The Stanford Systems Optimization Laboratory (SOL): Some Applications of our Large-scale Optimization Software

Michael Saunders

Management Science and Engineering (MS&E)
Institute for Computational and Mathematical Engineering (iCME)
Stanford University

Optimization Day — Research and Applications
Mechanical Engineering, Thermal and Fluid Sciences

Affiliates and Sponsors Program
Stanford University, Feb 1, 2011

SOL

Systems Optimization Lab

Models, Algorithms, Software

- George Dantzig

  1974–1988 PILOT energy-economic model

  Linear program

Models, Algorithms, Software

- George Dantzig

1974–1988	PILOT energy-economic model Linear program

- Alan Manne

1976	ETAMACRO energy model nonlinear objective
1996–2006	MERGE greenhouse-gas model nonlinear objective and constraints

Models, Algorithms, Software

- George Dantzig

	1974–1988	PILOT energy-economic model
		Linear program

- Alan Manne

	1976	ETAMACRO energy model
		nonlinear objective
	1996–2006	MERGE greenhouse-gas model
		nonlinear objective and constraints

- Ideal test problems for our optimization solvers

Models, Algorithms, Software

- George Dantzig

1974–1988	PILOT energy-economic model
	Linear program

- Alan Manne

1976	ETAMACRO energy model
	nonlinear objective
1996–2006	MERGE greenhouse-gas model
	nonlinear objective and constraints

- Ideal test problems for our optimization solvers
- Murtagh and Saunders / Gill, Murray, and Saunders / Infanger

Murtagh and Saunders	MINOS	LP/NLP
Gill, Murray, and Saunders	SQOPT, SNOPT	QP, NLP
Infanger	DECIS	Stochastic LP

Models, Algorithms, Software

- George Dantzig

1974–1988	PILOT energy-economic model Linear program

- Alan Manne

1976	ETAMACRO energy model nonlinear objective
1996–2006	MERGE greenhouse-gas model nonlinear objective and constraints

- Ideal test problems for our optimization solvers

Murtagh and Saunders	MINOS	LP/NLP
Gill, Murray, and Saunders	SQOPT, SNOPT	QP, NLP
Infanger	DECIS	Stochastic LP

- Part of modeling systems GAMS and AMPL (also TOMLAB)

Iterative Solvers for

$$Ax = b \qquad \min \|Ax - b\|$$

http://www.stanford.edu/group/SOL/software.html

CG-type solvers for symmetric $Ax = b$

Krylov subspace	$\mathcal{K}_k(A, b) = \text{range}\{b, Ab, \dots, A^{k-1}b\}$
Lanczos process generates	$V_k = \begin{bmatrix} v_1 & v_2 & \dots & v_k \end{bmatrix} \in \mathcal{K}_k$
	using products Av_j
kth approximation	$x_k = V_k y_k$ for some y_k

CG-type solvers for symmetric $Ax = b$

Krylov subspace	$\mathcal{K}_k(A,b) = \text{range}\{b, Ab, \ldots, A^{k-1}b\}$
Lanczos process generates	$V_k = \begin{bmatrix} v_1 & v_2 & \ldots & v_k \end{bmatrix} \in \mathcal{K}_k$
	using products Av_j
kth approximation	$x_k = V_k y_k$ for some y_k

Choose y_k to minimize something

CG-type solvers for symmetric $Ax = b$

Krylov subspace	$\mathcal{K}_k(A, b) = \text{range}\{b, Ab, \dots, A^{k-1}b\}$
Lanczos process generates	$V_k = \begin{bmatrix} v_1 & v_2 & \dots & v_k \end{bmatrix} \in \mathcal{K}_k$
	using products Av_j
kth approximation	$x_k = V_k y_k$ for some y_k

Choose y_k to minimize something

CG	min $\frac{1}{2}x_k^T A x_k - b^T x_k$	(A posdef)
SYMMLQ	min $\|e_k\|$	error $e_k = x - x_k$
MINRES	min $\|r_k\|$	residual $r_k = b - Ax_k$
MINRES-QLP	min $\|r_k\|$	for singular incompatible $Ax \approx b$

