The Stanford Systems Optimization Laboratory (SOL): Some Applications of our Large-scale Optimization Software

Michael Saunders

Management Science and Engineering (MS&E)
Institute for Computational and Mathematical Engineering (iCME)
Stanford University

Optimization Day — Research and Applications
Mechanical Engineering, Thermal and Fluid Sciences

Affiliates and Sponsors Program
Stanford University, Feb 1, 2011
SOL
Systems Optimization Lab
• **George Dantzig**  
  1974–1988  
  PILOT energy-economic model  
  Linear program
Models, Algorithms, Software

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- **Alan Manne**
  - 1976 ETAMACRO energy model
    - nonlinear objective
  - 1996–2006 MERGE greenhouse-gas model
    - nonlinear objective and constraints
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- **Ideal test problems for our optimization solvers**
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  **Gill, Murray, and Saunders**  **SQOPT, SNOPT**  QP, NLP
  **Infanger**  **DECIS**  Stochastic LP
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- **Part of modeling systems**
  GAMS and AMPL (also TOMLAB)
Iterative Solvers for

\[ Ax = b \quad \text{min} \| Ax - b \| \]

http://www.stanford.edu/group/SOL/software.html
CG-type solvers for symmetric $Ax = b$ 

Krylov subspace

Lanczos process generates $V_k = [v_1 \ v_2 \ \ldots \ v_k] \in \mathcal{K}_k$ using products $Av_j$

$k$th approximation

$x_k = V_k y_k$ for some $y_k$

\[ \mathcal{K}_k(A, b) = \text{range}\{b, Ab, \ldots, A^{k-1}b\} \]
CG-type solvers for symmetric $Ax = b$  

**Krylov subspace**  
$K^k(A,b) = \text{range}\{b, Ab, \ldots, A^{k-1}b\}$

**Lanczos process** generates  
$V_k = [v_1 \ v_2 \ \ldots \ v_k] \in K^k$  
using products $Av_j$

$k$th approximation  
$x_k = V_k y_k$ for some $y_k$

Choose $y_k$ to minimize something
CG-type solvers for symmetric $Ax = b$

Krylov subspace

Lanczos process generates

$k$th approximation

Choose $y_k$ to minimize something

- **CG**
  \[
  \min \frac{1}{2} x_k^T Ax_k - b^T x_k \quad (A \text{ posdef})
  \]

- **SYMMLQ**
  \[
  \min \| e_k \| \quad \text{error } e_k = x - x_k
  \]

- **MINRES**
  \[
  \min \| r_k \| \quad \text{residual } r_k = b - Ax_k
  \]

- **MINRES-QLP**
  \[
  \min \| r_k \| \quad \text{for singular incompatible } Ax \approx b
  \]

Paige, Saunders, Choi
CG-type solvers for $\min \|Ax - b\|$ 

Golub-Kahan process generates 

$$U_k = \begin{bmatrix} u_1 & u_2 & \ldots & u_k \end{bmatrix},$$

$$V_k = \begin{bmatrix} v_1 & v_2 & \ldots & v_k \end{bmatrix}$$

using products $Au_j, A^Tu_j$

$k$th approximation $x_k = V_k y_k$ for some $y_k$
CG-type solvers for $\min \| Ax - b \|

Golub-Kahan process generates

$$U_k = [u_1 \ u_2 \ \ldots \ u_k],$$

$$V_k = [v_1 \ v_2 \ \ldots \ v_k]$$

using products $A v_j$, $A^T u_j$

$k$th approximation

$$x_k = V_k y_k$$ for some $y_k$

Choose $y_k$ to minimize something
Iterative Solvers
Optimization Solvers
Aerospace Applications

Symmetric $Ax = b$
Unsymmetric or rectangular $Ax \approx b$

**CG-type solvers for** $\min \|Ax - b\|$

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$x_k = V_k y_k$ for some $y_k$

Choose $y_k$ to minimize something

**LSQR** $\min \|r_k\|$ residual $r_k = b - Ax_k$

**LSMR** $\min \|A^Tr_k\|$ residual for $A^TAx = A^Tb$

Paige, Saunders
David Fong, iCME
Jon Claerbout, Geophysics
LSQR vs LSMR on $\min \|Ax - b\|$

