40 Years of Linear Algebra and Optimization at Stanford

NA History @ Stanford Lecture

SIAM Student Chapter

Stanford University

March 14, 2007

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Abstract

I came to Stanford in 1967 as a very green graduate student (not in today's ecological sense). Computer Science was a new department, as was Operations Research. The CS qualifying exams allowed 3 out of 5 topics, including numerical analysis. Alan George and I obtained permission to take one of the OR exams. Thus began a career of applying stable matrix methods to numerical optimization (as pioneered by Gene Golub, Philip Gill, and Walter Murray).

We trace the impact of Gene inviting numerous researchers to Serra House (including Chris Paige and Bruce Murtagh), as well as George Dantzig's creation of the Systems Optimization Laboratory in the OR Department, and Gene's founding of the SCCM Program.

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- 1966 **DSIR, Wellington**
  1967 Stanford CS

CS Dept 1966

George Forsythe, A university's educational program in computer science, Report CS39, May 1966.

Numerical analysis Algorithms for linear algebra, linear PDEs, nonlinear systems, libraries

Computer control Linear accelerator, data reduction, portable computing

Time-sharing How to use TV display instead of typewriters?

Artificial intelligence Speech recognition, picture recognition, classification of organic molecules, simulation of human learning

Programming languages Successors to ALGOL 60, languages for persons designing algorithms in a conversational mode at a console, operating systems, languages for compilers

Hardware Modular arithmetic

Computational linguistics Manipulating transformational grammars

Theorem proving

Psychiatry

CS Dept 1967–68

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  CS204: **A fun course**

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  McKeeman: Compiler-compiler

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- Wylbur line editor
  - IBM Selectric typewriter
  - Printing a letter (Don Knuth observing)

Serra House 1970–72

- Gene

  LU updating earlier (Bartels)

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- Visitors
  - Mike Osborne, Roger Fletcher, Iain Duff
  - Chris Paige
  - Bruce Murtagh

LU Terminology

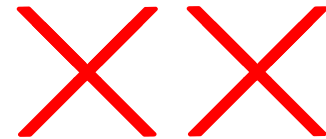(Since LU and QR updates have been Gene's favorites)

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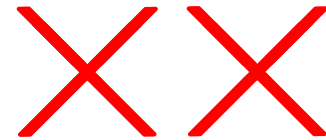(Since LU and QR updates have been Gene's favorites)

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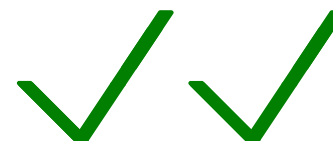
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- LDU with L and U well-conditioned
  Rank-revealing
  LUSOL (rook pivoting), MA27, MA57

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- Gerd Infanger
  1989 Visit from Vienna for 1 year (now 18 years)
  George Dantzig's right-hand man
  DECIS stochastic optimization software

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- Star students (and codes)
  1987 Irv Lustig   → CPLEX Barrier
  2002 Michael Friedlander Knossos Stabilization of MINOS
  2005 Holly Jin SpaseLoc Scalable sensor localization
  2006 Sou-Cheng Choi MINRES-QLP Singular symmetric $Ax = b$

SCCM

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- History
  A whole 'nother talk . . .

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- Long live the King (Gene)
  Chair of CS, President of SIAM
  Founder of SISSC, SIMAX
  Founder and Director of SCCM
  Universal coauthor, Host to Hundreds

Unsung heroes

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Generalized Least Squares, GSVD, TLS, GPS, etc

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Covariance matrix representation in linear filtering, *Contemporary Mathematics* 47, 1985

$$
\begin{aligned}
x_{k+1} &= A_k x_k + B_k w_k, & E(w_k w_k^T) &= Q_k \\
y_k &= C_k x_k + v_k, & E(v_k v_k^T) &= R_k
\end{aligned}
$$

Covariances Q_k and R_k may have **any rank**

Aerospace Applications

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    1974–2007 NASA Ames

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- Projects
    - OAW Oblique All-Wing supersonic airliner
    - HSCT Supersonic airliner
    - SHARP Shuttle design
    - CEV Apollo-type capsule to ISS, moon

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  F-4 Minimum time-to-climb

  DC-X SSTO Prototype Minimum-fuel landing maneuver

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- Antony Jameson, Juan Alonso Aerospace Computing Lab
    - MDO Multidisciplinary Design Optimization
    - ASO Aerodynamic Shape Optimization

AEROSPACE.pdf

News Flash, 3 March 2007

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- GMT 062:19:26
  The International Space Station was successfully maneuvered
  using DIDO/TOMLAB/SNOPT
  Found zero propellant solutions (globally optimal)
  Saved NASA $1M fuel cost

The Lighter Side of Optimization

In New Zealand, the equivalent of the TV guide is called **The Listener**. Every week a Life in New Zealand column publishes clippings describing local events. The first sender receives a $5 Lotto Lucky Dip. The following clippings illustrate some characteristics of optimization problems in the real(?) world.

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Robust solutions

RECOVERY CARE gives you financial protection from specified sudden illness. You get cash if you live ... and cash if you don't.

No objective function

People have been marrying and bringing up children for centuries now. Nothing has ever come of it.   (Evening Post, 1977)

Multiple objectives

"I had the choice of running over my team-mate or going onto the grass, so I ran over my team-mate then ran onto the grass", Rymer recalled later.