Paige, Saunders, Choi

CG-type solvers for min $\|Ax - b\|$

Golub-Kahan process generates $U_k = \begin{bmatrix} u_1 & u_2 & \dots & u_k \end{bmatrix}$,
$V_k = \begin{bmatrix} v_1 & v_2 & \dots & v_k \end{bmatrix}$
using products Av_j, $A^T u_j$

kth approximation $x_k = V_k y_k$ for some y_k

CG-type solvers for min $\|Ax - b\|$

Golub-Kahan process generates $U_k = \begin{bmatrix} u_1 & u_2 & \ldots & u_k \end{bmatrix}$,
$V_k = \begin{bmatrix} v_1 & v_2 & \ldots & v_k \end{bmatrix}$
using products Av_j, $A^T u_j$

kth approximation $x_k = V_k y_k$ for some y_k

Choose y_k to minimize something

CG-type solvers for min $\|Ax - b\|$

Golub-Kahan process generates $U_k = \begin{bmatrix} u_1 & u_2 & \dots & u_k \end{bmatrix}$,
$V_k = \begin{bmatrix} v_1 & v_2 & \dots & v_k \end{bmatrix}$
using products Av_j, $A^T u_j$

kth approximation $x_k = V_k y_k$ for some y_k

Choose y_k to minimize something

LSQR	min $\|r_k\|$	residual $r_k = b - Ax_k$
LSMR	min $\|A^T r_k\|$	residual for $A^T A x = A^T b$

Paige, Saunders
David Fong, iCME
Jon Claerbout, Geophysics

LSQR vs LSMR on $\min \|Ax - b\|$

Measure of Convergence

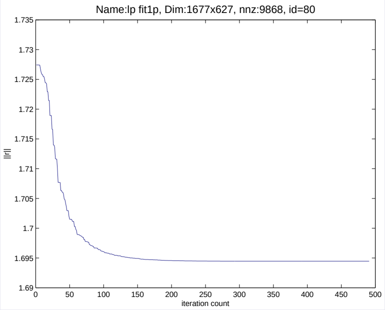
- $r_k = b - Ax_k$
- $\|r_k\| \to \|\hat{r}\|, \quad \|A^T r_k\| \to 0$

LSQR vs LSMR on min $\|Ax - b\|$

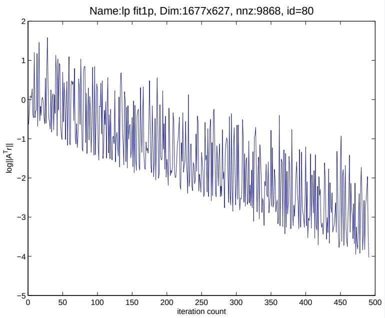
Measure of Convergence

- $r_k = b - Ax_k$
- $\|r_k\| \to \|\hat{r}\|, \quad \|A^T r_k\| \to 0$

LSQR $\|r_k\|$



LSQR log $\|A^T r_k\|$



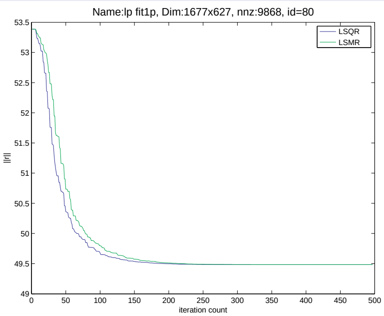
LSQR vs LSMR on min $\|Ax - b\|$

Measure of Convergence

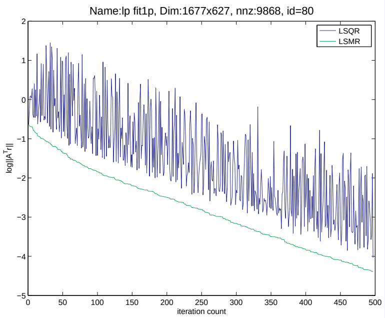
- $r_k = b - Ax_k$
- $\|r_k\| \to \|\hat{r}\|, \quad \|A^T r_k\| \to 0$

