Computing Sparse PageRank Vectors by Basis Pursuit

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Outline

1. Abstract

Outline

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2. Sparse solutions
3. PageRank

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Abstract1

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where $\quad 0 < \alpha < 1 \quad He \le e \quad x \ge 0$, sparse

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Perhaps BPDN can find a very sparse approximate x

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- Sometimes $x \geq 0$ naturally (e.g. PageRank)

- Apply optimization to choose nonzero x_j one by one

Sparse solutions to $Ax = b$

Sparse x

Lasso(ν) Tibshirani 1996

$$\min \tfrac{1}{2}\|Ax - b\|^2 \quad \text{st} \quad \|x\|_1 \le \nu \qquad\qquad A = \boxed{}$$

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Basis Pursuit Chen, Donoho & Saunders 1998

$$\min \|x\|_1 \quad \text{st} \quad Ax = b \qquad\qquad A = \boxed{\text{fast operator}}$$

(Ax, A^Ty are efficient)

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(Ax, A^Ty are efficient)

BPDN(λ) Same paper

$$\min \lambda\|x\|_1 + \tfrac{1}{2}\|Ax-b\|^2$$

fast operator, any shape

$\|x\|_1$ is closest convex approximation to $\|x\|_0$

BP and BPDN algorithms

$$\min \lambda \|x\|_1 + \tfrac{1}{2}\|Ax - b\|^2$$

OMP	Davis, Mallat et al 1997	Greedy
BPDN-interior	Chen, Donoho & S, 1998, 2001	Interior, CG
PDSCO, PDCO	Saunders 1997, 2002	Interior, LSQR
BCR	Sardy, Bruce & Tseng 2000	Orthogonal blocks
Homotopy	Osborne et al 2000	Active-set, all λ
LARS	Efron, Hastie, Tibshirani 2004	Active-set, all λ
STOMP	Donoho, Tsaig, et al 2006	Double greedy
l1_ls	Kim, Koh, Lustig, Boyd et al 2007	Primal barrier, PCG
GPSR	Figueiredo, Nowak & Wright 2007	Gradient Projection
SPGL1	van den Berg & Friedlander 2007	Spectral GP, all λ

Basis Pursuit Denoising (BPDN)

Pure LS

$$\min_{x,\,r}\ \tfrac{1}{2}\|r\|^2 \qquad \mathrm{st} \qquad r = b - Ax$$

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Smaller $\|x\|$, bigger $\|r\|$

Let $r = \lambda y$

$$\text{BPprimal}(\lambda) \quad \min_{x,\,y} \; \|x\|_1 + \tfrac{1}{2}\lambda\|y\|^2 \quad \text{st} \quad Ax + \lambda y = b$$

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Suggests regularized LP problems:

LPprimal(λ) $\quad \min_{x,\,y} \; c^T x + \frac{1}{2}\lambda\|y\|^2 \quad \text{st} \quad Ax + \lambda y = b,\ x \ge 0$

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Friedlander and S 2007–2008: New active-set MATLAB codes

PageRank

PageRank

Langville and Meyer 2006

$$H = \begin{pmatrix} 1/3 & & 1/3 & 1/3 \\ & 1/2 & 1/2 & \\ 0 & 0 & 0 & 0 \\ & 1/2 & & 1/2 \end{pmatrix}$$ Hyperlink matrix $He \le e$

PageRank

Langville and Meyer 2006

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$G = \alpha S + (1-\alpha) e v^T$	Google matrix $\alpha = 0.85$ say	$Ge = e$

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$$\boxed{\begin{array}{lcl} \text{eigenvector} & \equiv & \text{linear system} \\ G^T x = x & \equiv & (I - \alpha H^T)x = v \end{array}}$$

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$$\boxed{\begin{array}{lcl} \text{eigenvector} & \equiv & \text{linear system} \\ G^T x = x & \equiv & (I - \alpha H^T) x = v \end{array}}$$

In both cases, $x \leftarrow x / e^T x$ (and $x \geq 0$)

Web matrices

Web matrices H

Name	n (pages)	nnz (links)	i	$(v = e_i)$
csStanford	9914	36854	4	cs.stanford.edu
Stanford	275689	1623817	23036	cs.stanford.edu
			6753	calendus.stanford.edu
Stanford-Berkeley	683446	7583376	6753	unknown