Obvious objective

He said the fee was increased from $5 to $20 because some people had complained it was not worth writing a cheque for $5.

Equilibrium condition

"The pedestrian count was not considered high enough to justify an overbridge", Helen Ritchie said. "And if there continues to be people knocked down on the crossing, the number of pedestrians will dwindle."

Constraints

ENTERTAINERS, DANCE BAND, etc. Vocalist wanted for New Wave rock band, must be able to sing.

DRIVING INSTRUCTOR Part-time position. No experience necessary.

HOUSE FOR REMOVAL in excellent order, $800. Do not disturb tenant.

Exactly one feasible solution

MATTHEWS RESTAURANT, open 365 nights. Including Mondays.

Buying your own business might mean working 24 hours a day. But at least when you're self-employed you can decide which 24.

Peters: Oh, it's not that I don't want to be helpful. But in this case the answer is that I don't want to be helpful. (Listener, 1990)

Sergeant J Johnston said when Hall was stopped by a police patrol the defendant denied being the driver, but after it was pointed out he was the only person in the car he admitted to being the driver.

His companion was in fact a transvestite, X, known variously as X or X.

Bound your variables

By the way, have you ever seen a bird transported without the use of a cage? If you don't use a cage it will fly away and maybe the same could happen to your cat. Mark my words, we have seen it happen.

Redundant constraints

If you are decorating before the baby is born, keep in mind that you may have a boy or a girl.

EAR PIERCING while you wait.

CONCURRENT TERM FOR BIGAMY   (NZ Herald, 1990)

Infeasible constraints

I chose to cook myself to be quite sure what was going into the meals.

We apologize to Wellington listeners who may not be receiving this broadcast.

The model 200 is British all the way from its stylish roofline to its French-made Michelin tyres. (NZ Car Magazine)

BALD, 36 yr old, handsome male seeking social times and fun with bald 22 years and upwards female Napier Courier, 28/2/02

$\ge$ or $\le$?

BUY NOW! At $29.95 these jeans will not last long!

NOT TOO GOOD TO BE TRUE! We can sell your home for much less than you'd expect! (NZ Property Weekly)

The BA 146's landing at Hamilton airport was barely audible above airport background noise, which admittedly included a Boeing 737 idling in the foreground.

Yesterday Mr Palmer said "The Australian reports are not correct that I've seen, although I can't say that I've seen them".

It will be a chance for all women of this parish to get rid of anything that is not worth keeping but is too good to throw away. Don't forget to bring your husbands.

$\geq$ or $\leq$?

The French were often more blatant and more active, particularly prop X and number eight Y, but at least one All Black was seen getting his retaliation in first.

WHAT EVERY TEENAGER SHOULD KNOW — PARENTS ONLY

"Love Under 17" Persons under 18 not admitted.

"Keeping young people in the dark would not stop them having sex—in fact it usually had the opposite effect," she said.

NELSON, approximately 5 minutes from airport. Golf course adjacent. Sleeps seven all in single beds. Ideal for honeymoons. (Air NZ News, 1978)

Hard or soft constraints

The two have run their farm as equal partners for 10 years, with Jan in charge of grass management, Lindsay looking after fertilizer, and both working in the milk shed. "We used to have our staff meetings in bed. That got more difficult when we employed staff!" (NZ country paper)

Elastic constraints

The Stationary Engine Drivers Union is planning rolling stoppages.

When this happens there are set procedures to be followed and they are established procedures, provided they are followed.

APATHY RAMPANT? Not in Albany—the closing of the electoral rolls saw fully 103.49 percent of the area's eligible voters signed up.

Auckland City ratepayers are to be reminded that they can pay their rates after they die.   (Auckland Herald, 1990)

He was remanded in custody to appear again on Tuesday if he is still in the country.

Convergence

"There is a trend to open libraries when people can use them", he says.

Mayor for 15 years, Sir Dove-Myer wants a final three years at the helm "to restore sanity and stability in the affairs of the city".

Applications

(Yachting) It is not particularly dangerous, as it only causes vomiting, hot and cold flushes, diarrhoea, muscle cramping, paralysis, and sometimes death ...   (Boating New Zealand, 1990)

(Ecological models) CAR POLLUTION SOARS IN CHRISTCHURCH—BUT CAUSE REMAINS MYSTERY

Nappies wanted for window cleaning. Must be used.

(Optimal control) Almost half the women seeking fertility investigations at the clinic knew what to do to get pregnant

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(Optimal control) Almost half the women seeking fertility investigations at the clinic knew what to do to get pregnant, but not when to do it.

Binary variables

0 or 1   Sometimes neither is optimal

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Integer variables

0 or 1 or 2 ...

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When Taupo police arrested a Bay of Plenty man
for driving over the limit,
they discovered he was a bigamist.   Nelson Mail, 5/04

Always room for improvement

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SC̦iCME

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Postscript

Curve Smoother

Holly Jin and MS

- Motivated by DARPA Grand Challenge 2005
  Won by Stanford's robot vehicle Stanley

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- Nonlinear objective function, bounds on the variables
  Solved by SNOPT

Smooth curve from A to B with given directions at endpoints