— LSQR
— LSMR

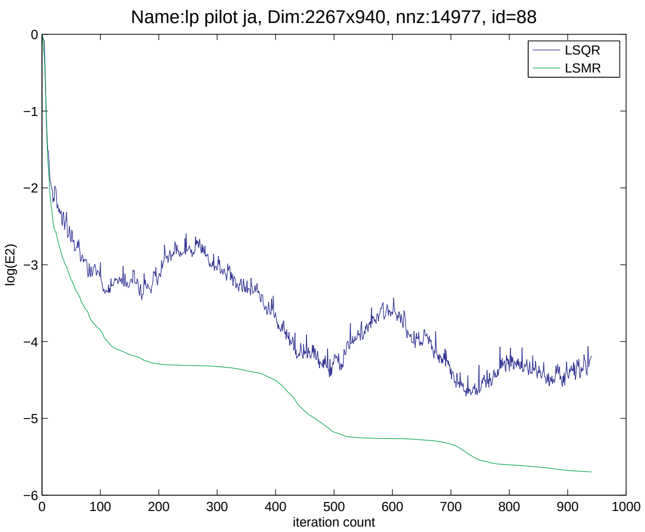
$\|r_k\|$



log $\|A^T r_k\|$



$\log_{10} \frac{\|A^T r_k\|}{\|r_k\|}$ for LSQR and LSMR – typical



Optimization Solvers

Active-set solvers for LP, NLP

$$\min_x \ \varphi(x) \quad \text{st} \quad \ell \le \begin{pmatrix} x \\ Ax \\ c(x) \end{pmatrix} \le u$$

MINOS	Sparse LP, NLP	
LSSOL	Dense constrained least-squares	
NPSOL	Dense NLP	
QPOPT	Dense QP	
SQOPT	Sparse QP	also QPBLUR, Chris Maes, iCME
SNOPT	Sparse NLP	Philip Gill, UCSD

PDCO: An optimizer for convex objectives

Nominally:

$$\min_x \ \varphi(x) \quad \text{st} \quad Ax = b, \qquad x \geq 0$$

where A may be a sparse matrix or an operator

PDCO: An optimizer for convex objectives

Nominally:

$$\min_{x} \ \varphi(x) \quad \text{st} \quad Ax = b, \qquad x \geq 0$$

where A may be a sparse matrix or an operator

More useful:

$$\min_{x,r} \ \varphi(x) + \tfrac{1}{2}\|D_1 x\|^2 + \tfrac{1}{2}\|r\|^2$$
$$Ax + D_2 r = b, \qquad \ell \leq x \leq u,$$

where D_1 and D_2 are posdef diagonal matrices

- Regularized LP, QP, . . .

PDCO: An optimizer for convex objectives

Nominally:

$$\min_{x} \ \varphi(x) \quad \text{st} \quad Ax = b, \qquad x \geq 0$$

where A may be a sparse matrix or an operator

More useful:

$$\min_{x,\,r} \ \varphi(x) + \tfrac{1}{2}\|D_1 x\|^2 + \tfrac{1}{2}\|r\|^2$$

$$Ax + D_2 r = b, \qquad \ell \leq x \leq u,$$

where D_1 and D_2 are posdef diagonal matrices

- Regularized LP, QP, ...
- Basis Pursuit DeNoising

David Donoho

PDCO: An optimizer for convex objectives

Nominally:

$$\min_x \ \varphi(x) \quad \text{st} \quad Ax = b, \qquad x \ge 0$$

where A may be a sparse matrix or an operator

More useful:

$$\min_{x,r} \ \varphi(x) + \tfrac{1}{2}\|D_1 x\|^2 + \tfrac{1}{2}\|r\|^2$$
$$Ax + D_2 r = b, \qquad \ell \le x \le u,$$

where D_1 and D_2 are posdef diagonal matrices

- Regularized LP, QP, . . .
- Basis Pursuit DeNoising — David Donoho
- LP feasibility ($D_2 = I$) — Jon Dattorro

PDCO: An optimizer for convex objectives

Nominally:

$$\min_{x} \ \varphi(x) \quad \text{st} \quad Ax = b, \qquad x \geq 0$$

where A may be a sparse matrix or an operator

More useful:

$$\min_{x,r} \ \varphi(x) + \tfrac{1}{2}\|D_1 x\|^2 + \tfrac{1}{2}\|r\|^2$$
$$Ax + D_2 r = b, \qquad \ell \leq x \leq u,$$

where D_1 and D_2 are posdef diagonal matrices

- Regularized LP, QP, . . .
- Basis Pursuit DeNoising — David Donoho
- LP feasibility ($D_2 = I$) — Jon Dattorro
- NMR analysis — Zeev Wiesman, Ofer Levi

