

Computing Sparse PageRank Vectors by Basis Pursuit

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Joint work with

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Outline

1 Abstract

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- 2 Sparse solutions

Outline

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- 3 PageRank

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- 4 Web matrices

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- 1 Abstract
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- 5 Sparse PageRank
- 6 BP solvers

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- 4 Web matrices
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- 7 Numerical results

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- 6 BP solvers
- 7 Numerical results
- 8 Preconditioning

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- 1 Abstract
- 2 Sparse solutions
- 3 PageRank
- 4 Web matrices
- 5 Sparse PageRank
- 6 BP solvers
- 7 Numerical results
- 8 Preconditioning
- 9 Summary

Abstract1

- Original problem:

Solve $Ax = b$ A square, sparse

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- Approximate solution:

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- Sometimes $x \geq 0$ naturally (e.g. PageRank)
- Apply optimization to choose nonzero x_j one by one

Abstract2

Many imaging and compressed sensing applications seek
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Personalized PageRank eigenvectors involve square systems

$$(I - \alpha H^T)x = v \quad H, v \text{ sparse}$$

where $0 < \alpha < 1$ $He \leq e$ $x \geq 0$, sparse

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Perhaps **BPDN** can find a *very sparse approximate* x

Sparse solutions to $Ax = b$

Sparse x Lasso(ν) Tibshirani 1996

$$\min \frac{1}{2} \|Ax - b\|^2 \quad \text{st} \quad \|x\|_1 \leq \nu$$

$$A = \boxed{}$$

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(Ax , $A^T y$ are efficient)

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BPDN(λ) Same paper

$$\min \lambda \|x\|_1 + \frac{1}{2} \|Ax - b\|^2$$

fast operator, any shape

$\|x\|_1$ is closest convex approximation to $\|x\|_0$

BP and BPDN algorithms

OMP	Davis, Mallat et al 1997	Greedy
BPDN-interior	Chen, Donoho & S, 1998, 2001	Interior, CG
PDSCO, PDCO	Saunders 1997, 2002	Interior, LSQR
BCR	Sardy, Bruce & Tseng 2000	Orthogonal blocks
Homotopy	Osborne et al 2000	Active-set, all λ
LARS	Efron, Hastie, Tibshirani 2004	Active-set, all λ
STOMP	Donoho, Tsai, et al 2006	Double greedy
l1_ls	Kim, Koh, Lustig, Boyd et al 2007	Primal barrier, PCG
GPSR	Figueiredo, Nowak & Wright 2007	Gradient-projection

Basis Pursuit Denoising (BPDN)

Pure LS

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Smaller $\|x\|$, bigger $\|r\|$

Let $r = \lambda y$

$$\text{BPprimal}(\lambda) \quad \min_{x,y} \|x\|_1 + \frac{1}{2}\lambda\|y\|^2 \quad \text{st} \quad Ax + \lambda y = b$$

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Suggests regularized LP problems:

$$\text{LPprimal}(\lambda) \quad \min_{x,y} c^T x + \frac{1}{2}\lambda\|y\|^2 \quad \text{st} \quad Ax + \lambda y = b, \quad x \geq 0$$

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Friedlander and S 2007: New active-set MATLAB codes

PageRank

PageRank

Langville and Meyer 2006

$$H = \begin{pmatrix} 1/3 & & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 & 0 \\ & 0 & 0 & 0 \\ & 1/2 & & 1/2 \end{pmatrix}$$

Hyperlink matrix

$$He \leq e$$

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$$v = \frac{1}{n}e \text{ or } e_i$$

Teleportation vector
or personalization vector

$$e^T v = 1$$

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$$Se = e$$

$$G = \alpha S + (1 - \alpha)ev^T$$

Google matrix
 $\alpha = 0.85$ say

$$Ge = e$$

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eigenvector \equiv linear system

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eigenvector	\equiv	linear system
$G^T x = x$	\equiv	$(I - \alpha H^T)x = v$

In both cases, $x \leftarrow x/e^T x$ (and $x \geq 0$)

