

$f_i(x_1)\{\gtreqless\} f_i(x_2)$ iff, with respect to goal K_i, there holds for a person supporting K_i: $x_1 \{\succ \sim \prec\} x_2$, x_1, $x_2 \in X$ (e.g., $x_1 \succ x_2$ means that x_1 is preferred to x_2). Further on transitivity is assumed to hold. Equivalent to problem (1.4.2), we formulate:

Let F be the set of vectors $(f_i(x) | x \in I)$ (2.4)

for each $x \in X$. Then the problem is to find a mapping

$z_2:F \to \mathbb{R}$

with $z_2((f_i(x)|i \in I)) = q(x)$ such that $q(x_1)\{\gtreqless\} q(x_2)$ iff for the entity of peoples supporting different goals K_i:

$x_1 \{\gtrless\} x_2$, $x_1, x_2 \in X$ (and transitivity holds).

With reference to the original formulated problem (1.3), a subproblem of (2.4) already is interesting in itself:

to find an $x \in X$ which maximizes $q(x)$, (2.5)

or symbolically let us say that we want to maximize a vector subject to x:

$$\max_{x \in X} (f_i(x) \mid i \in I).$$

See, e.g.,[5].

We will concentrate our investigations to problem (2.5). In most cases it is easy to see how a complete order on X could be obtained by the rules provided to solve (2.5). Short notes to this occasionally will be made below. We further will put the following specifications on our problem (2.5) and its solution:

As problem (1.4.2) is concerned with controversial goals, we assume that for each goal there exists a pressure group which clearly would know an option to meet its and only its specific goal. Let X^i be the subset of X whose elements maximize $f_i(x)$ subject to $x \in X$ with $i \in I$. We require for any $i_1, i_2 \in I$, $i_1 \neq i_2$, that there is no $x' \in X^{i_1}$ and no $x'' \in X^{i_2}$ such that $x' = x''$. (2.5.1)

On the basis of (2.5.1), we demand a solution (2.5.2)
set of problem (2.5), say $\hat{x}$, $\hat{x} \subset X$, to be unique, so the decision maker finds a clear decision. There is an underlying assumption that the option $x \in X$ should be regarded as an alternative, that is, it may make no sense to choose a mix of them.

If there were a lexicographic order defined on G with $K_{i_1} \succ K_{i_2} \succ \ldots$, $i_1, i_2, \ldots \in I$, then a solution of problem (2.5) would be to choose the $m' < m$ for which $\bigcap_{n=1,\ldots,m'} x^{i_n}$ has exactly one element, which then constitutes $\hat{x}$. It is easy to see how successively a complete order on X could be established. For various less distinct information on the relative importance of the goals in I, there exist methods in the utility theory to establish $\hat{x}$ or an ordered X. See, e.g., [6].

The decision maker's problem, however, is to solve (2.5.3)
(2.5) without any information about the relative importance of the goals in I. In this situation it is wise to regard each goal at least somewhat important. This we will regard as an additional specification of problem (2.5).

The decision maker is said to act responsibly if his choice is among those options, say $\bar{x}$, $\bar{x} \subset X$, for which it holds for each $x' \in \bar{x}$ there is no $x \in X$ such that $f_i(x) \geq f_i(x')$ for all $i \in I$ and $f_i(x) > f_i(x')$ for at least one $i \in I$. That is, $\bar{x}$ contains all dominant $x \in X$ and is said to be Pareto optimal. As long as f_i, $i \in I$, is not specified more than in (2.3), the decision maker can choose

$\hat{x}$ only randomly from $\bar{x}$. (We denote the element of the unique solution set $\hat{x}$ with the identical symbol.)

If for all $i \in I$ f_i in (2.3) is specified (2.6)
to represent an interval scale, that is, for $x, x', x_0, x_1, x_2 \in X$ and $x \succ x_0$, $x' \succ x_0$, $x_2 \succ x_1$ holds

$$\frac{f_i(x) - f_i(x_0)}{f_i(x_2) - f_i(x_1)} \{ \gtreqless \} \frac{f_i(x') - f_i(x_0)}{f_i(x_2) - f_i(x_1)}$$

$$\iff x \{ \succ \sim \prec \} x' \text{ for a person supporting goal } K_i,$$

then we can define linearly transformed goal functions

$$r_i(x) = (f_i(x) - f_i^*)/(f_{i*} - f_i^*)$$

for any fixed f_i^* and f_{i*} where $f_i^* > f_{i*}$. Problem (2.5) then reads

$$\min_{x \in X} (r_i(x) | i \in I) \qquad (2.6.1)$$

regarding (2.5.1) through (2.5.3).

With x^i defined as in (2.5.1), the decision (2.6.2)
maker will reasonably assume for any $i_1, i_2 \in I$, $x' \in x^{i_1}$ and $x'' \in x^{i_2}$ to be equally desirable; that is, he assumes assessment like equivalence $x' \sim x''$. Let $x_i \subset X$ with $x' \in x_i$ iff

x' minimizes $f_i(x)$ subject to $x \in \bigcup_{i \in I} x^i$, $i \in I$. Then for any $i_1, i_2 \in I$, we assume $x' \in x_{i_1}$ and $x'' \in x_{i_2}$ to be equally undesirable for the decision maker; that is, $x' \sim x''$.

We say $r_i(x)$ is normalized if $f_i^* = f_i(x')$ with $x' \in x^i$ and $f_{i*} = f_i(x'')$ with $x'' \in x_i$ for all $i \in I$; then $r_i(x') = 0$ and $r_i(x'') = 1$ and $r_i(x) \geq 0$ for all $x \in X$. (2.6.3)

Note that the definition of x_i with $r_i(x'') = 1$ for $x'' \in x_i$ differs from the one in [4] where each x'' minimizes $f_i(x)$ subject to $x \in X$ instead of subject to $x \in \bigcup_{i \in I} x^i$.

In other words, we want to make up the dissatisfaction degrees regarding realistic worst cases, that is, the worst cases under these alternatives which are best for one competing pressure group; we won't take into consideration the absolute worst cases which may not be favored by any one of the combatants. The degree of dissatisfaction, one naturally feels, is made up, in fact, rather by expectable alternatives than by merely fictive ones.

So definition (2.6.2) and (2.6.3) come out quite natural if we assume that we only pay attention to the nondominated subset in X (2.6.4)

and that the goals are controversial in a way that the minimum satisfaction with respect to a goal, say i_1, occurs where a goal, say i_2, is maximally fulfilled. (This condition supplements (2.5.1).)

With (2.6.3) for all i_1, $i_2 \in I$ and any $x, x' \in X$ full cardinal comparability holds between $r_{i_1}(x)$ and $r_{i_2}(x^1)$, defined as in (2.6.3). (Note that (2.6) and (2.6.3) do not limit the utility like representation of x, as the shape of f_i directly determines the shape of r_i.) We call $r_i(x)$, thus normalized, the dissatisfaction degree of $x \in X$ with respect to goal $i \in I$ and arrange them in the matrix $R = [r_i(x)]$, $i \in I$, $x \in X$. We use R_x to denote a column and R_i to denote a row of R. (For simplicity, we denote the indexset corresponding to X by the same symbol X.)

We now look for a solution for two different cases regarding the properties of X and R.

2.3.1 Convex Goal Functions

In order to solve problem (2.6.1) under special conditions, (2.7)
let us assume a game (I, X, R), where the column player, that is, the decision maker, controls the choice among an infinite set of options X and the row player controls the weightings of the goals in G. R is the payoff matrix as defined in (2.6.3). It is known from (2.1.2) how to interpret such a game with respect to the decision maker's problem. Note that a utility function f_i, forming R, is assumed to be the individual utility function of the power group related to goal K_i. No compromised utility functions, representing different opinions

each related to a different goal K_i, $K_i \in G$, are necessary. To establish a compromised action (the choice of the decision maker among X) is the object of the subsequent evaluations.

Combining theorems 2.3.3.1 and 2.5.1 of [7] says that the game (I,X,R) has a value, the row player has a good, mixed strategy, and the column player has a good, pure stategy if I is finite, X is convex (closed and bounded), and for all $i \in I$ it holds that $r_i(x)$ is convex in X, $r_i(x) \in R$. (2.7.1)

Let ξ be a set of probability distributions on G and $\bar{\xi} \in \Xi$ denote a good strategy for the row player; let $\mathcal{N}$ be a set of probability distributions on X and $\bar{\eta} \in \mathcal{N}$ denote a good strategy for the column player. Let $\mathcal{N}' \subset \mathcal{N}$ be all those strategies for which there holds $\eta(x) = 0$ except for one $x \in X$, that is, all $\eta' \in \mathcal{N}'$ are pure strategies. We know from (2.7.1) that $\bar{\eta} \in \mathcal{N}'$.