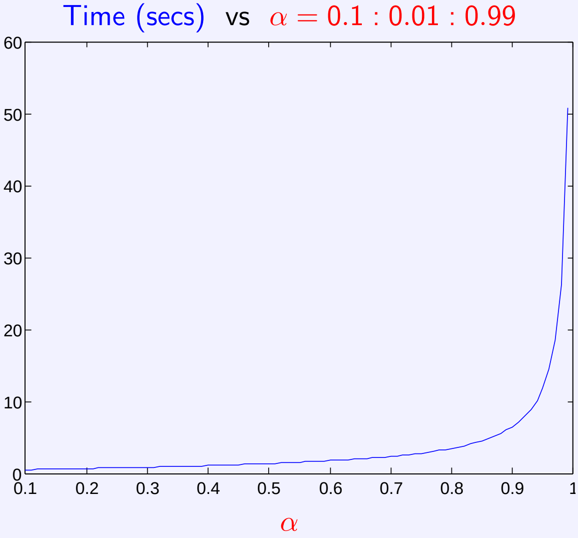
All data collected around 2001

Cleve Moler
Sep Kamvar
David Gleich

$$(I - \alpha H^T)x = v$$

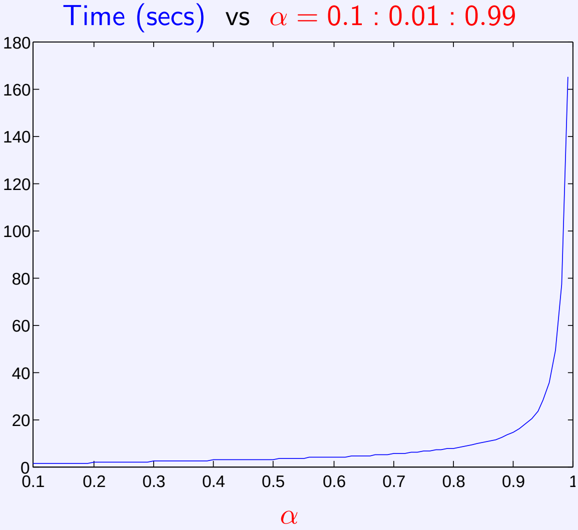
Power method on $G^T x = x$

Stanford, $n = 275000$, $v = e_{6753}$ (calendus.stanford.edu)



Power method

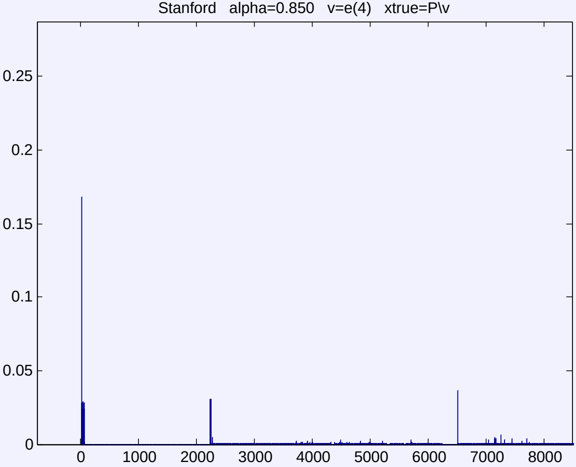
Stanford-Berkeley, $n = 683000$, $v = e_{6753}$



Exact $x(\alpha)$

csStanford, $n = 9914$, $v = e_4$ (cs.stanford.edu)

$x_j(\alpha), \; j = 1 : n$

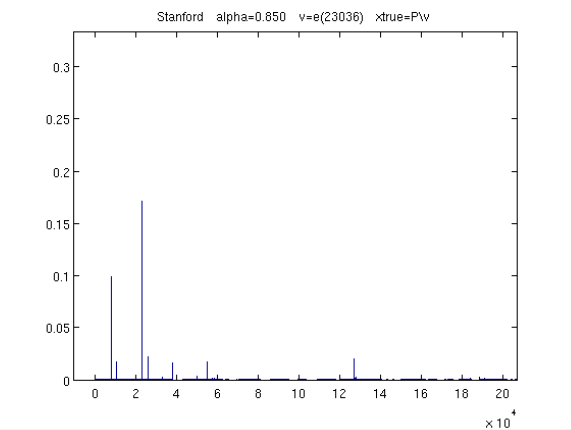


$\alpha = 0.85$

Exact $x(\alpha)$

Stanford, $n = 275000$, $v = e_{23036}$ (cs.stanford.edu)