Web matrices

Web matrices H

Name	n (pages)	nnz (links)	i	$(v = e_i)$
csStanford	9914	36854	4	cs.stanford.edu
Stanford	275689	1623817	23036	cs.stanford.edu
			6753	calendus.stanford.edu
Stanford-Berkeley	683446	7583376	6753	unknown

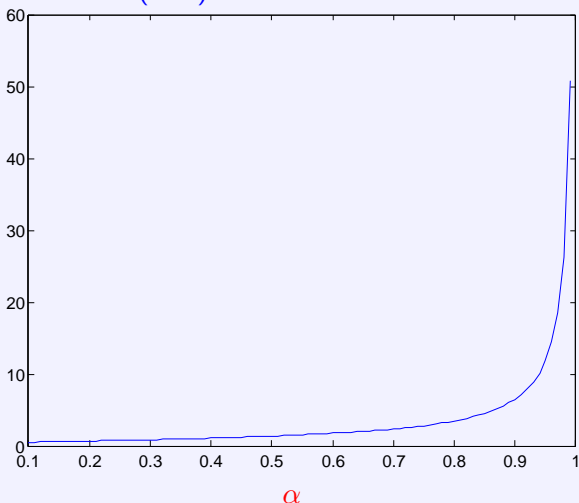
All data collected around 2001

Cleve Moler

Sep Kamvar

David Gleich

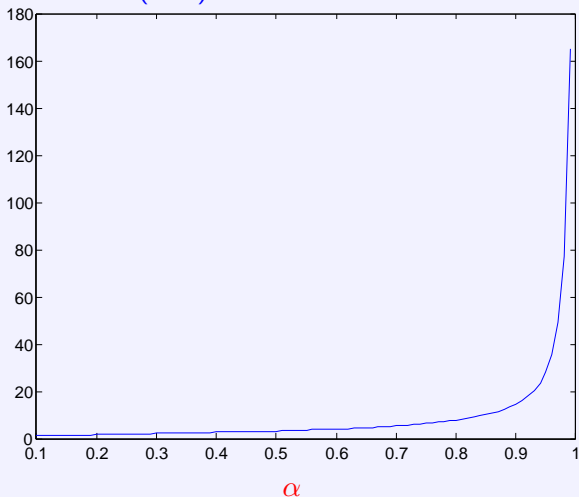
$$(I - \alpha H^T)x = v$$

Power method on $G^T x = x$ Stanford, $n = 275000$, $v = e_{6753}$ (calendus.stanford.edu)Time (secs) vs $\alpha = 0.1 : 0.01 : 0.99$ 

Power method

Stanford-Berkeley, $n = 683000$, $v = e_{6753}$

Time (secs) vs $\alpha = 0.1 : 0.01 : 0.99$

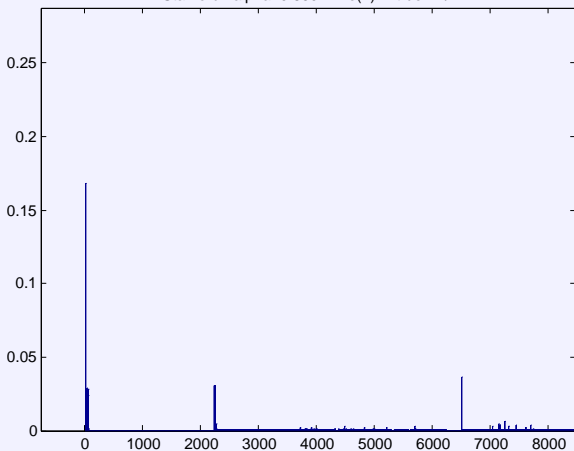


α

Exact $x(\alpha)$ csStanford, $n = 9914$, $v = e_4$ (cs.stanford.edu)

