Input-Output Multipliers, General Purpose Technologies, and Economic Development

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Intermediate goods are another produced factor of production, like capital. Simple examples suggest that the multiplier associated with intermediate goods can be substantial, even larger than the one resulting from physical capital accumulation. This paper evaluates this insight using a model in which N goods are produced using all of the other goods as intermediate inputs. Calibrating the model using detailed input-output tables from the United States and 34 other countries confirms the importance of the input-output structure of an economy for economic growth and development. Eleven-fold income differences in a standard neoclassical model become 32-fold differences when intermediate goods are taken into account.

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1. INTRODUCTION

Modern economies involve very sophisticated input-output structures. Goods like electricity, financial services, transportation, information technology and healthcare are both inputs and outputs. A wide range of inter-

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mediate goods are used to produce most goods in the economy, and these goods in turn are often used as intermediates.

Despite our intuitive recognition of this point, standard models of macroeconomics and economic growth typically ignore intermediate goods.[1] The conventional wisdom seems to be that as long as we are concerned about overall value-added (GDP) in the economy, one can specify the model entirely in terms of value added and ignore intermediate goods. Hence the neoclassical growth model.

This conventional wisdom is incorrect, and the goal of this paper is to explore some of the implications of the input-output structure of the economy for economic growth and development.

The first insight that emerges from thinking about intermediate goods is that they are very similar to capital. In fact, the only difference between intermediate goods and capital is one of short-run timing: intermediate goods can be installed more quickly than capital and "depreciate" fully during the course of production, while capital takes a bit longer to install and only partially depreciates during production. From the point of view of the long run — the perspective relevant in most of this paper — intermediate goods and capital are essentially the same. In particular, both are produced factors of production.

The key implications of intermediate goods for economic growth, development, and macroeconomics arise from seeing them as another form of capital. It has long been recognized that the share of capital in production is a fundamental determinant of the quantitative predictions of macro models. When the capital share is 1/3, the intrinsic propogation mechanism of the neoclassical growth model is weak, convergence to the steady state is rapid, and the model generates a small multiplier on changes in productivity or the investment rate. In contrast, when the capital share is higher, like 2/3, these

[1] Exceptions are given at the end of the introduction.

deficiencies are largely remedied. A fairly large portion of the literature on economic growth can be viewed as an attempt to justify using a (broad) capital share of 2/3 when the data for (narrow) capital loudly proclaim that the right number empirically is only 1/3.[2]

As documented carefully below, the intermediate goods share of gross output is about 1/2 across a large number of countries. The share of capital in value-added is about 1/3, so its share in gross output is 1/6. Combining these two kinds of capital, the share of capital-like goods in gross output is our magic number, $1/2 + 1/6 = 2/3$. Incorporating intermediate goods into macroeconomic models, then, has the potential to help us understand a range of economic phenomenon, including the propogation of business cycle shocks and the speed of transition dynamics. These applications will not be explored here. Instead, the main application in this paper will be to the great puzzle of understanding why some countries are 50 times richer than others, as opposed to only 10 times richer.

The paper is organized as follows. The first part provides a simple example to illustrate how and why intermediate goods lead to large multipliers. In this example, a single final output good is used as the single intermediate good in the economy, so the input-output structure is very simple. The second part of the paper builds an N-sector model of economic activity, where each sector uses the outputs from the other sectors as intermediate goods. This model is very similar to the original multi-sector business cycle model of Long and Plosser (1983). The only technological difference is that we include international trade, allowing sectors to import intermediate goods

[2]For examples of these points in various contexts, see Rebelo (1991), Mankiw, Romer and Weil (1992), Cogley and Nason (1995), and Chari, Kehoe and McGrattan (1997). Mankiw et al. (1992) make many of these points, adding human capital to boost the capital share. Chari et al. (1997) introduced "organizational capital" for the same reason. Howitt (2000) and Klenow and Rodriguez-Clare (2005) consider the accumulation of ideas, another produced factor. More recently, Manuelli and Seshadri (2005) and Erosa, Koreshkova and Restuccia (2006) have resurrected the human capital story in a more sophisticated fashion.

from abroad. The substantive difference is in the application to economic growth and development.