Applying the way to solve a game according to [3, p. 286] (the column player's problem) to the game (2.7), we can determine his good strategy $\bar{\eta}$ by

$$\min_{\eta(x)\in\mathcal{N}} v \quad \text{subject to} \quad \int_X \eta(x)\cdot r_i(x)dx \leq v \quad \text{for all} \quad i \in I \,. \tag{2.7.2}$$

But as $\bar{\eta} \in \mathcal{N}'$, we have the identical program saying

$$\min_{x\in X} v \quad \text{subject to} \quad r_i(x) \leq v \quad \text{for all} \quad i \in I \,. \tag{2.7.3}$$

The procedure (2.7.3) to solve the game (2.7) according to (2.7.1) shows clearly that the solution of the game provides for a pure choice among the options X, say $\hat{x}$, in a way which minimizes the maximum of the m dissatisfaction degrees, each related to a goal $K_i \in G$, and $card(G) = m$. This again corresponds to (2.1.1) and so shows that (2.1.2) leads to the same solution as (2.1.1).

In general, degeneracy may occur and the solution of (2.7.3) may be a set $\bar{x}$ from which $\hat{x}$ is to choose. With (2.8.7) and (2.8) of the supplementary note 1, one can verify that $r_i(x)$ can be assumed to have the same (optimal) value for all $i \in I$ if $x \in \bar{x}$, and, therefore, it is quite natural to regard each $x \in \bar{x}$ as equally valuable. Hence, one can choose $\hat{x}$ randomly from $\bar{x}$.

(2.7.3) corresponds to

$$\bar{x} = \{x | \min_{x \in X} \{\rho(x) | \rho(x) = \max_{i \in I} R_x\}\} \qquad (2.7.4)$$

Note that for the evaluation in section 2.2, the conditions (2.7.1) are met as X is a (closed, bounded) convex polyhedron there and f and g are linear.

More generally, we see that (2.7.3) provides for a solution of problem (2.6.1) if the conditions (2.7.1) apply; especially, referring to section 2.2, f_1, the utility of the economic situation, should be concave in X and g_2, the disutility with respect to the environmental situation, should be convex in X in order to get a (unique) compromise of the decision maker's problem. If $r_i(x)$ for all $i \in I$ is approximated by a broken line fit and X by a polyhedron, then (2.7.3) can be solved by a common linear program. (For algorithms, see [3].)

Note, that (2.7.3) and (2.7.4) provide for a quite intuitive approach to our problem to get a compromise, stating that the dissatisfaction degrees for all pressure groups (goals) should be as low as simultaneously possible.

Supplementary Note 1:

Though (2.7.3) provides for sufficient tools to solve the decision maker's problem and to establish a compromise, we will discuss some variants of the program (2.7.3).

For at least one $i \in I$, say i_0, $\bar{\xi}_{i_0} > 0$ will hold so that (2.8)
according to the strong minimax theorem (see 6.2.33 in [8]; see also corollary 1, p. 287, resp. theorem 4, in [3, p. 136]) $r_{i_0}(\bar{x}) = \bar{v}$.
This leads from (2.7.3) to

$$\min_{x \in X} r_{i_0}(x) \quad \text{subject to} \quad r_i(x) \leq r_{i_0}(\hat{x}) \quad \text{for all} \quad i \in I,\ i \neq i_0 \qquad (2.8.1)$$

or equivalently

$$\max_{x \in X} f_{i_0}(x) \quad \text{subject to} \quad r_i(x) \leq r_{i_0}(\hat{x}) \quad \text{for all} \quad i \in I,\ i \neq i_0.$$

As $\hat{x}$ is the optimal solution of (2.7.3), there can be no $x' \in X$, $x' \notin \hat{x}$, for which $r_i(x) \leq r_{i_0}(x')$ holds for all $i \in I$, but $r_{i_0}(x') < r_{i_0}(\hat{x})$. So we finally may replace (2.7.3) by

$$\max_{x \in X} f_{i_0}(x) \quad \text{subject to} \quad r_i(x) \leq r_{i_0}(x) \quad \text{for all} \quad i \in I,\ i \neq i_0. \tag{2.8.2}$$

However, as the difference of two convex functions, here $r_i(x) - r_{i_0}(x)$ in the restriction set $r_i(x) - r_{i_0}(x) \leq 0$ for all $i \in I$, $i \neq i_0$, need not be convex and may cause trouble in performing (2.8.2), we must add to (2.7.1) the condition

choose i_0 such that $\xi_{i_0} > 0$ and r_{i_0} (resp. f_{i_0}) is a linear function in X. (2.8.3)

Then (2.8.2) can be performed. If an i_0, $i_0 \in I$, meeting (2.8.3) cannot be found, we have to work with (2.7.3).

If $X \subset \mathbb{R}^n$, we have, according to theorem 2.3.2 and extensions [7] (see also theorem 2.5.1) defining a subset $I' \subset I$ with $i \in I' \iff \bar{\xi}_i > 0$, $i \in I$:

$$\text{card}(I') \leq \text{Min}\{n, \text{card}(I)\} . \tag{2.8.4}$$

With $n \geq \text{card}(I)$, which is true for the most realistic models, (2.8.5)
$\bar{\xi}_i$ may be positive for all $i \in I$.

In fact, assuming that (2.8.5) holds, the conditions in (2.5.1) and the properties of R according to (2.6.3) lead to

there exists an $x' \in X$ such that for any $i_1, i_2 \in I$ (2.8.6)

$$r_{i_1}(x') < r_{i_2}(x')$$

(which is at least true for $x' \in x^{i_1}$ where $r_{i_1}(x') = 0$ and $r_{i_2}(x')$ cannot be zero) and, therefore, equivalently there exists an $x'' \in X$ such that for any $i_1, i_2 \in I$, $r_{i_1}(x'') > r_{i_2}(x'')$ (which is at least true for $x'' \in x^{i_2}$ where $r_{i_2}(x'') = 0$ and $r_{i_1}(x'')$ cannot be zero).

So no pure strategy $i_1 \in I$ will be dominated by any different strategy $i_2 \in I$, and as can be seen easily with the same arguments of (2.8.6), no pure strategy $i_1 \in I$ will be dominated by any mixed strategy (not containing i_1).

Hence, according to theorem 7, in [9, p. 235], it holds (2.8.7) $\bar{\xi}_i > 0$ for all $i \in I$.

To make it clear, assume $\text{card}(\xi) = 2$ and $I = \{i_1, i_2\}$, $(n \geq 2)$, then

$$0 = \underset{i \in I}{\text{Max}} \{\rho_i \mid \rho_i = \underset{x \in X}{\text{Min}} R_i\} < \underset{x \in X}{\text{Min}} \{\rho(x) \mid \rho(x) = \underset{i \in I}{\text{Max}} R_x\}$$

As shown, e.g., in [3, p. 283], this forces mixed strategies with $\bar{\xi}_{i_1}, \bar{\xi}_{i_2} > 0$.

Summarizing, we have (for a game (I,X,R) as defined in (2.7.1) and if (2.8.5) applies) that i_0 in the solution concept (2.7.6) may be any $i \in I$ for which f_{i_0} is linear in X. Hence we simply maximize one utility function subject to $x \in X$ and subject to card$(I)-1$ additional restrictions according to (2.8.2), which guarantee that the dissatisfaction with respect to none of the goals exceeds the one of i_0 and so guarantee to obtain a compromise with respect to all goals in G. This result also proves (2.2.6) as a solution of problem (1.3).

Supplementary Note 2:

We note that under several assumptions (e.g., those listed in 2.6, 2.6.2, 2.6.3, 2.6.4, 2.7.1) we have solved the problem (2.5). We now will discuss the problem (2.4), that is, to establish a complete ordering on X, applying the same conditions.

First we will discuss a pitfall. We have seen that the game (I,X,R) can be assumed to provide for a good, strictly mixed strategy for the row player, that is $\bar{\xi} \in \Xi$ with $\bar{\xi}_i > 0$ for all $i \in I$. One might interpret $\bar{\xi}_i$ as the weight of goal K_i and calculate a dissatisfaction degree for any $x \in X$ according to

$$v(x) = \sum_{i \in I} \bar{\xi}_i \cdot r_i(x). \tag{2.9}$$

There is, however, a difficulty to state that $v(x)$ would represent the order on X. It happens that

$$\bar{\bar{v}} = \sum_{i \in I} \bar{\xi}_i \cdot r_i(\bar{x}) \quad \text{as}$$

$$\bar{v} = \underset{\xi \in \Xi}{\text{Max}} \{v_\xi | v_\xi = \underset{x \in X}{\text{Min}} \{ \sum_{i \in I} \xi_i \cdot r_i(x) \} \}$$

$$= \underset{x \in X}{\text{Min}} \{ v(x) | v(x) = \underset{\xi \in \Xi}{\text{Max}} \{ \sum_{i \in I} \xi_i \cdot r_i(x) \} \}$$

or in terms of [5, proposition 5], that if $\bar{x}$ is efficient (as each $x \in X$ with $\bar{\eta}_x > 0$ is efficient, so is any pure strategy and $\hat{x}$ in our case), then there exists a $\bar{\xi}$ such that $\bar{x}$ minimizes $\sum_{i \in I} \xi_i \cdot r_i(x)$ subject to $x \in X$ with $\xi = \bar{\xi}$.