Aerospace Applications

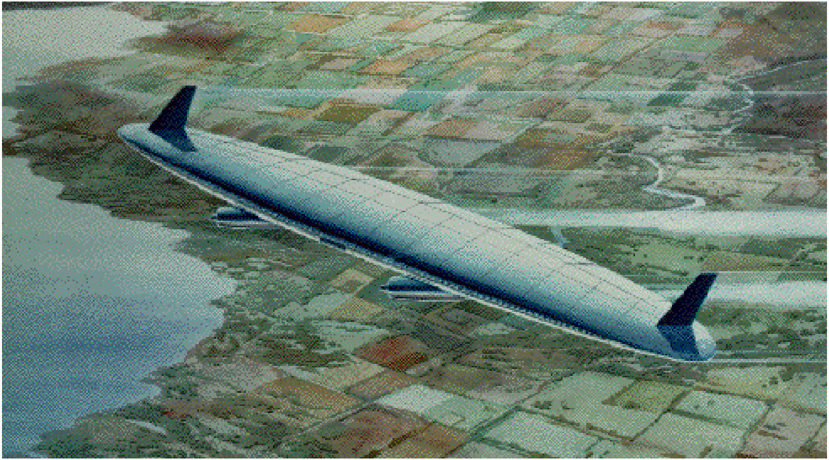
NASA Aerospace Applications

- David Saunders
  - 1970 Visit Stanford for 1 month (now 40 years)
  - 1974–present NASA Ames

NASA Aerospace Applications

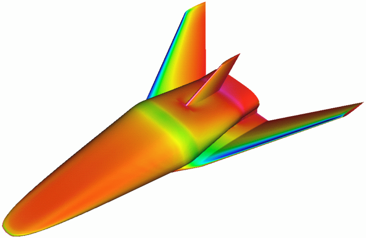
- David Saunders
  - 1970 Visit Stanford for 1 month (now 40 years)
  - 1974–present NASA Ames
- Projects
  - OAW Oblique All-Wing supersonic airliner
  - HSCT Supersonic airliner
  - CTV SHARP shuttle design
  - CEV Apollo-type capsule to ISS, moon

OAW oblique all wing airliner



HSCT high speed civil transport

CTV crew transfer vehicle

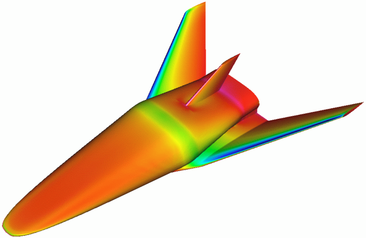


SHARP design (Slender Hypervelocity Aerothermodynamic Research Probes)

Aerothermal performance constraint in (Velocity, Altitude) space, used during trajectory optimization

with UHTC materials (Ultra High Temperature Ceramics) to avoid exceeding material limits

CTV crew transfer vehicle



SHARP design (Slender Hypervelocity Aerothermodynamic Research Probes)

Aerothermal performance constraint in (Velocity, Altitude) space, used during trajectory optimization with UHTC materials (Ultra High Temperature Ceramics) to avoid exceeding material limits

- Trajectory optimization with SNOPT
- Could always abort to Kennedy, Boston, Gander, or Shannon
- 4000-mile cross-range capability during reentry

Image credit: David Kinney, NASA Ames Research Center

CEV crew exploration vehicle

- Tried shape optimization of heat shield and shoulder curvature (but the Apollo folk were pretty close already)

McDonnell-Douglas Aerospace Applications

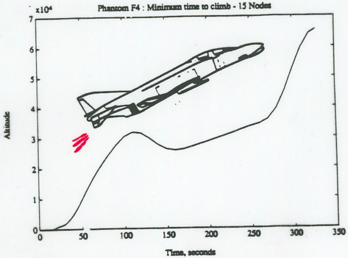
- Philip Gill, Rocky Nelson
  1979–1988 SOL QPSOL, LSSOL, NPSOL
  1988–2007 UC San Diego QPOPT, SQOPT, SNOPT
  McDonnell-Douglas Space Systems, LA (now Boeing)