The third section connects this model to the wealth of input-output data that exist. Data from 35 countries — including not only the currently rich countries but also Argentina, Brazil, China, and India — allows us to quantify the multiplier associated with the input-output structure of the economy.

The exploration of a last idea is only touched on briefly: connecting the input-output structure to general purpose technologies. In principle, the model and data in this paper can answer questions such as, "If there is a one percentage point improvement in productivity in electric power generation or in the production of information technology goods, what is the long-run overall gain in GDP?" To the extent that a good is associated with a "general purpose technology" that benefits the economy as a whole, one would expect these sectoral multipliers to be large. Input-output analysis therefore potentially offers one way to quantify the impact of general purpose technologies. Unfortunately, this analysis has not yet been completed, so we can only offer some tentative hints at the results that may emerge.[3]

Before continuing, it is worth noting that there is a very important branch of the economics literature that has studied the impact of intermediate goods. Historically, the input-output literature reigned in economics from the 1930s through the 1960s and is most commonly associated with Leontief (1936) and his followers. Hirschman (1958) emphasized the importance of sectoral linkages to economic development, which itself spawned a large literature. Hulten (1978) is also closely related, in showing how intermediate goods should properly be included in growth accounting. More recently,

[3]There is a large literature on general purpose technologies, including David (1990), Bresnahan and Trajtenberg (1995), Jorgenson and Stiroh (1999), Crafts (2004), and many others.

the intermediate goods multiplier shows up most clearly in the economic fluctuations literature; see Long and Plosser (1983), Basu (1995), Horvath (1998), Dupor (1999), Conley and Dupor (2003), and Gabaix (2005). In the international trade context, Yi (2003) argues that tariffs can multiply up in much the same way when goods get traded multiple times during the stages of production. Ciccone (2002) is closely related to the current paper, deriving a multiplier formula through intermediate goods for a very particular input-output structure. Jones (2007) emphasizes the importance of the intermediate goods multiplier for a different but very particular input-output structure.

2. A SIMPLE EXAMPLE

A simple example is quite helpful for understanding how intermediate goods generate a multiplier. Suppose final output Y_t is produced using capital K_t, labor L_t, and intermediate goods X_t.

$$Y_t = \bar{A}\left(K_t^{\alpha} L_t^{1-\alpha}\right)^{1-\sigma} X_t^{\sigma}. \tag{1}$$

Final output can be used for consumption or investment or it can be carried over to the next period and used as an intermediate good. To keep things simple, assume a constant fraction $\bar{s}$ of final output is used for investment and a constant fraction $\bar{x}$ is used as an intermediate good. Therefore

$$K_{t+1} = \bar{s}Y_t + (1-\delta)K_t, \tag{2}$$

$$X_{t+1} = \bar{x}Y_t. \tag{3}$$

Consumption is then given by $C_t = (1 - \bar{s} - \bar{x})Y_t$, and GDP in this economy is consumption plus investment, or output net of intermediate goods: $(1-\bar{x})Y_t$. In other words, all interesting quantities are proportional to Y_t. Assume labor is exogenous and constant.

This model features a steady state, where the level of output per worker $y_t \equiv Y_t/L_t$ is

$$y^* \equiv \frac{Y}{L} = \left(\bar{A}\bar{x}^{\sigma} \left(\frac{\bar{s}}{\delta}\right)^{\alpha(1-\sigma)} \right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \tag{4}$$

A key implication of this result is that a 1% increase in productivity $\bar{A}$ increases output by more than 1% because of the multiplier, $\frac{1}{(1-\alpha)(1-\sigma)}$. In the absence of intermediate goods ($\sigma = 0$), this multiplier is just the familiar $\frac{1}{1-\alpha}$: an increase in productivity raises output, which leads to more capital, which leads to more output, and so on. The cumulation of this virtuous circle is $1 + \alpha + \alpha^2 = \frac{1}{1-\alpha}$.