$x_j(\alpha), \quad j = 1 : n$

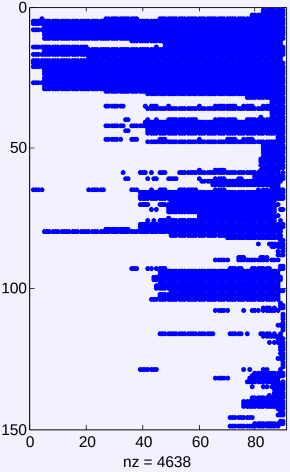


$\alpha = 0.85$

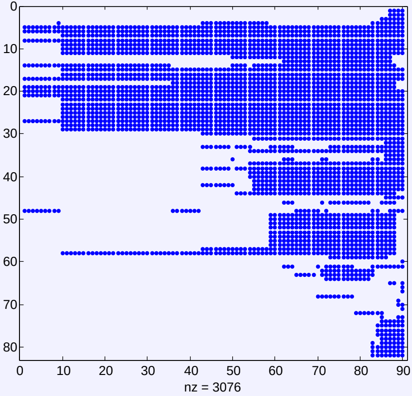
Sparsity of $x(\alpha)$

Stanford, $n = 275000$, $v = e_{23036}$ (cs.stanford.edu)

$x > 250 * (1/n)$

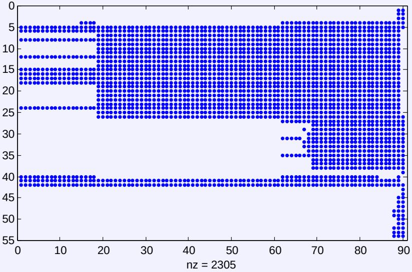


$x > 500 * (1/n)$

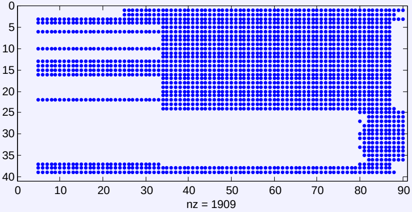


$\alpha = 0.1 : 0.01 : 0.99$ (90 values)

$x > 1000 * (1/n)$



$x > 2000 * (1/n)$



$\alpha = 0.1 : 0.01 : 0.99$ (90 values)

Sparse PageRank

Sparse PageRank

$$\boxed{(I - \alpha H^T)x = v \qquad\qquad v \ \text{ sparse}}$$

Regard as

$$\boxed{Ax = v \qquad\qquad v = e_i \ \text{ say}}$$

Sparse PageRank

$$\boxed{(I - \alpha H^T)x = v \qquad\qquad v \;\; \text{sparse}}$$

Regard as

$$\boxed{Ax = v \qquad\qquad v = e_i \;\; \text{say}}$$

Apply active-set solver to

$$\boxed{\min_{x,\,r} \;\; \lambda e^T x + \tfrac{1}{2}\|r\|^2 \quad \text{st} \quad Ax + r = v, \quad x \geq 0}$$

BP solvers

LPprimal solver

$$\boxed{\min_{x,\,y}\;\; e^T x + \tfrac{1}{2}\lambda\|y\|^2 \quad \text{st} \quad Ax + \lambda y = v, \quad x \ge 0}$$

LPprimal solver

$$\boxed{\min_{x,y} \ e^T x + \tfrac{1}{2}\lambda\|y\|^2 \quad \text{st} \quad Ax + \lambda y = v, \quad x \geq 0}$$

Seems to proceed in a *greedy way*

Starts with $x = 0$, chooses one x_j at a time

s nonzero $x_j \quad \Rightarrow \quad s$ iterations

LPprimal solver

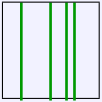
$$\boxed{\min_{x,\,y}\ e^T x + \tfrac{1}{2}\lambda\|y\|^2 \quad \text{st} \quad Ax + \lambda y = v, \quad x \ge 0}$$

Seems to proceed in a *greedy way*

Starts with $x = 0$, chooses one x_j at a time

s nonzero $x_j \quad \Rightarrow \quad s$ iterations

$B =$ columns of A with $x_j > 0$

$A =$ 

LPprimal solver

$$\boxed{\min_{x,\,y}\ e^T x + \tfrac{1}{2}\lambda\|y\|^2 \quad \text{st} \quad Ax + \lambda y = v, \quad x \ge 0}$$