But nothing can be said about (2.9) if $x \neq \bar{x}$ except that then $v(x) > v(\bar{x})$. There is no guarantee that the derivative

$$\frac{dv(x)}{dx} = \sum_{i \in I} \bar{\xi}_i \frac{dr_i(x)}{dx} \qquad (2.9.1)$$

gives a meaningful measure of the slope of the disutility at point x. The tradeoff between competing goals can be highly nonlinear. The game theory approach with the linearly calculated value of the game only provides for a meaningful "optimal" solution, the compromise. The weights of the goals are controversial and so not generally known. What the weights are found to be in (2.9) is valid only for the compromise case itself.

Regarding the program (2.7.3) and its interpretation, especially (2.7.4), we can state, however, on the basis of the decision maker's problem as stated in (2.1.1), that it might be a useful rule to say for any $x_1, x_2 \in X$ (2.9.2)

$$x_1 \{ \succ \sim \prec \} x_2 \iff \underset{i \in I}{\text{Max}} R_{x_1} \{ \leq = \geq \} \underset{i \in I}{\text{Max}} R_{x_2} .$$

This principally provides for an ordering on X regarding all goals K_i in G. So we have a solution now for the problem (2.4) too. We will add here that

$$\text{if} \quad \underset{i \in I}{\text{Max}} R_{x_2} = \underset{i \in I}{\text{Max}} R_{x_2} \tag{2.9.3}$$

with $r_{i_1}(x_1) = \underset{i \in I}{\text{Max}} R_{x_1}$ and $r_{i_2}(x_2) = \underset{i \in I}{\text{Max}} R_{x_2}$, a decision $x_1 \{ \succ \prec \} x_2$ can be attempted by deleting $r_{i_1}(x_1)$ and $r_{i_2}(x_2)$ in R_{x_1} respective R_{x_2} and repeating (2.9.2). If after card(I)-1 iterations equality still holds, then $x_1 \sim x_2$.

2.3.2. A Finite Set of Options

Let G be a finite set of goals and so I is finite; (2.10)
let X be finite and R a general payoff matrix, defined as in (2.6.3). For example, let $x_\ell \in X$ be a feasible, let us say optimal, solution of a linear program constrained by X_ℓ, $\ell = 1,2,\dots$. Let X_ℓ represent a scenario of the general economy with constant technical coefficients and consumption patterns. Then we now will find a compromise with respect to different goals regarding the optimal solutions of different scenarios.

The best strategy for the decision maker would be to choose a mix of options if there is defined what $\eta_x \cdot x$ for $x \in X$, $0 < \eta_x < 1$ means or if $\sum_{x \in X} \bar{\eta}_x \cdot x$ also is an element of X.

But the latter is not guaranteed by (2.10) and to use a mix of options is not allowed by assumption (2.5.2).

To play the game more often in a time sequence and, therefore, to regard η_x as the probability $x \in X$ is chosen, is not the problem. Instead one act of decision, choosing one $x \in X$, is required.

In those cases, game theory advises the decision maker to choose randomly, regarding $\bar{\eta}$, one $x \in X$ to be $\hat{x}$. This assures the expected dissatisfaction degree for the decision maker to be

$$v \leq \underset{x \in X}{\text{Min}}\{\rho(x) \,|\, \rho(x) = \underset{i \in I}{\text{Max}}\ R_x\} .$$

See, e.g., [10, pp. 14-19,78].

But we now have to specify our game further, if it should meet our decision maker's problem fully, by stating

that the row player, interpreted as the holding company of the goal related pressure groups, need not choose his actual (pure) strategy out of his mixed stategy randomly, as advised, if he has to choose before the decision maker makes public his choice, rather can react to the chosen x. In those cases, the decision maker again has to choose among pure strategies and it is advisable to do it corresponding to (2.7.4). That is, (2.10.1)

$$\bar{x} = \{x | \underset{x \in X}{\mathrm{Min}}\{\rho(x) | \rho(x) = \underset{i \in I}{\mathrm{Max}}\, R_x\}\} \quad . \qquad (2.10.2)$$

$\bar{x}$ now need not be unique. But by deleting in each R_x with $x \in \bar{x}$ one element $r_i(x)$ for which $r_i(x) = \rho(x)$, so generating R'_x, and then constituting a new $\bar{x}$ according to (2.10.3)

$$\bar{x}_{(new)} = \{x | \underset{x \in \bar{x}_{(old)}}{\mathrm{Min}} \{\rho(x) | \rho(x) = \underset{i \in I}{\mathrm{Min}}\, R'_x\}\} \ ,$$

diminishes $\mathrm{card}(\bar{x})$.

(2.10.3) can be repeated until $\mathrm{card}(\bar{x}) = 1$, maximal $\mathrm{card}(I)-1$ times; then $\hat{x} = \bar{x}$, or in case of degeneracy $\hat{x} \in \bar{x}$, to be chosen randomly.

We realize that (2.10.2) is the minimax principal of the original game (that is, not considering its mixed extension) and that it corresponds to what was found for the decision maker to be reasonable in (2.1.1). (2.10.3) in addition states that in cases where the minimax leads to no decision, one has to choose among the drawn elements regarding the minimum of the second worst satisfaction degrees, similar to (2.9.3).

The rules (2.10.2) and (2.10.3) can be performed easily, as shown by an example in the next section. For the case of infinite X as treated in Section 2.3.1, we will give an example at the end of the paper.

A complete ordering in X is obtained directly by $\underset{i \in I}{\mathrm{Max}}\, R_x$ and in case by repetitions in the sense of (2.10.3). So we have solved the problems (2.4) and (2.5) for the case of finite X too.

2.4. An Example

To prepare for an example, we will take rough figures for the representation of an economic utility as well as an environmental disutility for a comparison of different power plants.

That is, X is finite and represents various power plants. Let us take the environmental disutility from results of some calculations in Section 3.4 regarding the air pollution of different power stations under normal operating conditions and let us assume that the tentative supply horizon of different fuels, if uniquely used for a growing world's total energy demand, represents the economic utility. We compare coal fired (Coal), oil fired (Oil), and natural gas fired (Gas) power plants and nuclear power stations, namely, light water reactors assuming no reprocessing (LWR), the same with reprocessing and plutonium recycling (PuR) and fast breeder reactors (FBR). Utility estimates and resulting dissatisfaction degrees are shown in Table 1, which follows.

This example is only for demonstration purposes and no serious conclusions can be drawn. For example, we are lacking the contributions of the various fuel cycles. Some principles can be shown, however.

Note, in this sense, that the best economic option is the FBR, the best environmental one is Gas. So 30 is the economic result if the environmental goal is optimized, and 55 is the environmental result if the economic goal is optimized. Oil is regarded as a compromise between an economist and an environmentalist as it has $\min_{x \in X} \max_{i \in I} r_i(x)$. In general, we get

$$\text{Oil} \succ \text{PuR} \succ \text{Gas, FBR} \succ \text{LWR} \succ \text{Coal} .$$

Table 1

UTILITY ESTIMATES AND DISSATISFACTION DEGREES

	Coal	Oil	Gas	LWR	PuR	FBR
Utility Estimates						
Economic Utility (in years supply horizon)	150	70	30	10	100	300
Environmental Disutility (in $m^3/kWh_{e\ell}$ dilution demand)	112	26	20	53	53	55
Resulting Dissatisfaction Degrees						
Economic	0.56	0.85	1.0	1.07	0.74	0
Environmental	2.63	0.17	0	0.94	0.94	1.0
With $\underset{i \in I}{\text{Max}}\ R_x$	2.63	0.85	1.0	1.07	0.94	1.0

Note that the change is not drastic if the environmental effects of the nuclear power plants are assumed to equal:

$$\text{Oil} \succ \text{Gas, FBR} \succ \text{PuR} \succ \text{LWR} \succ \text{Coal} .$$

In other words, sensitivity analysis easily may show, e.g.,

$$\text{Oil} \succ \text{Gas, FBR, PuR} \succ \text{LWR} \succ \text{Coal} .$$

If in those cases for some reason Oil is no alternative and therefore dropped, we see that one can choose randomly from Gas, PuR, and FBR to obtain a compromise.

What the methodology is able to explain further is that Oil remains a compromise as long as the dissatisfaction degrees are lower than 0.94 for either goal. This means that the environmental index may go up to near 53 and the economic index down to near 46. So the choice Oil would be shown to be sufficiently stable.