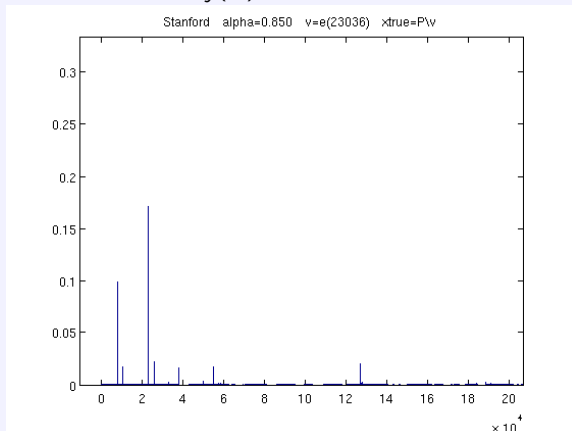
$$x_j(\alpha), j = 1 : n$$

Stanford alpha=0.850 v=e(4) xtrue=P\v

 $\alpha = 0.85$

Exact $x(\alpha)$ Stanford, $n = 275000$, $v = e_{23036}$ (cs.stanford.edu)

$$x_j(\alpha), \quad j = 1 : n$$

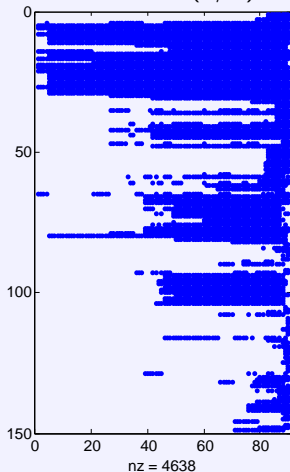


$$\alpha = 0.85$$

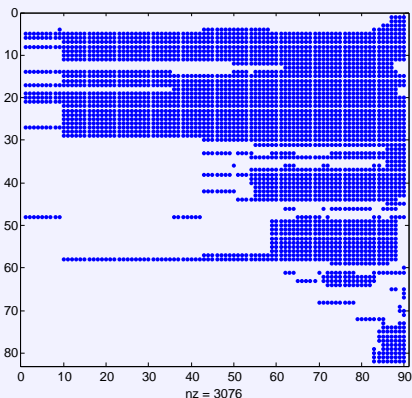
Sparsity of $x(\alpha)$

Stanford, $n = 275000$, $v = e_{23036}$ (cs.stanford.edu)

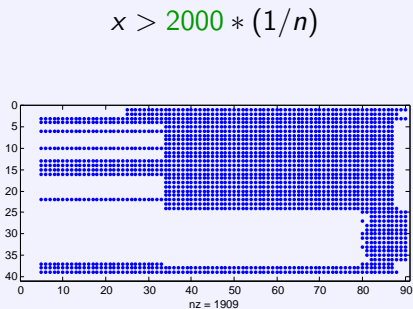
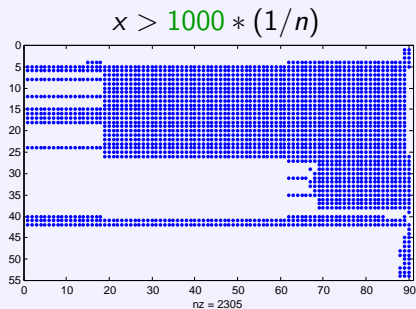
$$x > 250 * (1/n)$$



$$x > 500 * (1/n)$$



$\alpha = 0.1 : 0.01 : 0.99$ (90 values)



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Sparse PageRank

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Regard as

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Apply active-set solver to

$$\min_{x,r} \lambda e^T x + \frac{1}{2} \|r\|^2 \quad \text{st} \quad Ax + r = v, \quad x \geq 0$$

BP solvers

LP primal solver

$$\min_{x,y} e^T x + \frac{1}{2} \lambda \|y\|^2 \quad \text{st} \quad Ax + \lambda y = v, \quad x \geq 0$$

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Seems to proceed in a *greedy way*

Starts with $x = 0$, chooses one x_j at a time

s nonzero $x_j \Rightarrow s$ iterations

LP primal solver

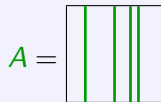
$$\min_{x,y} e^T x + \frac{1}{2} \lambda \|y\|^2 \quad \text{st} \quad Ax + \lambda y = v, \quad x \geq 0$$

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Starts with $x = 0$, chooses one x_j at a time

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B = columns of A with $x_j > 0$