In the presence of intermediate goods, there is an additional multiplier: higher output leads to more intermediate goods, which raises output (and capital), and so on. The overall multiplier is therefore $\frac{1}{(1-\alpha)(1-\sigma)}$.

Quantitatively, the addition of intermediate goods has a large effect. For example, consider the multipliers using conventional parameter values, a capital exponent of $\alpha = 1/3$ and an intermediate goods share of gross output of $\sigma = 1/2$. In the absence of intermediate goods the multiplier is $\frac{1}{1-\alpha} = 3/2$, and a doubling of TFP raises output by a factor of $2^{3/2} = 2.8$. But with intermediate goods, the multiplier is $\frac{1}{(1-\alpha)(1-\sigma)} = \frac{3}{2} \cdot 2 = 3$, and a doubling of TFP raises output by a factor of $2^3 = 8$. As discussed in Jones (2007), if we think of the standard neoclassical factors (like $\bar{s}$ and $\bar{x}$ in the example) as generating a 4-fold difference in incomes across rich and poor countries, then this 2-fold difference in TFP leads to an 11.3-fold difference in the model with no intermediate goods, but to a 32-fold difference once intermediate goods are taken into account, close to what we see in the data.[4]

[4]An implication of this reasoning that is worthy of further exploration is related to transition dynamics. A puzzle in the growth literature is why speeds of convergence are so slow, on the order of 2% per year; see Hauk and Wacziarg (2004) for a recent summary of the evidence. The standard neoclassical growth model with a capital share of 1/3 leads to

The deeper question in this paper is whether this multiplier carries over into a model with a rich and realistic input-output structure. Perhaps the input-output structure in practice does not lead to these large feedback effects. Or perhaps importing intermediate goods dilutes the multiplier substantially in practice. In fact, the remainder of this paper shows that these concerns are not important in practice. The simple "one over one minus the intermediate goods share" formula suggested by this example turns out to be a very good approximation to the true input-output multiplier in modern economies.

3. THE FULL INPUT-OUTPUT MODEL

Assume the economy consists of N sectors. Each sector uses capital, labor, domestic intermediate goods, and imported intermediate goods to produce output. In turn, this output can be used for final consumption or as an intermediate good in production.

Given this general picture, we specialize to a particular structure with two goals in mind: analytic tractability and obtaining a model that can be closely connected to the rich input-output data. To these ends, the model augments the original Long and Plosser (1983) business cycle model, based on Cobb-Douglas production functions, by embedding it in a model with trade.

We begin by describing the economic environment and then allocate resources using a competitive equilibrium with taxes.

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a speed of convergence of about 7% per year. The presence of intermediate goods would slow this rate down, just as it raises the multiplier. (A difficulty in quantifying this effect is the question of how long it takes to produce and use intermediate goods: one week, one month, or one year? That is, how long is a period?)

3.1. The Economic Environment

Each of the N sectors produces with the following Cobb-Douglas technology:

$$Y_i = A_i \left(K_i^{\alpha_i} H_i^{1-\alpha_i}\right)^{1-\sigma_i-\lambda_i} \underbrace{d_{i1}^{\sigma_{i1}} d_{i2}^{\sigma_{i2}} \cdot ... \cdot d_{iN}^{\sigma_{iN}}}_{\text{domestic IG}} \underbrace{m_{i1}^{\lambda_{i1}} m_{i2}^{\lambda_{i2}} \cdot ... \cdot m_{iN}^{\lambda_{iN}}}_{\text{imported IG}} \tag{5}$$

where i indexes the sector. A_i is an exogenous productivity term, which itself is the product of aggregate productivity A and sectoral productivity η_i: $A_i \equiv A\eta_i$. K_i and H_i are the quantities of physical and human capital used in sector i. Two kinds of intermediate goods are used in production: d_{ij} is the quantity of domestic good j used by sector i, and m_{ij} is the quantity of the imported intermediate good j used by sector i. (We assume imported intermediate goods are different, so that they are not perfect substitutes; this fits with the empirical fact that countries both import and produce intermediate goods in narrow 6-digit categories.) We abuse notation by assuming there are N different intermediate goods that can be imported and by indexing these by j as well. The parameter values in this production function satisfy $\sigma_i \equiv \sum_{j=1}^{N} \sigma_{ij}$ and $\lambda_i \equiv \sum_{j=1}^{N} \lambda_{ij}$ and $0 < \alpha_i < 1$, so the production function features constant returns to scale.