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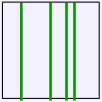
$A =$ 

Main work per itn:

Form $z = A^T y$

Solve $B^T B\, dx = c_B - B^T y$

Form $dy = B\, dx$

LPprimal solver

$$\boxed{\min_{x,\,y}\ \ e^Tx + \tfrac{1}{2}\lambda\|y\|^2 \quad \text{st} \quad Ax + \lambda y = v, \quad x \ge 0}$$

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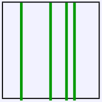
$A =$ 

Main work per itn:

Form $z = A^Ty$

Solve $B^TB\,dx = c_B - B^Ty$ Update $QB = \binom{R}{0}$ without Q

Form $dy = B\,dx$

LPdual solver

$$\boxed{\min_y \ -v^T y + \tfrac{1}{2}\lambda\|y\|^2 \quad \mathrm{st} \quad A^T y \le e}$$

LPdual solver

$$\boxed{\min_y \ -v^T y + \tfrac{1}{2}\lambda\|y\|^2 \quad \text{st} \quad A^T y \le e}$$

B = columns of A corresponding to $B^T y = e$ (active constraints)

LPdual solver

$$\boxed{\min_y \ -v^T y + \tfrac{1}{2}\lambda\|y\|^2 \quad \text{st} \quad A^T y \le e}$$

B = columns of A corresponding to $B^T y = e$ (active constraints)

Main work per itn:

Solve $\min \|Bx - g\|$
Form $dy = (g - Bx)/\lambda$
Form $dz = A^T dy$

LPdual solver

$$\boxed{\min_y \; -v^T y + \tfrac{1}{2}\lambda\|y\|^2 \quad \text{st} \quad A^T y \le e}$$

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Main work per itn:

Solve $\min \|Bx - g\|$ Update $QB = \begin{pmatrix} R \\ 0 \end{pmatrix}$ without Q

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Main work per itn:

Solve $\min \|Bx - g\|$ Update $QB = \begin{pmatrix} R \\ 0 \end{pmatrix}$ without Q

Form $dy = (g - Bx)/\lambda$

Form $dz = A^T dy$

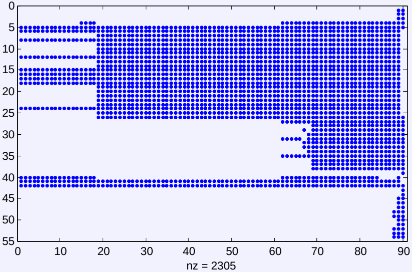
Selects B in same *greedy* manner as LPprimal in almost same order

Numerical results with BP solvers

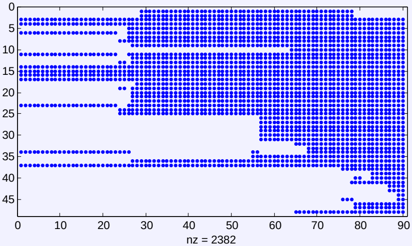
Sparsity of $xBP(\alpha)$

Stanford, $n = 275000$, $v = e_{23036}$ (cs.stanford.edu)

$xPower > 1000 * (1/n)$



$xBP > 1 * (1/n)$



$\alpha = 0.1 : 0.01 : 0.99$ (90 values)

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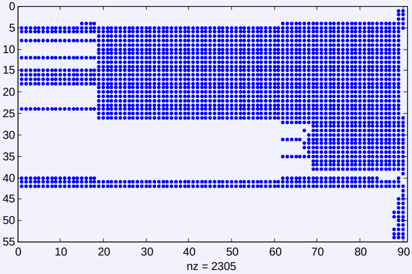
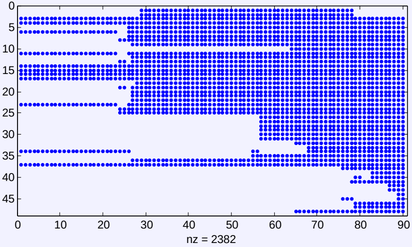