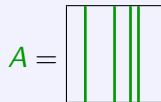
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Main work per itn:

$$\text{Form } z = A^T y$$

$$\text{Solve } B^T B dx = c_B - B^T y$$

$$\text{Form } dy = B dx$$

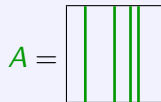
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Main work per itn:

Form $z = A^T y$

Solve $B^T B dx = c_B - B^T y$ Update $QB = \begin{pmatrix} R \\ 0 \end{pmatrix}$ without Q

Form $dy = B dx$

LPdual solver

$$\min_y -v^T y + \frac{1}{2} \lambda \|y\|^2 \quad \text{st} \quad A^T y \leq e$$

LPdual solver

$$\min_y -v^T y + \frac{1}{2} \lambda \|y\|^2 \quad \text{st} \quad A^T y \leq e$$

B = columns of A corresponding to $B^T y = e$ (active constraints)

LPdual solver

$$\min_y -v^T y + \frac{1}{2} \lambda \|y\|^2 \quad \text{st} \quad A^T y \leq e$$

B = columns of A corresponding to $B^T y = e$ (active constraints)

Main work per itn:

Solve $\min \|Bx - g\|$

Form $dy = (g - Bx)/\lambda$

Form $dz = A^T dy$

LPdual solver

$$\min_y -v^T y + \frac{1}{2} \lambda \|y\|^2 \quad \text{st} \quad A^T y \leq e$$

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Main work per itn:

$$\begin{aligned} &\text{Solve } \min \|Bx - g\| && \text{Update } QB = \begin{pmatrix} R \\ 0 \end{pmatrix} \text{ without } Q \\ &\text{Form } dy = (g - Bx)/\lambda \\ &\text{Form } dz = A^T dy \end{aligned}$$

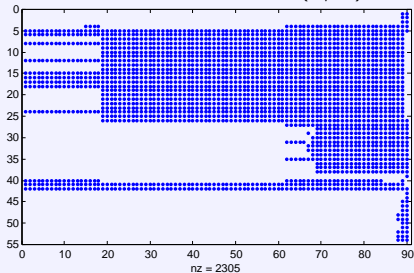
Selects B in same *greedy* manner as LPprimal in almost same order

Numerical results with BP solvers

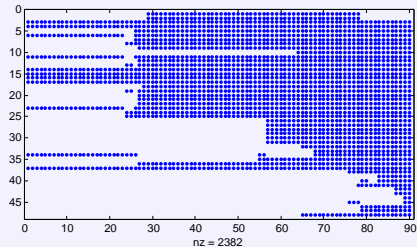
Sparsity of $xBP(\alpha)$

Stanford, $n = 275000$, $v = e_{23036}$ (cs.stanford.edu)

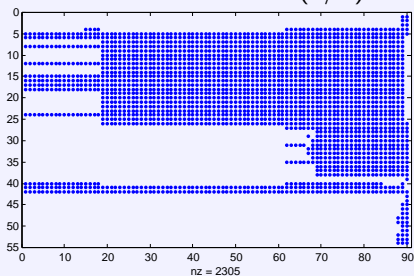
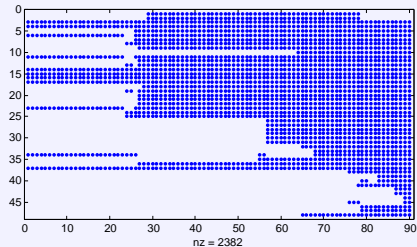
$xPower > 1000 * (1/n)$



$xBP > 1 * (1/n)$



$\alpha = 0.1 : 0.01 : 0.99$ (90 values)

Sparsity of $xBP(\alpha)$ Stanford, $n = 275000$, $v = e_{23036}$ (cs.stanford.edu) $xPower > 1000 * (1/n)$  $xBP > 1 * (1/n)$  $\alpha = 0.1 : 0.01 : 0.99$ (90 values)