With respect to Coal, we could see that it enters to become a compromise as soon as abatement measures provide for an environmental dissatisfaction degree smaller than 0.85, that is, an environmental disutility index smaller than 50. With the information of Section 3.4, this could be correlated to a necessary SO_2 abatement level.

Note that if the environmental disutility for the FBR would come down to 20 or less, our decision problem does not exist anymore, as then the FBR would be best with respect to either goal. No "compromise" is necessary then.

3. A Normative Utility Measure

This chapter will give an idea of how to perform (1.4.1), the establishment of an aggregated utility measure for nonconflicting aspects. That is, e.g., an economic or an environmental index must be developed which represents an overall utility or disutility on an interval scale. In the case of the economy, many ways of calculating such an index exist.

PILOT [1], e.g., regards the consumption level as an overall measure of the economic situation. A corresponding measure for the environmental situation must be developed. According to problem (1.1), this chapter will address itself to the evaluation of an environmental disutility measure. The maximum permissible ambient concentrations will play a major role in it, which is discussed in Section 3.1. Section 3.2 constructs the shape of an environmental disutility function for each pollutant and Section 3.3 gives conditions for an appropriate aggregation.

It should be noted that only the environmental effects under normal operational conditions are treated here. Accident conditions are excluded since the theory on these is not sufficiently developed at present, especially applying the expectation concept, which in fact would be possible, is questionable in those cases (see [19]). It also should be noted that only burdens related to human beings and caused by chemical pollutants are addressed but none of the other impacts of economic activities. In some cases, however, it would be easy to see how the concept to be evaluated also could treat those different impacts.

As there is no real competition among the various contributing factors within the environmental field, the hazards of different pollutants cannot be played off one against the other. Our approach to perform (1.4.1), therefore, must be different to that used in Chapter 2 to perform (1.4.2).

3.1. The Role of Environmental Quality Norms

The frequency distribution of emissions of certain pollutants is assumed to be fairly well known. Pollutants normally get to human bodies in different ways and in doing so their chemical composition may change. Up to now, there exist only rough models for calculating food or drinking water pollution or ambient concentrations in air. Again, knowledge about the frequency distribution of these concentrations would be desirable. With those imperfections, one might be able to deduce from medical and epidemiological knowledge an estimate of the probability for different kinds of health hazards. Mostly such knowledge neglects synergisms of different pollutants. That is, one adds up the expectations of a specific kind of disease over all contributing pollutants. In addition to the major difficulties of finding the necessary data, there is a technical difficulty in the aggregation of the different hazards: how to weigh short term disease against long term disease, illness against death, effects to children against those to workers or old people, etc.

Another problem is whether to count the average frequency with which each kind of disease occurs, that is, to take under consideration the exposed size of population as a whole, or to relate to the individuum only and consider the probability with which each kind of disease may occur for the most affected person. This is a political question.

Now consider how the maximum permissible ambient concentration should be established and how this affects the procedure to solve our problem. What is said below with respect to air pollution should in the same sense apply to water and food pollution.

It is expected that ambient concentrations are calculable or at least measurable, and so are their frequency distributions, but assume now their expectation to be an ordinal measure of the health hazard assessment. That is, we assume that higher expectation levels of the concentration of one pollutant lead to increased health hazards and, consequently, to a worse assessment. (3.1)

Instead of showing explicitly now the various diseases caused, establishing an aggregate measure of them, and defining an upper acceptability limit of those, an upper limit of ambient concentrations should be fixed. This will be called a norm. To those norms for each pollutant as much information as possible about the above mentioned expected diseases and their assessments as well as acceptance levels is desirable. However, responsible analysts, who have an intuitive notion of disease possibilities and acceptable disease levels, may help to establish those norms by analysis of present incomplete calculations based on very rough data. Representing some knowledge and a good deal of ignorance, the norms could be a realistic utility measure of the potential harms done by a pollutant. In this sense the norms should become better assessments the more the process of establishing them interlinks factual and scientific knowledge with political values.

It is desired to be able to assume that such norms are comparable for different pollutants with respect to the assessment of diseases caused, if the ambient concentrations would meet these norms. (3.1.1)

These norms are not considered here as strict thresholds with respect to assessing diseases or with respect to allowable pollution. There still are emission standards to be set depending on the individual case of the polluting activity and its environment. But these norms are regarded as a help, as they prevent us from weighting single diseases and help us to compare the disutility of different pollutants. Norms often have not been established in a comparable manner, due to the influence of different power groups. Nevertheless, there is improvement and unofficial norms exist, which have been proposed by quite responsible groups. Moreover, what was outlined above indicates a philosophy directed to adequate procedures in establishing these norms. With respect to analysis, the assessment problem has been eased since it is possible to consider one figure per pollutant and perform simulations with respect to changes of these norms. It is not necessary to be involved with details of a mathematical model, with numerous parameters to calculate and to aggregate diseases, where each of those parameters needs to withstand opposing opinions. One parameter per pollutant can more easily be the subject of fruitful discussions and so improve a model. Thus an environmental index can be established along those lines.

3.2 Quantifying Environmental Disutilities

Denote a norm in the above mentioned sense with respect to a pollutant k by n_k^2, where the superscript is to distinguish between other norms to be discussed below. Let $u(y_k)$ represent a measure of the disappointment level caused by the diseases from an ambient concentration y_k. Then

a norm n_k^1 is defined with $u(y_k \leq n_k^1) = 0$, we normalize $u_k(n_k^2) = 1$, (3.1.2)
and assume that a norm n_k^3 can be established with $u_k(y_k \geq n_k^3) = \infty$.

n_k^3 can be defined as a protest limit and n_k^1 as the disease threshold, whereas n_k^2 will be denoted as the acceptance level. From a psychological standpoint it is reasonable to assume further that the change in the disappointment level with respect to the change in the ambient concentration varies inversely with the amount the concentration is below the protest limit:

$$\frac{du(y_k)}{dy_k} \text{ proportional } \frac{1}{n_k^3 - y_k} \tag{3.1.3}$$

for $n_k^1 \leq y_k \leq n_k^3$. Within these limits this leads to

$$u_k(y_k) = \frac{\ln \dfrac{1 - n_k^1/n_k^3}{1 - y_k/n_k^3}}{\ln \dfrac{1 - n_k^1/n_k^3}{1 - n_k^2/n_k^3}} . \tag{3.1.4}$$

To establish n_k^1 and n_k^3 may be even easier than to establish n_k^2, as n_k^1 and n_k^3 reflect stronger scientific evidence. So in (3.1.4) we have a convex, fully fixed cardinal disutility function for each pollutant k.

We will show now that also a linear disutility function often is a sufficient approximation. It can be expected in scenarios treated by models like PILOT that $n_k^1 < 0.1 \cdot n_k^2 < 0.01 \cdot n_k^3$ and that ambient concentrations normally would not exceed a mean of $2.5 \cdot n_k^2$; if in addition the slope of $u_k(y_k)$ for $y_k < 0.5 \cdot n_k^2$ is not considered to be important, $u_k(y_k)$ can be developed to get a first approximation: (3.2)

$$u_k(y_k) = y_k/n_k^2 \qquad (3.2.1)$$

with an error less than 15% if $0.5 \cdot n_k^2 \leq y_k \leq 2.5 \cdot n_k^2$.

Thus it has been shown which conditions lead to a simple measure of environmental impacts, known as the hazard index [11].

Though the formulae (3.1.4) and (3.2.1) are only approximations of the real disutilities of various hazards, they should work better than a procedure to perform u_k by individual questioning as, e.g., proposed in [12], because in those interindividual assessment problems it should be easier to elaborate on political consensus about some norms than about the detailed measuring of u_k or to find a representative whose hazard-utility relationship would be commonly agreed upon.

It should be noted further that the aspects gathered in K_i with $i = 2$, K_2 representing the environmental goal, are assumed to be synonymous with the various pollutants under investigation, that is, $k \in K_2$.

3.3 Conditions for an Aggregation

Now that a single pollutant can be measured, the next task is the aggregation of a number of pollutants for an overall measure of environmental impacts. Either of the above formulae for u_k can be used for this. By definition, see (3.1.1), for any two different pollutants k and ℓ

$$n_k^1 \text{ is equally assessed to } n_\ell^1 \text{ with } u_k(n_k^1) = u_\ell(n_\ell^1) = 0, \tag{3.3}$$

$$n_k^2 \text{ is equally assessed to } n_\ell^2 \text{ with } u_k(n_k^2) = u_\ell(n_\ell^2) = 1,$$

$$n_k^3 \text{ is equally assessed to } n_\ell^3 \text{ with } u_k(n_k^3) = u_\ell(n_\ell^3) = \infty.$$

To show what we can make of this for the aggregation problem, formulated in (1.4.1), we will proceed as follows. For the background thereto see [13,14].