McDonnell-Douglas Aerospace Applications

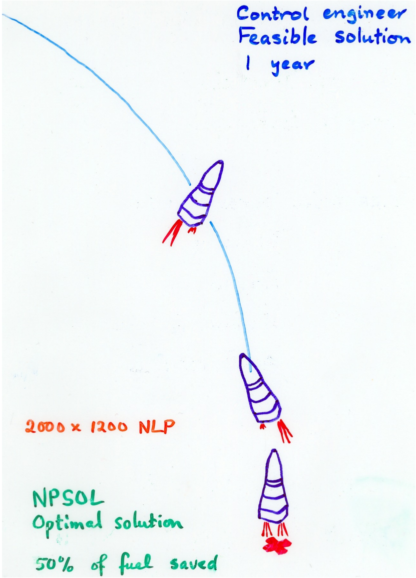
- Philip Gill, Rocky Nelson
  - 1979–1988 SOL QPSOL, LSSOL, NPSOL
  - 1988–2007 UC San Diego QPOPT, SQOPT, SNOPT
  - McDonnell-Douglas Space Systems, LA (now Boeing)
- Projects
  - F-4 Minimum time-to-climb
  - DC-Y SSTO Minimum-fuel landing maneuver

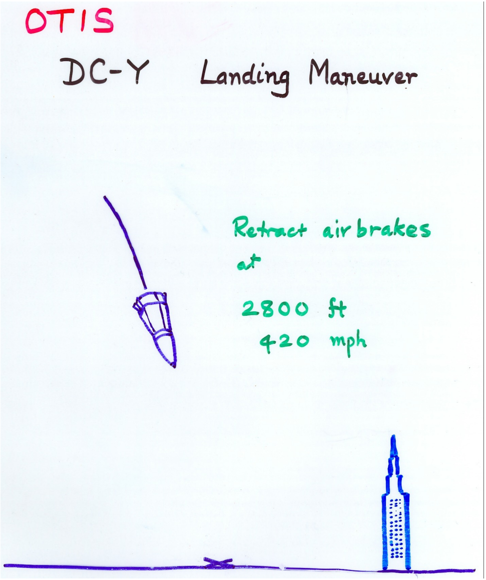
Aerospace Applications
of NPSOL and SNOPT

OTIS #1



DC-Y single-stage-to-orbit


Control engineer
Feasible solution
1 year
2000 x 1200 NLP
NPSOL
Optimal solution
50% of fuel saved


OTIS
DC-Y Landing Maneuver
Retract air brakes
at
2800 ft
420 mph

DC-Y landing, 2nd OTIS/NPSOL optimization

- 1st optimization: starting altitude = 2800ft

DC-Y landing, 2nd OTIS/NPSOL optimization

- 1st optimization: starting altitude = 2800ft
- 2nd optimization: starting altitude = variable

DC-Y landing, 2nd OTIS/NPSOL optimization

- 1st optimization: starting altitude = 2800ft
- 2nd optimization: starting altitude = variable
- New constraint needed:

DC-Y landing, 2nd OTIS/NPSOL optimization

- 1st optimization: starting altitude = 2800ft
- 2nd optimization: starting altitude = variable
- New constraint needed: Don't exceed 3g

DC-Y landing, 2nd OTIS/NPSOL optimization

- 1st optimization: starting altitude = 2800ft
- 2nd optimization: starting altitude = variable
- New constraint needed: Don't exceed 3g

Optimum starting altitude = 1400ft(!)

DC-Y landing, 2nd OTIS/NPSOL optimization

- 1st optimization: starting altitude = 2800ft
- 2nd optimization: starting altitude = variable
- New constraint needed: Don't exceed 3g

Optimum starting altitude = 1400ft(!)

Come back Alan Shephard!