$xPower > 1 * (1/n)$ has
6000 rows, 155000 nonzeros

 $\lambda = 4e-3$

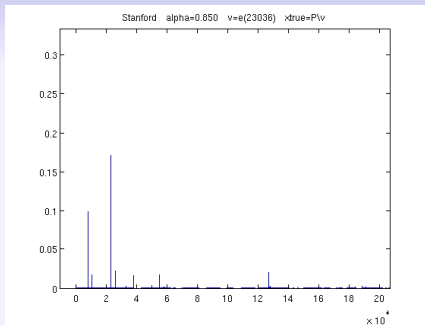
Stanford web, $n = 275000$

$\alpha = 0.85$

$v = e_{23036}$ (cs.stanford.edu)

Direct solve:

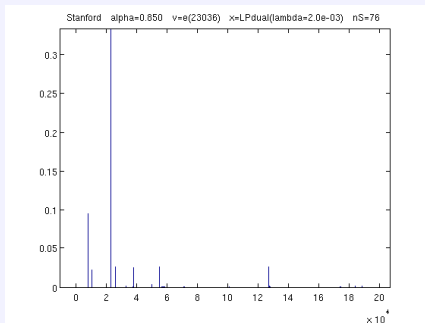
$x_{\text{true}} = A \setminus v$



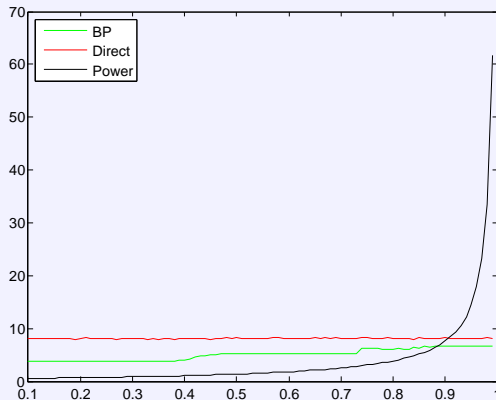
Basis Pursuit

$\lambda = 2e-3$

76 nonzero x_j



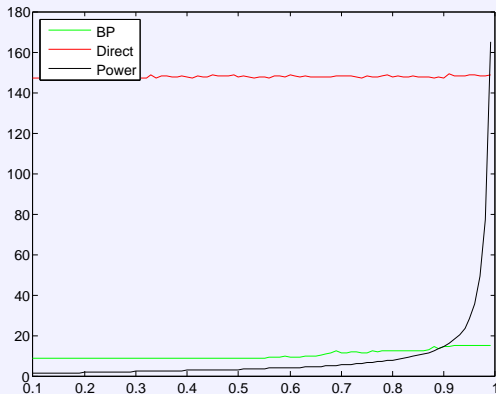
Power method vs BP

Stanford, $n = 275000$, $v = e_{23036}$ (cs.stanford.edu)Time (secs) for each α  $\alpha = 0.1 : 0.01 : 0.99$ (90 values) $\lambda = 4e-3$

Power method vs BP

Stanford-Berkeley, $n = 683000$, $v = e_{6753}$ (unknown)

Time (secs) for each α



$\alpha = 0.1 : 0.01 : 0.99$ (90 values)

$\lambda = 5e-3$

Varying α and λ

$$\min_{x, r} \lambda \|x\|_1 + \frac{1}{2} \|r\|^2 \quad \text{st} \quad (I - \alpha H^T)x + r = v$$

H = 9914 \times 9914 csStanford web

v = e_4 (cs.stanford.edu)

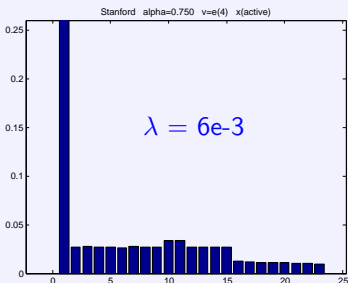
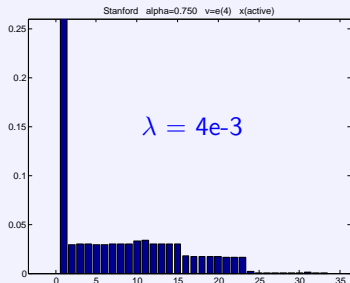
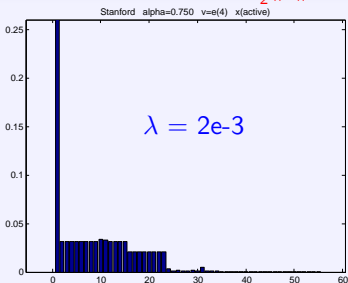
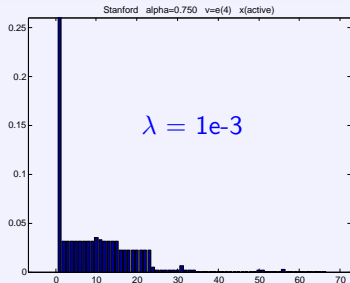
α = 0.75, 0.85, 0.95