Let y_k^* be the best and y_{k*} the worst ambient concentration of a pollutant k which can occur in a given situation. Assume a lottery $L_p(y_k^*, y_{k*})$ where y_k^* should occur with probability p and y_{k*} with $1-p$. We call $\hat{y}_k$ a certainty equivalent to $L_p(y_k^*, y_{k*})$ if $\hat{y}_k \sim L_p(y_k^*, y_{k*})$.

It is assumed in the corresponding theory that the utility w_k' of the lottery is expressed by its expectation, which is equal per definition to the utility of the certainty equivalent. That means

$$w_k'(\hat{y}_k) = p \cdot w_k'(y_k^*) + (1-p) \cdot w_k'(y_{k*}) \tag{3.4}$$

if and only if $\hat{y}_k \sim L_p(y_k^*, y_{k*})$.

(3.4) holds for any linear transform of w_k', so we can normalize $w_k'(y_{k*}) = 0$ and $w_k'(y_k^*) = 1$ with $w_k'(\hat{y}_k) = p$. This means, e.g., if one is indifferent for p to live in area A where y_k^* related body burdens occur with probability p and with $1-p$ those related to y_{k*} or to live in area B with a $\hat{y}_k$ related body burden for sure, then p is the utility index. In repeating those questions for p for different $\hat{y}_k$ a utility function w_k', normalized to 0 and 1, can be built up. The same is true for a disutility function

$$w_k(y_k) = 1 - w_k'(y_k) \tag{3.4.1}$$

for which holds $w_k(y_k^*) = 0$ and $w_k(y_{k*}) = 1$ which we prefer subsequently to the utility function.

Now y_{k_1*} may be preferred to y_{k_2*}, $k_1, k_2 \in K_2$; or we say $y_{k_1*} \not\sim y_{k_2*}$ though $w_{k_1}(y_{k_1*}) = w_{k_2}(y_{k_2*}) = 1$ per definition (3.4.1). On the other hand, e.g., $n_{k_1}^2 \sim n_{k_2}^2$ per definition (3.3), but generally $w_{k_1}(n_{k_1}^2) \neq w_{k_2}(n_{k_2}^2)$. Correspondingly $u_{k_1}(y_{k_1*}) \neq u_{k_2}(y_{k_2*})$ and $u_{k_1}(n_{k_1}^2) = u_{k_2}(n_{k_2}^2)$.

Assume that $u_k(n_k^3)$ in (3.1.2) is not indefinite but equals b. b shall be a large figure such that (3.1.4) is a reasonable approximation of a thus redefined u_k. Now, if the theory in Section 3.2 is realistic, we can assume that a questioning process of "a representative" along (3.4) leads to a utility shape, which is approximately the same as shown by u_k. In particular, it should hold approximately that for any y_k and fixed $k \in K_2$

$$u_k(y_k) = w_k(y_k)/w_k(n_k^2) \tag{3.4.2}$$

Vice versa, as u_k in (3.1.4) or its first approximation (3.2.1) is easier to get, we know w_k approximately to be a linear transform of u_k, normalized to 0 and 1.

Let $k_1, k_2 \in K_2$ and ease notation in saying $\alpha = y_{k_1}$ and $\beta = y_{k_2}$. We will evaluate now some conditions, which lead to an aggregation of k_1 and k_2, that is, to a disutility measure denoted by $u(\alpha,\beta)$.

We have to assume that expected utility is an adequate way to calculate a utility judgment of a lottery $L_p((\alpha_1,\beta_1),(\alpha_2,\beta_2))$, that is, if (3.5)

$$(\hat{\alpha},\hat{\beta}) \sim L_p((\alpha_1,\beta_1),(\alpha_2,\beta_2)), \text{ then}$$

$$u(\hat{\alpha},\hat{\beta}) = p\cdot u(\alpha_1,\beta_1) + (1-p)\cdot u(\alpha_2,\beta_2).$$

Let us assume $u(\alpha^*,\beta^*) = 0$, (3.5.1)

further for all β (3.5.2)

$$(\hat{\alpha},\beta) \sim L_p((\alpha_1,\beta),\ (\alpha_2,\beta))$$

and for all α

$$(\alpha,\hat{\beta}) \sim L_p((\alpha,\beta_1),(\alpha,\beta_2)).$$

In the case (3.5.2) we call k_1 and k_2 to be mutually utility independent, that is, the pollutants show no synergism. As detailed knowledge is not available, it is not possible to consider them. In case that (3.5.2) is an acceptable assumption, then with (3.5) and linear transformability it holds

$$u(\alpha,\beta) = c_1(\beta) + d_1(\beta)\cdot u(\alpha,\beta_1)$$

$$u(\alpha,\beta) = c_2(\alpha) + d_2(\alpha)\cdot u(\alpha_1,\beta)$$

or

$$u(\alpha,\beta_1) = c_1(\beta_1) + d_1(\beta_1)\cdot u(\alpha,\beta_1) \qquad (3.5.3)$$

$$u(\alpha_1,\beta) = c_2(\alpha_1) + d_2(\alpha_1)\cdot u(\alpha_1,\beta)$$

and

$$u(\alpha_1,\beta) = c_1(\beta) + d_1(\beta)\cdot u(\alpha_1,\beta_1) \qquad (3.5.4)$$

$$u(\alpha, \beta_1)= c_2(\alpha) + d_2(\alpha)\cdot u(\alpha_1,\beta_1).$$

With $\alpha_1 = \alpha^*$ and $\beta_1 = \beta^*$, with (3.5.1) and substituting (3.5.4) in (3.5.3) we get

$$u(\alpha,\beta^*) = c_1(\beta^*) + d_1(\beta^*)\cdot c_2(\alpha)$$

$$u(\alpha^*,\beta) = c_2(\alpha^*) + d_2(\alpha^*)\cdot c_1(\beta)$$

Regarding (3.5.1) $c_1(\beta^*) = c_2(\alpha^*) = 0$, (3.5.5)

so we have

$$u(\alpha,\beta^*) = d_1(\beta^*)\cdot c_2(\alpha)$$

$$u(\alpha^*,\beta) = d_2(\alpha^*)\cdot c_1(\beta)$$

or with $\alpha = n^2_{k_1}$ and $\beta = n^2_{k_2}$ we can calculate $d_1(\beta^*)$ and $d_2(\alpha^*)$ and substitute it, so we get

$$u(\alpha,\beta^*) = \frac{u(n^2_{k_1},\beta^*)}{c_2(n^2_{k_1})}\cdot c_2(\alpha) \qquad (3.5.6)$$

$$u(\alpha^*,\beta) = \frac{u(\alpha^*,n^2_{k_2})}{c_1(n^2_{k_2})}\cdot c_1(\beta) .$$

The most important assumption now is that (3.5.7)

$$L_{0.5}((\alpha,\beta),(\alpha^*,\beta^*)) \sim L_{0.5}((\alpha,\beta^*),(\alpha^*,\beta)).$$

This means that one should be indifferent, e.g., in living in an area A or an area B if

A gives (high) concentrations α for pollutant k_1 and β for pollutant k_2 half the time and low concentrations α^* and β^* for both pollutants the rest of the time, B gives half the time (high) concentration for one pollutant, say α and k_1, with contemporary low concentrations for the other pollutant, say β^* and k_2

and vice versa for the other half of the time.

This assumption includes that no synergism applies as in (3.5.2). It states further that also psychologically no "synergism" should apply.

With (3.5) and (3.5.1) we get from (3.5.7)

$$u(\alpha,\beta) = u(\alpha,\beta^*) + u(\alpha^*,\beta) \qquad (3.5.8)$$

and with (3.5.6) (3.5.9)

$$u(\alpha,\beta) = \frac{u(n^2_{k_1},\beta^*)}{c_2(n^2_{k_1})} \cdot c_2(\alpha) + \frac{u(\alpha^*,n^2_{k_2})}{c_1(n^2_{k_2})} \cdot c_1(\beta)$$

With (3.4.2) and (3.3) we have β^* equaling $n^1_{k_2}$ and α^* equaling $n^1_{k_1}$ with utility zero each. With (3.3) we also have $n^2_{k_1} \sim n^2_{k_2}$ and so expect $u(n^2_{k_1},\beta^*) = u(\alpha^*,n^2_{k_2})$. So a linear transform of (3.5.9) with still $u'(\alpha^*,\beta^*) = 0$ runs

$$u'(\alpha,\beta) = \frac{c_2(\alpha)}{c_2(n^2_{k_1})} + \frac{c_1(\beta)}{c_1(n^2_{k_2})} \quad .$$

With (3.4.1) and (3.5.5) this is identical to

$$u'(y_{k_1}, y_{k_2}) = \frac{w_{k_1}(y_{k_1})}{w_{k_1}(n^2_{k_1})} + \frac{w_{k_2}(y_{k_2})}{w_{k_2}(n^2_{k_2})}$$

or with (3.4.2)

$$u'(y_{k_1}, y_{k_2}) = u_{k_1}(y_{k_1}) + u_{k_2}(y_{k_2}) \ . \qquad (3.5.10)$$

Note that we know now the conditions under which we are allowed to add the disutilities of two pollutants in order to get a disutility measure regarding both pollutants:

First, we have to say $u_k(y_k)$ are to be normative disutility functions.