Each domestically produced good can be used for final consumption, c_j, or can be used as an intermediate good:

$$c_j + \sum_{i=1}^{N} d_{ij} = Y_j, \quad j = 1, \ldots, N. \tag{6}$$

Rather than specifying a utility function over the N different consumption goods and performing a formal national income accoutning exercise, it is more convenient to aggregate these final consumption goods into a single final good through another log-linear production function:

$$Y = c_1^{\beta_1} \cdot ... \cdot c_N^{\beta_N}, \tag{7}$$

where $\sum_{i=1}^{N} \beta_i = 1$.

This aggregate final good can itself be used in one of two ways, as consumption or exported to the rest of the world:

$$C + X = Y. \tag{8}$$

It is these exports that pay for the imported intermediate goods. We think of this (static) model as describing the long-run steady state of a model, so we impose balanced trade:

$$\bar{P}X = \sum_{i=1}^{N}\sum_{j=1}^{N} \bar{p}_j m_{ij}, \tag{9}$$

where $\bar{P}$ is the exogenous world price of the final good and $\bar{p}_j$ is the exogenous world price of the imported intermediate goods.

Finally, we assume fixed, exogenous supplies of physical and human capital (for now):

$$\sum_{i=1}^{N} K_i = K, \tag{10}$$

$$\sum_{i=1}^{N} H_i = H. \tag{11}$$

3.2. A Competitive Equilibrium with Taxes

To allocate resources in this economy, we will focus on a competitive equilibrium with tax distortions. As in Chari, Kehoe and McGrattan (forthcoming), Hsieh and Klenow (2006), Lagos (2006), and Restuccia and Rogerson (2007), tax distortions at the micro (here sectoral) level can aggregate up to provide differences in TFP. Sector-specific taxes could literally be taxes, but they could also represent any kind of policy that favors one sector over another (regulations, special consideration for credit, and so on). The additional insight here is that these differences can be multiplied by the input-output structure of the economy.

DEFINITION 1. A *competitive equilibrium with taxes* in this environment is a collection of quantities C, Y, X, Y_i, K_i, H_i, c_i, d_{ij}, m_{ij} and prices p_j, w, and r for $i = 1, \ldots, N$ and $j = 1, \ldots, N$ such that

1. $\{c_i\}$ solves the profit maximization problem of a representative firm in the perfectly competitive final goods market:

$$\max_{\{c_i\}} \bar{P} c_1^{\beta_1} \cdot \ldots \cdot c_N^{\beta_N} - \sum_{i=1}^{N} p_i c_i$$

taking $\{p_i\}$ as given.

2. $\{d_{ij}, m_{ij}\}, K_i, H_i$ solve the profit maximization problem of a representative firm in the perfectly competitive sector i for $i = 1, \ldots, N$:

$$\max_{\{d_{ij}, m_{ij}\}, K_i, H_i} (1-\tau_i) p_i A_i \left(K_i^{\alpha_i} H_i^{1-\alpha_i} \right)^{1-\sigma_i-\lambda_i} d_{i1}^{\sigma_{i1}} d_{i2}^{\sigma_{i2}} \cdot \ldots \cdot d_{iN}^{\sigma_{iN}} m_{i1}^{\lambda_{i1}} m_{i2}^{\lambda_{i2}} \cdot \ldots \cdot m_{iN}^{\lambda_{iN}}$$
$$- \sum_{j=1}^{N} p_j d_{ij} - \sum_{j=1}^{N} \bar{p}_j m_{ij} - rK_i - wH_i,$$

taking $\{p_i\}$ as given (τ_i, A_i, and $\bar{p}_j$ are exogenous).