$xBP > 1 * (1/n)$

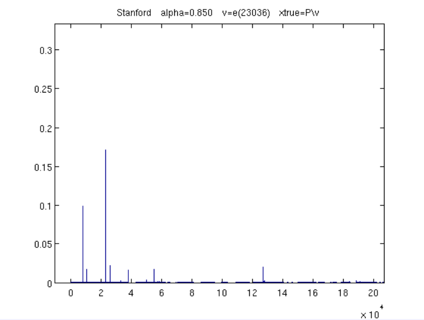


$\alpha = 0.1 : 0.01 : 0.99$ (90 values)

$xPower > 1 * (1/n)$ has
6000 rows, 155000 nonzeros

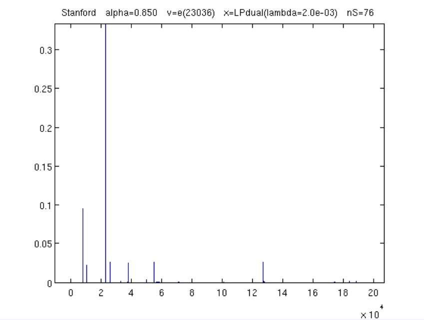
$\lambda = 4\text{e-}3$

Stanford web, $n = 275000$
$\alpha = 0.85$
$v = e_{23036}$ (cs.stanford.edu)



Direct solve:
`xtrue = A\v`

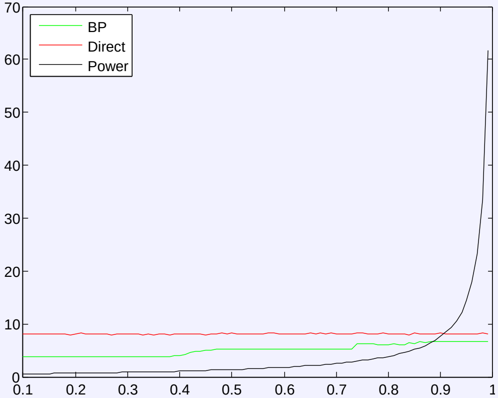
Basis Pursuit
$\lambda = 2\text{e-}3$
76 nonzero x_j



Power method vs BP

Stanford, $n = 275000$, $v = e_{23036}$ (cs.stanford.edu)

Time (secs) for each α



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$\lambda =$ 4e-3

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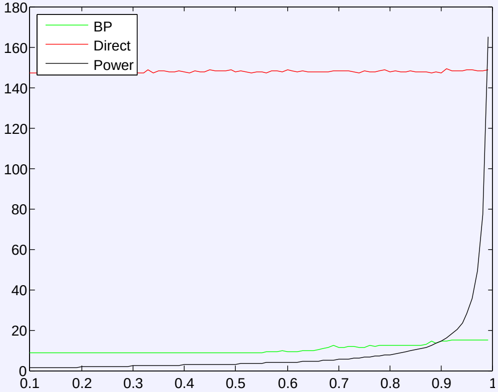
Stanford-Berkeley, $n = 683000$, $v = e_{6753}$ (unknown)

Time (secs) for each α



$\alpha = 0.1 : 0.01 : 0.99$ (90 values)

$\lambda =$ 5e-3

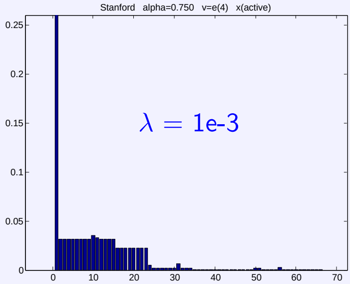
Varying α and λ

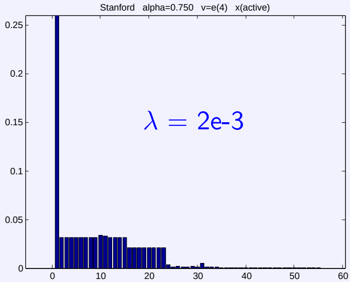
$$\min_{x,\,r}\ \lambda\|x\|_1 + \tfrac{1}{2}\|r\|^2 \qquad \text{st} \qquad (I - \alpha H^T)x + r = v$$

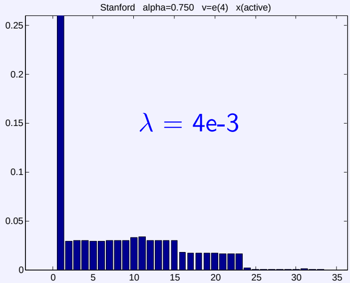
$$\begin{aligned} H &= 9914 \times 9914 \text{ csStanford web} \\ v &= e_4 \text{ (cs.stanford.edu)} \\ \alpha &= 0.75,\ 0.85,\ 0.95 \\ \lambda &= \text{1e-3, 2e-3, 4e-3, 6e-3} \end{aligned}$$