Measure of Convergence

- $r_k = b - Ax_k$
- $\|r_k\| \rightarrow \|\hat{r}\|$, $\|A^T r_k\| \rightarrow 0$
**LSQR vs LSMR on** \( \min \| Ax - b \| \)

**Measure of Convergence**

- \( r_k = b - Ax_k \)
- \( \| r_k \| \to \| \hat{r} \|, \quad \| A^T r_k \| \to 0 \)

---

**LSQR**

- \( \| r_k \| \)

---

**LSQR**

- \( \log \| A^T r_k \| \)
LSQR vs LSMR on \( \min \|Ax - b\| \)

Measure of Convergence

- \( r_k = b - Ax_k \)
- \( \|r_k\| \rightarrow \|\hat{r}\|, \quad \|A^Tr_k\| \rightarrow 0 \)

![Graphs comparing LSQR and LSMR](image)

Michael Saunders
Symmetric $Ax = b$

Unsymmetric or rectangular $Ax \approx b$

$$\log_{10} \frac{\|A^T r_k\|}{\|r_k\|}$$ for LSQR and LSMR – typical

Name: lp pilot ja, Dim: 2267x940, nnz: 14977, id=88
Optimization Solvers
Active-set solvers for LP, NLP

\[
\min_x \varphi(x) \quad \text{st} \quad \ell \leq \begin{pmatrix} x \\ Ax \\ c(x) \end{pmatrix} \leq u
\]

MINOS    Sparse LP, NLP
LSSOL     Dense constrained least-squares
NPSOL     Dense NLP
QPOPT     Dense QP
SQOPT     Sparse QP also QPBLUR, Chris Maes, iCME
SNOPT     Sparse NLP Philip Gill, UCSD
PDCO: An optimizer for convex objectives

Nominally:

$$\min_x \varphi(x) \quad \text{st} \quad Ax = b, \quad x \geq 0$$

where $A$ may be a sparse matrix or an operator
PDCO: An optimizer for convex objectives

Nominally:

\[ \min_x \varphi(x) \quad \text{st} \quad Ax = b, \quad x \geq 0 \]

where \( A \) may be a sparse matrix or an operator

More useful:

\[ \min_{x, r} \varphi(x) + \frac{1}{2} \|D_1 x\|^2 + \frac{1}{2} \|r\|^2 \]

\[ Ax + D_2 r = b, \quad \ell \leq x \leq u, \]

where \( D_1 \) and \( D_2 \) are posdef diagonal matrices

- Regularized LP, QP, . . .
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- Regularized LP, QP, …
- Basis Pursuit DeNoising

David Donoho
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- Basis Pursuit DeNoising
- LP feasibility ($D_2 = I$)

David Donoho
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- Regularized LP, QP, . . .
- Basis Pursuit DeNoising
- LP feasibility ($D_2 = I$)
- NMR analysis

David Donoho
Jon Dattorro
Zeev Wiesman, Ofer Levi
Aerospace Applications
David Saunders

1970 Visit Stanford for 1 month (now 40 years)
1974–present NASA Ames
David Saunders

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Projects

OAW  Oblique All-Wing supersonic airliner
HSCT  Supersonic airliner
CTV  SHARP shuttle design
CEV  Apollo-type capsule to ISS, moon
OAW oblique all wing airliner
HSCT high speed civil transport
CTV crew transfer vehicle

SHARP design (Slender Hypervelocity Aerothermodynamic Research Probes)

Aerothermal performance constraint in (Velocity, Altitude) space, used during trajectory optimization with UHTC materials (Ultra High Temperature Ceramics) to avoid exceeding material limits
CTV crew transfer vehicle

SHARP design (Slender Hypervelocity Aerothermodynamic Research Probes)

Aerothermal performance constraint in (Velocity, Altitude) space, used during trajectory optimization with UHTC materials (Ultra High Temperature Ceramics) to avoid exceeding material limits

- Trajectory optimization with SNOPT
- Could always abort to Kennedy, Boston, Gander, or Shannon
- 4000-mile cross-range capability during reentry