Second, we say the pollutants k_1 and k_2 are to be additive utility independent.

If additive utility independence holds for any $k_1, k_2 \in K_2$, then we can derive similar to (3.5.10)

$$u'(y_k | k \in K_2) = \sum_{k \in K_2} u_k(y_k) \ . \qquad (3.6)$$

Referring to Chapters 1 and 2, we denote

$$u'(y_k | k \in K_2) = g(x)$$

if $(y_k | k \in K_2)$ represents the environment part of an option $x \in X$. So we have solved problem (1.4.1) and (1.1) with the aggregation advice (3.6). With (3.4.2) we use as $u_k(y_k)$ either formula (3.1.4) or (3.2.1). The first approximation (3.2.1) leads to

$$u'(y_k | k \in K_2) = \sum_{k \in K_2} y_k / n_k^2 \; . \tag{3.6.1}$$

Action based on these measures should be better anyway than decisions based on the present way of handling pollution information.

It should be noted here that as $u_k(y_k)$ is convex for (3.1.4) and (3.2.1) for all $k \in K_2$ so is $g_2(x)$ along (3.6), and we have provided for the conditions which were necessary in Chapter 2 to get a compromise between different controversial goals K_i, $i = 1,2,\ldots$.

3.4 Comparing Power Stations Environmentally

An example will be provided for the methodology given in the previous part of the Chapter 3. Emphasis is oriented to demonstrate the methodology and not to interpret results. So the data used are not claimed to be correct. We will calculate an air pollution index of the conventional power stations, fired by hard coal (Coal H), lignite (Coal L), oil heavy (Oil H), oil light (Oil L), natural gas (Gas) and of the nuclear power stations: light water reactor (LWR) and fast breeder reactors (FBR). As pollutants in the case of conventional power stations, we come up with sulfur dioxide (SO_2), nitrogenoxides (NO), carbon monoxide (CO), hydrocarbons (HC), fluorides (HF) and particulates (P). Pollutants considered in the nuclear case will be seen in the tables below.

The first task now should be to calculate a relevant number for ambient concentrations of those pollutants, related to the different power stations. To compare those power stations environmentally, we can assume the same site for all alternatively and therefore the average ambient concentration for a power station will be proportional to the average specific emission (e_k), as long as it is a good approximation to assume that the average residence time of the different pollutants in the relevant sphere with respect to ambient concentrations is about the same. As the dilution by wind in the average is the main factor regarding the residence time, this can be assumed. The environmental index needs only a correction, then, regarding the influence of the emission stack height. As we only want an interval scale, we can say y_k^j to be $\tau_j \cdot e_k^j$, if τ_j is the correction factor regarding the stack height of power station of type j with τ_j normalized, that is, this correction factor is one for the conventional power stations. This includes the effect of the additional virtual stack height, assumed to be about the same as the physical one of ca. 150 meters, and caused by the simultaneous heat emission. The lack of the additional virtual stack height in the nuclear case leads to a τ about 5 for them, assuming the same physical stack height. (For formulae see, e.g., [15].)

For normal operational conditions of conventional power stations, we have the following in Table 2.

Table 2

FOSSIL POWER STATIONS

	SO_2	NO	CO	CH	HF	P	
Specific Emissions kg/to SKE (1 to SKE is equivalent to $27.7 \cdot 10^6$ BTU)							
Coal H	22	10	0.2	0.02	0.15	1.5	
Coal L	23	10	0.2	0.3	0.3	2.5	
Oil L	7	2.5	0.3	0.2	0.01	0.1	
Oil H	25	7	0.3	0.3	0.01	0.8	
Gas	0.1	6	0.015	0.02	0	0.03	
Air Quality Norms (n_k in $\mu g/m^3$)							
	140	100	10^4	10^3	3	100	
Resulting Hazard Index (in m^3/kWh_{el} dilution demand)							
							Total
Coal H	51	32	0.007	0.007	24	4.7	112
Coal L	53	32	0.007	0.10	49	8.2	142
Oil L	16	8.2	0.010	0.07	1.6	0.32	26
Oil H	58	23	0.010	0.10	1.6	2.6	85
Gas	0.24	19	0.0005	0.007	0	0.10	20

The total column gives the air pollution index for the conventional power plants, with gas the best option. It can be seen further that HF in the coal cases has a major contribution to the disutility of the coal power stations and probably stands for other not so well known residuals. So at least the figures should be regarded as an indication to the direction of more detailed investigations rather than as complete numbers.

With [16] we assume in the nuclear case the emissions shown in the table below. The norms n_k^2 for the nuclear pollutants were calculated according to [17] and correspond to 150 mrem whole body dose equivalent. The resulting hazard indices (including $\tau = 5$) are shown in Table 3, which follows.

Table 3

NUCLEAR POWER STATIONS

Pollutant	LWR		FBR	
	Emission in $Ci/GW_e \cdot yr$	Hazard Index in m^3/kWh_{el} Dilution Demand	Emission in $Ci/GW_e \cdot yr$	Hazard Index in m^3/kWh_{el} Dilution Demand
H3	100	1.4	350	4.8
Kr 85	1300	11.7	350	3.2
Xe 131m	100	0.7	200	1.4
Xe 133	3000	27.2	4200	38.3
Xe 135	300	8.2	250	6.8
J 131	0.3	4.1	0.01	0.1
Total		53.3		54.6

For all other investigated pollutants (Sr 89, 90, Ru 103, 106, Cs 134, 137, C14) the hazard index remained below one. The main contribution comes from Xe 133. So the investigated power stations compare as follows:

Gas	Oil L	LWR	FBR	Oil H	Coal H	Coal L
20	26	53	55	85	112	142

One easily derives that if SO_2 in the Coal H case is fully abated, Coal H gets very close to the nuclear cases.

Note that a detailed comparison of those power stations also has to regard the total fuel cycles as well as to include the effects on water and food pollution. This is possible (see [18] for the nuclear case) and Chapter 4 will give some ideas, principally how to manage this. Safety problems, however, are better addressed by political discussion then as part of an aggregated environmental measure, as mentioned in Chapter 1 and the introduction to Chapter 3 (see also [19]). It should once more be mentioned that this example is only to demonstrate a methodology and cannot claim to have practical relevance. For the latter, more detailed investigations with respect to correct and complete data sets should be made.

4. An Approach to Environmental Modeling

In [1], the details of a linear program, that is, PILOT, modeling the interrelationships of the energy and the economy are described. From that it is clear that a variable environmental situation is obtained by different substitution technologies available and by assuming abatement industries. In each case, the emission of pollutants is changed and so the environmental hazard. How the environmental part can be incorporated in a model like PILOT on the basis of the previously explained theories will be discussed in the subsequent sections. Thus, problem (1.2) will be answered.

4.1. Estimation of Ambient Concentrations

The political problem is whether to count overall frequency of diseases or the probability of a single person getting the disease. Apparently, under the current political climate, the individual disease probability is used in establishing environmental quality norms, as those norms are applied without regard to the population density. In this case, y_k should represent the local maximum of time averaged ambient concentrations. However, there are some signs that a more epidemiological view of the problem may prevail. In this case, y_k should represent a locally averaged ambient concentration, weighted by the population density and possibly by the concentration's time dependence, reflecting daily, weekly and seasonal shifts in population and concentrations.

As the environmental disutility function g is an interval scale, u_k can be linearly transformed. Therefore, regarding (3.2.1), the absolute level of y_k is not important. That is, any reference measure for y_k can be chosen which shows a behavior proportional to what has been decided politically for y_k. It is clear, however, that with respect to the aggregation of the hazard index for different pollutants, the measure one takes for various y_k needs to be along the same approach.

One advantage of choosing to use only an interval scale is that in case of formula (3.2.1) the model builder needs no longer care which of these two approaches is used. In cases where pollutants occur from industries with site preferences which are very different with respect to the population density, the model user may apply a relative correction factor to y_k for those industries, as generally shown below, if going along the one approach; otherwise he would not. So, for ease of argument it can be assumed later that the local maximum of the time averaged ambient concentrations is the relevant figure (eventually corrected in the above mentioned sense), which means that the coordinates of the dilution process need not be considered. Remember that for these simplifications we have to assume that the linear disutility function (3.2.1) is good enough. Otherwise to choose a representative y_k grows more complicated or the approximations are rougher.