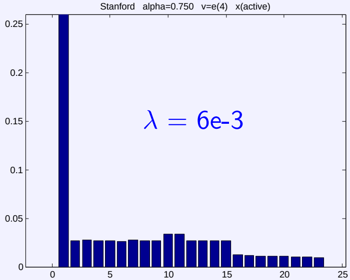
Plot nonzero x_j in the order they are chosen

csStanford $\quad v = e_4 \quad \alpha = 0.75 \quad \min \lambda e^T x + \frac{1}{2}\|r\|^2$

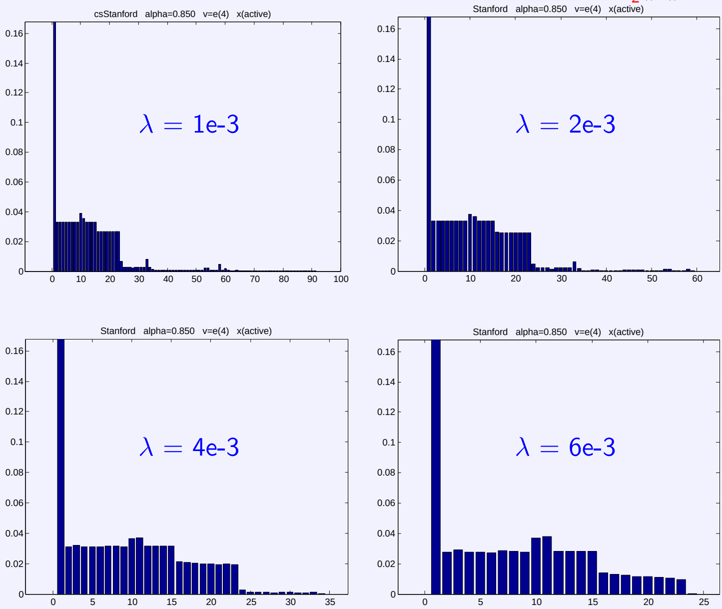




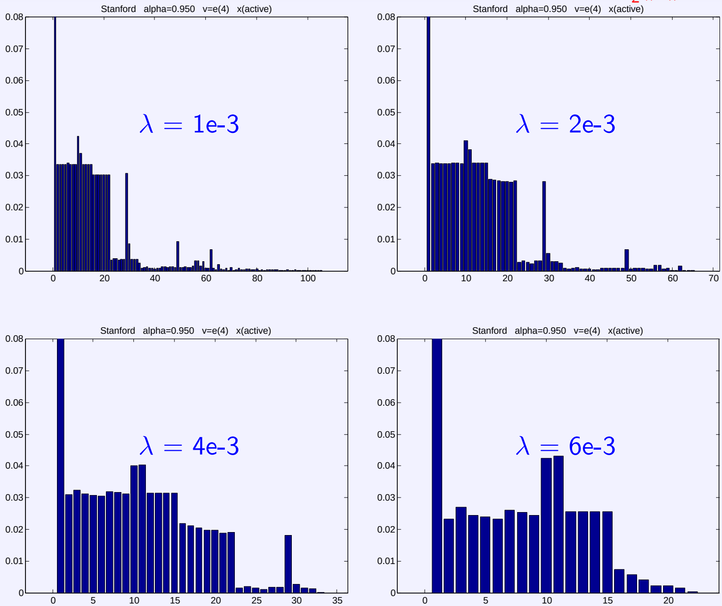




csStanford $v = e_4$ $\alpha = 0.85$ $\min \lambda e^T x + \frac{1}{2}\|r\|^2$



csStanford $v = e_4$ $\alpha = 0.95$ $\min \lambda e^T x + \frac{1}{2}\|r\|^2$



SPAI: Sparse Approximate Inverse

SPAI Preconditioning

A plausible future application

- Find sparse X such that $AX \approx I$

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Summary

$$(I - \alpha H^T)x \approx v$$

Comments

- Sparse v *sometimes* gives sparse x

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- cf. Andersen, Chung, and Lang $\approx$ 2005:
  Local graph partitioning using (approximate) PageRank vectors

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