Image credit: David Kinney, NASA Ames Research Center
CEV crew exploration vehicle

- Tried shape optimization of heat shield and shoulder curvature (but the Apollo folk were pretty close already)
Philip Gill, Rocky Nelson
1979–1988 SOL  QPSOL, LSSOL, NPSOL
1988–2007 UC San Diego  QPOPT, SQOPT, SNOPT
McDonnell-Douglas Space Systems, LA (now Boeing)
McDonnell-Douglas Aerospace Applications

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- Projects
  F-4  Minimum time-to-climb
  DC-Y SSTO  Minimum-fuel landing maneuver
Aerospace Applications of NPSOL and SNOPT

OTIS #1
DC-Y single-stage-to-orbit
Control engineer
Feasible solution
1 year

2000 x 1200 NLP

NPSOL
Optimal solution
50% of fuel saved
OTIS
DC-Y Landing Maneuver
Retract air brakes at
2800 ft
420 mph
DC-Y landing, 2nd OTIS/NPSOL optimization

- 1st optimization: starting altitude = 2800 ft

- 2nd optimization: starting altitude = variable

New constraint needed: Don’t exceed 3g

Optimum starting altitude = 1400 ft

(!) Come back Alan Shepard!
DC-Y landing, 2nd OTIS/NPSOL optimization

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Come back Alan Shepard!
Stanford Aerospace Applications

- Ilan Kroo  Aircraft Aerodynamics and Design Group
- Antony Jameson  Aerospace Computing Lab
- Juan Alonso  Aerospace Design Lab

  MDO  Multidisciplinary Design Optimization
  ASO  Aerodynamic Shape Optimization
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  - MDO  Multidisciplinary Design Optimization
  - ASO  Aerodynamic Shape Optimization
  
- Numerous completed projects
  - OAW  Oblique All-Wing supersonic airliner
  - Blended Wing-Body  Transonic airliner
  
...
Blended wing airliner
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- Ilan Kroo, Michael Holden  
  Aero/Astro, Stanford, 1999
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- Compute controls for stable flight of 17ft-span flying model
  Model trajectory of flexible body over time
Blended wing airliner

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- SNOPT today: Probably 3 hours (or much less)
Latest use of optimization

Conservative Meshless Scheme for Conservation Laws
Edmond Chiu, Qiqi Wang and Antony Jameson

PDCO:
\[
\min \|a_f\|
\]
\[
s.t. \begin{bmatrix} C_f & D \end{bmatrix} \begin{bmatrix} a_f \\ m \end{bmatrix} = -C_p a_p, \ m > 0
\]

LSQR:
\[
\min \|a_f\|
\]
\[
s.t. C_f a_f = -C_p a_p - D m
\]
Geophysics at Stanford

- Paul Segall, Dan Sinnett, Andrew Bradley

Geophysical inverse problem:
Determine the dislocation on a dike near Kilauea based on GPS data
Geophysics at Stanford

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  Regularization for smoothness of surface deformation
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- Apply SNOPT
News Flash, 3 March 2007

- Mike Ross  
  Naval Postgraduate School, Monterey

DIDO: A package for solving optimal control problems
Implemented in MATLAB
Calls TOMLAB/SNOPT for the optimization
News Flash, 3 March 2007

- **Mike Ross**  
  Naval Postgraduate School, Monterey

  **DIDO**: A package for solving optimal control problems
  Implemented in **MATLAB**
  Calls **TOMLAB/SNOPT** for the optimization

- **GMT 062:19:26**

  The International Space Station was successfully maneuvered using **DIDO/TOMLAB/SNOPT**
  Found **zero-propellant solutions** (globally optimal)
  Saved NASA **$1M fuel cost**
SNOPT Applications (Walter Murray)

Conventional Launcher:
Ariane 5 Dual Payload LEO/GEO

Direct optimization of Step-and-Shoot segments

Robot at JPL
Torque minimization
Daniel Clemente

Tumor radiation
Control problem
Paul Keall
Optimization

Stabilize aircraft
Minimize fuel
Reduce CO₂

Make the world a better place