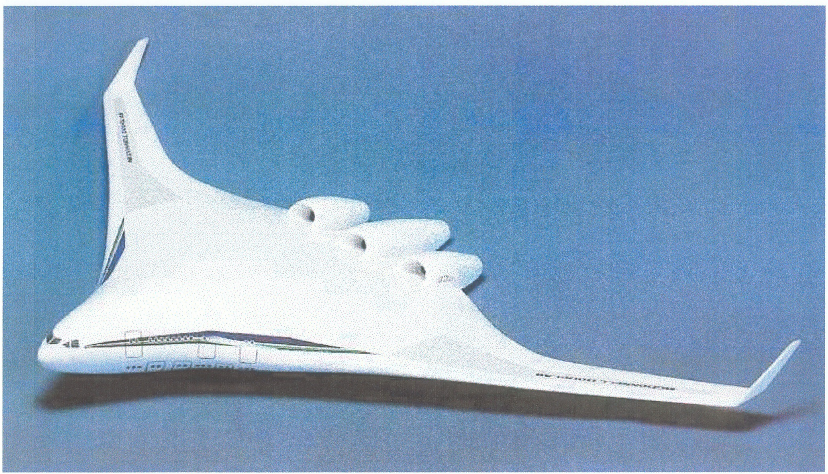
Stanford Aerospace Applications

- Ilan Kroo Aircraft Aerodynamics and Design Group
  Antony Jameson Aerospace Computing Lab
  Juan Alonso Aerospace Design Lab
  - MDO Multidisciplinary Design Optimization
  - ASO Aerodynamic Shape Optimization
  - . . .

Stanford Aerospace Applications

- Ilan Kroo   Aircraft Aerodynamics and Design Group
  Antony Jameson   Aerospace Computing Lab
  Juan Alonso   Aerospace Design Lab
  - MDO Multidisciplinary Design Optimization
  - ASO Aerodynamic Shape Optimization
  - ...
- Numerous completed projects
  - OAW Oblique All-Wing supersonic airliner
  - Blended Wing-Body Transonic airliner
  - ...

Blended wing airliner



Blended wing airliner

- Ilan Kroo, Michael Holden Aero/Astro, Stanford, 1999

Blended wing airliner

- Ilan Kroo, Michael Holden   Aero/Astro, Stanford, 1999
- Compute controls for stable flight of 17ft-span flying model
  Model trajectory of flexible body over time

Blended wing airliner

- Ilan Kroo, Michael Holden   Aero/Astro, Stanford, 1999
- Compute controls for stable flight of 17ft-span flying model
  Model trajectory of flexible body over time
- Minimize wing weight (or move CG aft as far as possible)
  subject to flutter constraints

Blended wing airliner

- Ilan Kroo, Michael Holden   Aero/Astro, Stanford, 1999
- Compute controls for stable flight of 17ft-span flying model
  Model trajectory of flexible body over time
- Minimize wing weight (or move CG aft as far as possible)
  subject to flutter constraints
- 9000 nonlinear eqns, 9000 state variables, 7 design variables
  400,000 gradients in the Jacobian (sparse finite differences)

Blended wing airliner

- Ilan Kroo, Michael Holden   Aero/Astro, Stanford, 1999
- Compute controls for stable flight of 17ft-span flying model
  Model trajectory of flexible body over time
- Minimize wing weight (or move CG aft as far as possible)
  subject to flutter constraints

- 9000 nonlinear eqns, 9000 state variables, 7 design variables
  400,000 gradients in the Jacobian (sparse finite differences)
- MINOS 1999: 26 major iterations, 4000 minor iterations
  1500 function and Jacobian evaluations, 3 days on SGI Octane

Blended wing airliner

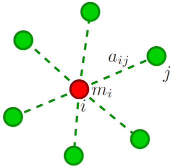
- Ilan Kroo, Michael Holden   Aero/Astro, Stanford, 1999
- Compute controls for stable flight of 17ft-span flying model
  Model trajectory of flexible body over time
- Minimize wing weight (or move CG aft as far as possible)
  subject to flutter constraints

- 9000 nonlinear eqns, 9000 state variables, 7 design variables
  400,000 gradients in the Jacobian (sparse finite differences)
- MINOS 1999: 26 major iterations, 4000 minor iterations
  1500 function and Jacobian evaluations, 3 days on SGI Octane
- SNOPT today: Probably 3 hours (or much less)