Next it is necessary to look for a reference measure for such a y_k. An ambient concentration on a certain point and related to one polluting object is proportional to the emission. The proportionality factor is called dilution factor. It depends on the pollutant, the medium of the distribution, the site of the polluting object, and special emission conditions. Let $E(\tau)$ be the amount of emission under dilution condition τ, where τ is known as the ratio of the actual dilution factor (for the maximum ambient concentration) with respect to the one of a basic dilution condition. As y_k can be linearly transformed, $\tau \cdot E(\tau)$ can be substituted for y_k. Regarding different pollutants the theory of distribution and dilution brings up the suggestion that the half-lifetime which they spend in hazard relevant areas should be one correction factor in τ. The comparison of calculated and empirical ambient concentrations shows, however, that it is a sufficiently good approximation not to change τ from a reference point, say $\tau = 1$, if the pollutant is changed. This means that to compare the air pollution impact of different fossil power stations the emission of pollutant k can be substituted for y_k because the site and emission conditions remain the same (see Section 3.4). $u'(y_k | k \in K_2)$ then is dimensionally "the amount of air needed to dilute to the norm concentration." To add nuclear power stations may make it advisable under certain circumstances, that is, if the potential hazards of pollutants with long half-lifetimes should be measured from a pessimistic point of view regarding a potential of their reentry in air, water, or food, to apply the physical half-lifetimes as a correction factor in τ to the emissions. Corresponding "half-lifetimes" then should be applied to the nonradioactive pollutants.

In any case, adding nuclear power stations to the comparison, we have to regard different emission conditions and apply a $\tau \neq 1$. The same is true for different sites (sites where meteorological conditions differ) and for different dilution media. (Assuming that emissions and norms also are known for pollutants in water, an average dilution factor for water can be compared with that of air to obtain a weighting factor τ_{Water}. Similar considerations could be made for food.) The corresponding τ can be calculated in separate models chosen from a broad literature in respect to those questions. This eases tremendously the calculation of $u'(y_k | k \in K_2)$ for PILOT.

It should be noted that this method implies that the time dependent economic course which PILOT projects into the future assumes the same proportions ecologically different regions are used for certain economic activities. If there is a shift, this could be reflected by applying different τ's to those industries for different time periods of PILOT.

Thus the following is proposed for PILOT: Let K_2 be a set of pollutants and J a set of polluting objects. Let M_m be a subset of J, where all $j \in M_m$ may have similar emission conditions. The subsets with different emission conditions are distinguished by the index m, summarized in M. Let L_ℓ be a subset of J with $j \in L_\ell$ showing similar sites and distinguish those sets by ℓ, summarized in L. As τ factorizes with respect to different causes of corrections, the indicator of ambient concentrations of pollutant $k \in K_2$ emitted by object $j \in J$, with $j \in M_m$ and $j \in L_\ell$, is $y_k^j = \tau_m \cdot \tau_\ell \cdot E_k^j$ with E_k^j the emission of pollutant k by object j. So one way to calculate $u'(y_k | k \in K_2)$ would be

$$u'(y_k | k \in K_2) = \sum_{k \in K_2} \frac{1}{n_k^2} \cdot (\sum_{\ell \in L} \tau_\ell \cdot (\sum_{m \in M} \tau_m \cdot (\sum_{j \in M_m \cap L_\ell} E_k^j))) .$$

If $x = (\ldots, x_j, \ldots)$, $j \in J$, shows the production level x_j of industry j and e_k^j denotes the specific emission, then $E_k^j = e_k^j \cdot x_j$. So the formula can be restated in a form which relates better to the variables of the linear program:

$$g(x) = \sum_{\ell, m, j} a_j \cdot x_j$$

with $a_j = \tau_\ell \cdot \tau_m \cdot (\sum_k e_k^j / n_k^2)$, $\ell \in L$, $m \in M$ and $j \in M_m \cap L_\ell$, $(M_m \cap L_\ell) \subset J$. This is a disutility function which enables calculation of the environmental impacts of each feasible solution $x \in X$ of a PILOT scenario.

4.2. Providing for Abatement Activities

So far we have discussed how to get utility functions for the environmental as well as the economic aspects of a model like PILOT and how to deal with those two different types of goal functions in order to get a solution which achieves a compromise between the desires of an environmental and economic point of view. Attention here is focused on interaction of industries which can improve or weaken the environmental situation. The easiest way to do this is to refer to the PILOT-E (E = Environment) matrix (Table 4) which follows. For simplification, assume that all industries show the same site patterns

Table 4

PILOT-E MATRIX

PRODUCTION INDUSTRIES				ABATEMENT "INDUSTRIES"								CONSUMPTION	ECONOMIC DEGREE OF DISSATISFACTION	AMBIENT CONCENTRATION INDICES	ENVIRONMENTAL INDEX	ENVIRONMENTAL DEGREE OF DISSATISFACTION	RIGHT HAND SIDE
ABATEMENT EASY		ABATEMENT DIFFICULT		LOW ABATEMENT MEASURES (UP TO QUOTA ξ)				HIGH ABATEMENT MEASURES (ABOVE ξ)									
LOW STACK	HIGH STACK	LOW STACK	HIGH STACK														
x_{11}	x_{12}	x_{21}	x_{22}	$z_{11}^{(\xi)}$	$z_{12}^{(\xi)}$	$z_{21}^{(\xi)}$	$z_{22}^{(\xi)}$	$z_{11}^{(1-\xi)}$	$z_{12}^{(1-\xi)}$	$z_{21}^{(1-\xi)}$	$z_{22}^{(1-\xi)}$	f	r_f	d	g	r_g	
Production Levels				Abatement Levels													
												1					MAX
$I-A_{11}$	$I-A_{12}$	$I-A_{21}$	$I-A_{22}$	$-G_{11}^{(\xi)}$	$-G_{12}^{(\xi)}$	$-G_{21}^{(\xi)}$	$-G_{22}^{(\xi)}$	$-G_{11}^{(1-\xi)}$	$-G_{12}^{(1-\xi)}$	$-G_{21}^{(1-\xi)}$	$-G_{22}^{(1-\xi)}$	$-Y$					≥ 0
ξE_{11}				$-I$													≥ 0
	ξE_{12}				$-I$												
		ξE_{21}				$-I$											
			ξE_{22}				$-I$										
$(1-\xi)E_{11}$								$-I$									
	$(1-\xi)E_{12}$								$-I$								
		$(1-\xi)E_{21}$								$-I$							
			$(1-\xi)E_{22}$								$-I$						
												α_f	-1				$= \beta_f$
$\tau_1 \cdot E_{11}$	$\tau_2 \cdot E_{12}$	$\tau_1 \cdot E_{21}$	$\tau_2 \cdot E_{22}$	$-\tau_1$	$-\tau_2$	$-\tau_1$	$-\tau_2$	$-\tau_1$	$-\tau_2$	$-\tau_1$	$-\tau_2$			$-I$			$= 0$
														N	-1		$= 0$
															α_g	-1	$= \beta_g$
													1			-1	≥ 0

with respect to environmental aspects, i.e., L has only one element. Further distinguish only between two emission conditions, for example, high stack and low stack, i.e., M has two elements. Also, with respect to abatement costs, distinguish only between two industry groups, and with respect to the desired abatement level, distinguish between two linear cost relations, thus linearizing the real cost relations depending on the abatement level. Extensions to more detailed breakdowns of industry groups or cost functions easily are possible.

$E_{mn} = [e_k^j]$, $k \in K_2$, $j \in J$ and j belonging to emission condition m and abatement cost structure n shows the specific emission e_k^j for the m and n related subset of J. ξ denotes the abatement level vector with respect to different cost functions, one for abatements between 0 and ξ and one between ξ and 1. $G_{mn}^{(\xi)}$ and $G_{mn}^{(1-\xi)}$ are matrices, which show the corresponding "costs" in industry j if one unit of pollutant k has to be abated, i.e., $G_{mn}^{(\xi)} = [g_{jk}^{(\xi)}(m,n)]$, $j \in J$, $k \in K$. It has to be assumed that $g_{jk}^{(1-\xi)}(m,n) > g_{jk}^{(\xi)}(m,n)$ for all i, k, m, and n. The abatement levels $z_{mn}^{(\xi)}(k)$ and $z_{mn}^{(1-\xi)}(k)$ are subject to the optimization procedure. A_{mn} shows the corresponding industry interrelationships of the model without environmental aspects. To obtain more information, the bookkeeping variables are introduced:

$$d_k = \sum_{\ell \in L} \tau_\ell \cdot (\sum_{m \in M} \tau_m (\sum_{j \in M_m \cap L_\ell} e_k^j \cdot x_j)) , \qquad k \in K ,$$

with the environmental disutility function

$$g(x) = \sum_{k \in K_2} d_k/n_k^2 .$$

With $d = (\ldots, d_k, \ldots)$ a column vector and $N = (\ldots, 1/n_k^2, \ldots)$ a row vector, $k \in K_2$, then $g(x) = N \cdot d$. Proceeding as shown previously, the best economic solution f^* and the worst ecological solution g_* will be obtained by calculating the linear program without the condition $r_g \leq r_f$ (last row in the matrix shown). The worst economic solution f_* and the best ecological solution g^* are obtained by asking for full abatement; that is, the inequalities related to the shaded area on the table should become equalities (still without condition $r_g \leq r_f$). This provides according to (2.2.2) for the calculation of

$$r_f(x) = \alpha_f \cdot f(x) - \beta_f \quad \text{and} \quad r_g(x) = \alpha_g \cdot g(x) - \beta_g$$

with

$$\alpha_f = 1/(f_* - f^*), \quad \alpha_g = 1/(g_* - g^*)$$

$$\beta_i = f_i^* \cdot \alpha_i \quad \text{for} \quad i \in [f,g],$$

and the compromise solution with $r_g \leq r_f$ can be calculated.