λ = 1e-3, 2e-3, 4e-3, 6e-3

Plot nonzero x_j in the order they are chosen

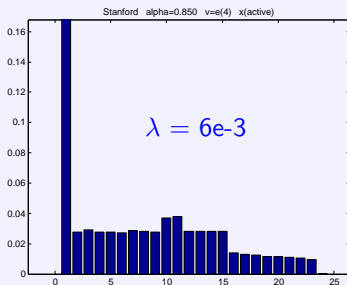
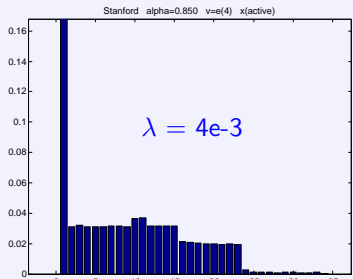
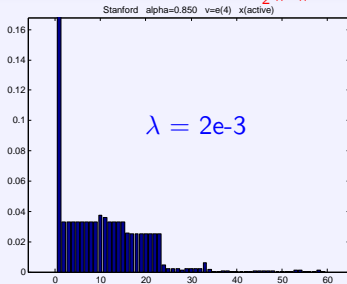
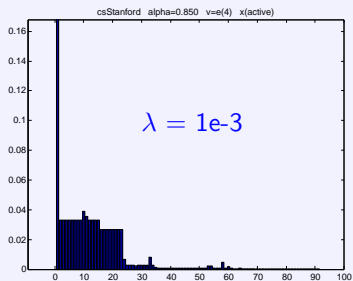
csStanford $v = e_4$

$\alpha = 0.75$ $\min \lambda e^T x + \frac{1}{2} \|r\|^2$



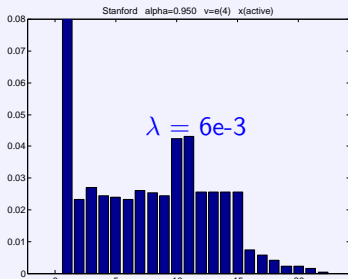
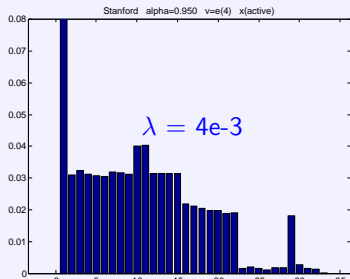
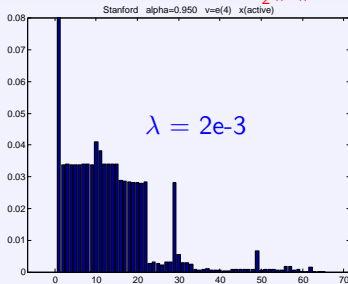
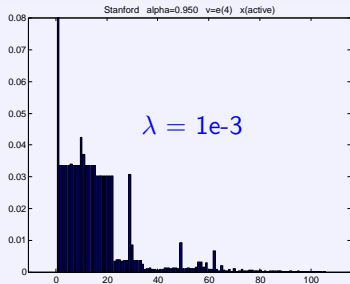
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SPAI: Sparse Approximate Inverse

SPAI Preconditioning

A plausible future application

- Find sparse X such that $AX \approx I$

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- Embarrassingly parallel use of sparse BP solutions

Summary

$$(I - \alpha H^T)x \approx v$$

Comments

- Sparse v *sometimes* gives sparse x

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- cf. Andersen, Chung, and Lang \approx 2005:
Local graph partitioning using (approximate) PageRank vectors

Thanks

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- Holly