Latest use of optimization

Conservative Meshless Scheme for Conservation Laws

Edmond Chiu, Qiqi Wang and Antony Jameson



$$\frac{\partial \mathbf{w}}{\partial t} + \nabla \cdot \mathbf{u} = \mathbf{0}$$

$$m_i \frac{\partial u_i}{\partial x_k} \approx a_{ii\,k} u_i + \sum_j a_{ij\,k} u_j$$

PDCO:

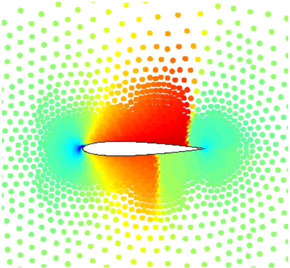
min $\|\boldsymbol{a}_f\|$

s.t. $\begin{bmatrix} \mathbf{C}_f & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_f \\ \boldsymbol{m} \end{bmatrix} = -\mathbf{C}_p \boldsymbol{a}_p,\ \boldsymbol{m} > \mathbf{0}$

LSQR:

min $\|\boldsymbol{a}_f\|$

s.t. $\mathbf{C}_f \boldsymbol{a}_f = -\mathbf{C}_p \boldsymbol{a}_p - \mathbf{D}\boldsymbol{m}$



Geophysics at Stanford

- Paul Segall, Dan Sinnett, Andrew Bradley
  Geophysical inverse problem:
  Determine the dislocation on a dike near Kilauea
  based on GPS data

Geophysics at Stanford

- Paul Segall, Dan Sinnett, Andrew Bradley
  Geophysical inverse problem:
  Determine the dislocation on a dike near Kilauea
  based on GPS data
- Least-squares matrix: half-space Green's functions
  Regularization for smoothness of surface deformation

Geophysics at Stanford

- Paul Segall, Dan Sinnett, Andrew Bradley
  Geophysical inverse problem:
  Determine the dislocation on a dike near Kilauea
  based on GPS data
- Least-squares matrix: half-space Green's functions
  Regularization for smoothness of surface deformation
- Kinematic consistency constraints on the dike
  Certain slip components must be nonnegative

Geophysics at Stanford

- Paul Segall, Dan Sinnett, Andrew Bradley
  Geophysical inverse problem:
  Determine the dislocation on a dike near Kilauea
  based on GPS data
- Least-squares matrix: half-space Green's functions
  Regularization for smoothness of surface deformation
- Kinematic consistency constraints on the dike
  Certain slip components must be nonnegative
- Apply SNOPT

News Flash, 3 March 2007

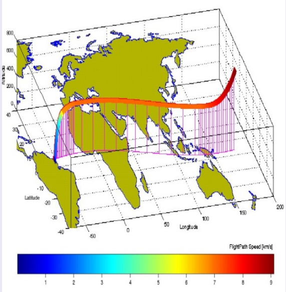
- Mike Ross Naval Postgraduate School, Monterey
  DIDO: A package for solving optimal control problems
  Implemented in MATLAB
  Calls TOMLAB/SNOPT for the optimization

News Flash, 3 March 2007

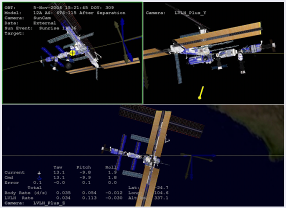
- Mike Ross   Naval Postgraduate School, Monterey
  DIDO: A package for solving optimal control problems
  Implemented in MATLAB
  Calls TOMLAB/SNOPT for the optimization

- GMT 062:19:26
  The International Space Station was successfully maneuvered using DIDO/TOMLAB/SNOPT
  Found zero-propellant solutions (globally optimal)
  Saved NASA $1M fuel cost

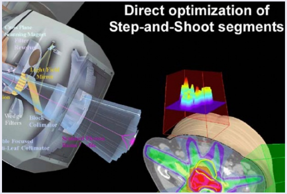
SNOPT Applications (Walter Murray)



Conventional Launcher:
Ariane 5 Dual Payload LEO/GEO



Robot at JPL
Torque minimization
Daniel Clemente



Tumor radiation
Control problem
Paul Keall

Optimization

Stabilize aircraft

Minimize fuel

Reduce CO_2

Make the world a better place