It should be clear from the matrix how parts of the model are linked. It may be worthwhile to impose the condition $r_g \leq r_f$ separately for each time interval. In addition, to allow only capacity buildups that do not result in any decline of consumption, it may be wise to delay the introduction of the condition $r_g \leq r_f$ until the year 2000; the periods before 2000 can be forced to prepare for no decline in f

or for less strong compromise conditions, introducing $\rho \cdot r_g \leq r_f$ with $0 < \rho < 1$, and $\rho = 0$ for the first period. A third way to provide for a reasonable, slow capacity buildup would be to introduce a separate and growth restricted environmental capacity industry.

In the matrix shown, there was no assumption of any pollution by the consumer. That pollution has been included in a supply industry. For example, the pollution caused by heating may be counted at the oil producing industry. As the oil industry also contributes to power stations with other emission conditions, the oil industry may be separated into two artificial, customer related industries. But this illustrates only one of two basic philosophies of attaching pollution. This procedure is easy only if one does not extensively distinguish between different emission conditions or abatement costs, as in the example below.

In the case where the environmental effects should be introduced in full detail, it is recommended that the pollution be attached to the industry where it is emitted. The emissions of the oil producing industry, for example, are then low, but, i.e., SO_2 from oil could be attached to all oil consuming industries and the consumer himself. No additional difficulties arise. Full detail then can be introduced easily with respect to different emission conditions, different abatement costs, and so on, without any artificial splitting of industries by user of its product.

It should be noted further that where a model provides for technology substitution, the optimal technological mix will be influenced also by the environmental goal; that is, the more a model provides for technology substitution possibilities with environmental advantages, the less abatement activities will be needed.

4.3. A Three Sector Economy with Two Pollutants

To provide a demonstration of the theory, TEST PILOT 1, described in [1, Appendix I] is used. It distinguishes only between electric and nonelectric industries and the rest of the industries. Only emissions of the energy sector and only sulfur dioxide and hydrocarbons will be regarded. The rather arbitrarily made up entries are shown in Table 5, which follows. A correction factor $\tau = 9$ for SO_2 in the nonelectric energy sector and $\tau = 20$ for HC, related to traffic only, is used because of a much lower average emission height than electric power plants would have. The aspiration level n_k^2 (maximum permissible concentration) for SO_2 is 80 $\mu g/m^3$ and for HC 160 $\mu g/m^3$.

The results are shown below in Table 6 for the case regarding only SO_2 and the case of SO_2 and HC. It shows that the results are depending on the level of detail at which the environmental sector is modeled. The demonstration example still shows small effects of environmental compromise measures on the economy; these effects will grow if more important pollutants and emissions (also in the nonenergy sectors) are regarded. In addition, if the theory is applied to a model like PILOT and its substitution technologies, major changes in the energy supply picture are expected to occur.

Although no practical conclusions can be drawn from this example, it may help to understand the procedure outlined in this paper.

Table 5

TEST PILOT 1-E: AN ENVIRONMENTAL FLOW MODEL OF THE ECONOMY

ROW \ COL	XENE	XELE	IENE	EENE	CONSV	XNS	XLS	NH8	NHO	XS	XH	XRC	XDS	XDH	XD	XRC	RHS
BNES	TEST PILOT 1					-0.4	-0.4	-0.1	-0.4								
NS	0.30		0.30	-0.30		-1											
LS		0.60					-1										≥ 0
NH8	0.30		0.30	-0.30				-1									
NHO	0.075		0.075	-0.075					-1								
DS	2.7	0.60	2.7	-2.7		-9	-1						-1				= 0
DH	7.5		7.5	-7.5				-20	-20					-1			= 0
D													$\frac{1}{80}$	$\frac{1}{160}$	-1		≤ 0
RD															α_g	-1	$= \beta_g$
RC					α_f							-1					$= \beta_f$
R												1				-1	≥ 0
S	0.30	0.60	0.30	-0.30		-1	-1			-1							= 0
H	0.375		0.375	-0.375				-1	-1		-1						= 0

LEGEND:

BNES, XENE, XELE, IENE, EENE, CONSV	as in TEST PILOT 1 of Appendix I
NS	SO_2 emission of nonelectric energy sector
LS	SO_2 emission of electric sector
NH8	80% of HC emission of nonelectric energy sector
NHO	20% of HC emission of nonelectric energy sector
DS, DH, D, RD, RC, S, H	equations to calculate the corresponding bookkeeping variables with the prefix X.
R	constraint to guarantee a compromise solution
XNS, XLS, XNH8, XNHO	abatement activity levels of the corresponding emission relations without the prefix X.
XS	SO_2 emission
XH	HC emission
XRC	dissatisfaction degree for economy
XDS	environmental index SO_2
XDH	environonental index HC
XD	environmental index for aggregated XDS and XDH
XRD	dissatisfaction degree for environment
RHS	right hand side of the relations

Economic data in 10^9\$/yr respectively 10^9\$/10^6 tons

Environmental data in 10^6 tons/yr respectively 10^6 tons/10^9\$

XD in $10^{18} m^3/10^9$\$.

Table 6

RESULTS OF THE DEMONSTRATION EXAMPLE

		WITH SO_2 ONLY		WITH SO_2 AND HC	
	NO ABATEMENT	FULL ABATEMENT	COMPROMISE	FULL ABATEMENT	COMPROMISE
Consumption 10^9\$/yr	676	665	672	662	671
SO_2-Emission 10^6 tons/yr	37	0	23	0	32
HC-Emission 10^6 tons/yr	26	--	--	0	0
Environment Index km^3/\$ — only SO_2	2.6	0	0.97	--	--
Environment Index km^3/\$ — SO_2 + HC	5.8	--	--	0	1.95
Abatement Level — SO_2 nonelectric	0	1	0.67	1	0.26
Abatement Level — SO_2 electric	0	1	0	1	0
Abatement Level — HC	0	--	--	1	1
Dissatisfaction Degree — Economy	0	1	0.38	1	0.33
Dissatisfaction Degree — Environment	1	0	0.38	0	0.33
α_f		-0.089606		-0.070028	
β_f		-60.616		-47.373	
α_g		0.0048744		0.17121	
β_g		0		0	

5. Concluding Remarks

Sometimes more philosophical questions arise regarding the benefit of models as described in this paper. In those cases frequently two positions can be regarded. Position one says, with respect to the incompleteness of models as the one described and the incorrectness of data, those models never can replace expert judgment in decision making; intuition is the better guide in complex decision situations. Position two claims the outcome of the model is more rational than any intuitive expert decision since it is a composite of diverse expert knowledge, processed in a logical manner.

The author of this paper wishes to stress the point that philosophy already has overcome those extreme and controversial positions and regards the benefit of models to lie in between, giving aid to intuitive judgment. This needs a brief clarification. The result of a model should not be regarded as showing the right decision. Uncertain parameters influencing the results often are better varied than seemingly improved. But using models is a good way to become an expert as they help to select and condense information and to make cause-effect chains obvious. Last not least they provide, especially where the expert feels unsatisfied with the model, for intuitive thinking. So models may be experimental tools for experts who advise decision makers (or for decision makers themselves, if they are able to interpret what the model says) rather than forecasting instruments, even if understood normatively. In this sense models rationalize thinking and stimulate intuition, especially if a decision requires reference to different kinds of expert knowledge, but they never replace a personalized decision. To make the point, one could even say that to deal with a model is a necessary but peculiar way of thinking that is inherent to scientists, but can become dangerous if transferred to one who wants to replace thinking by calculating. In other words, if a model has served to create the ideas leading to a decision, it should be possible and it is advisable to defend the decision without the help of the model in an argumentative way. So positions one and two need not be controversial. It is advisable that experts exercise and show their prudence by checking their theories with models but that they are able to and in practice do explain their advice not in referring to a model but to logical and intuitive aspects which are of political importance and so of value for the decision maker. Those arguments may easier include a wide range of imponderables, not regarded in models (but envisioned more easily by the use of models).

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