3. Markets clear

(i) r clears the capital market: $\sum_{i=1}^{N} K_i = K$

(ii) w clears the labor market: $\sum_{i=1}^{N} H_i = H$

(iii) p_j clears the sector j market: $c_j + \sum_{i=1}^{N} d_{ij} = Y_j$

4. Balanced trade pins down X:

$$\bar{P} X = \sum_{i=1}^{N} \sum_{j=1}^{N} \bar{p}_j m_{ij}.$$

5. Production functions for Y_i and Y:

$$Y_i = A_i \left(K_i^{\alpha_i} H_i^{1-\alpha_i} \right)^{1-\sigma_i-\lambda_i} d_{i1}^{\sigma_{i1}} d_{i2}^{\sigma_{i2}} \cdot \ldots \cdot d_{iN}^{\sigma_{iN}} m_{i1}^{\lambda_{i1}} m_{i2}^{\lambda_{i2}} \cdot \ldots \cdot m_{iN}^{\lambda_{iN}}$$

$$Y = c_1^{\beta_1} \cdot \ldots \cdot c_N^{\beta_N}.$$

6. Consumption is the residual:

$$C + X = Y.$$

Counting loosely, there are 12 equilibrium objects to be determined and 12 equations implicit in this equilibrium definition. Hiding behind the last equation is the fact that tax revenues are rebated lump sum to households. Because of balanced trade, however, there is no decision for households to make regarding final consumption C, and it is simply determined as the residual of final output less exports.[5]

3.3. Solving

In solving for the equilibrium of the model, it is useful to define some notation involving linear algebra. This is summarized in Table 1. Then the following proposition characterizes the equilibrium:

PROPOSITION 1 (Solution for Y and C). *In the competitive equilibrium, the solution for total production of the aggregate final good is*

$$Y = A^{\tilde{\mu}} K^{\tilde{\alpha}} H^{1-\tilde{\alpha}} \epsilon, \tag{12}$$

where the following notation applies:

$\mu' \equiv \frac{\beta'(I-B)^{-1}}{1-\beta'(I-B)^{-1}\lambda}$, *($N \times 1$ vector of multipliers)*
$\tilde{\mu} \equiv \mu' \mathbf{1}$
$\tilde{\alpha} \equiv \mu' \delta_K$
$\omega \equiv \frac{\beta' \omega_c + \beta'(I-B)^{-1}\omega_y}{1-\beta'(I-B)^{-1}\lambda}$
$\log \epsilon \equiv \omega + \mu' \bar{\eta}$.

Moreover, GDP for this economy is given by C, which equals

$$C = Y \left(1 - \sum_{i=1}^{N} \sum_{j=1}^{N} (1-\tau_i) \gamma_i \lambda_{ij} \right). \tag{13}$$

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[5] I presume this equation could be replaced by $\bar{P}C = wH + rK + T$, where T is the lump sum rebate. zzz Check.

TABLE 1.
Notation for Solving the Model

Notation	Typical Element	Comment
Matrices ($N \times N$):		
B	σ_{ij}	The input-output matrix of intermediate good shares.
$\bar{B}$	$(1-\tau_i)\sigma_{ij}$	The matrix of intermediate good exponents, adjusted for taxes.
I	—	Identity matrix.
Vectors ($N \times 1$):		
$\mathbf{1}$	1	Vector of ones.
β	β_i	Vector of exponents in final goods production.
γ	γ_i	$\gamma \equiv (I-\bar{B}')^{-1}\beta$; $\frac{p_i Y_i}{PY} = \gamma_i$
λ	λ_i	Vector of import shares, $\lambda_i \equiv \sum_{j=1}^N \lambda_{ij}$.
δ_K	$\alpha_i(1-\sigma_i-\lambda_i)$	Production elasticities for K_i
δ_H	$(1-\alpha_i)(1-\sigma_i-\lambda_i)$	Production elasticities for H_i
θ_K	$\frac{(1-\tau_i)\delta_{Ki}\gamma_i}{\sum_{j=1}^N (1-\tau_j)\delta_{Kj}\gamma_j}$	Solution for K_i/K
θ_H	$\frac{(1-\tau_i)\delta_{Hi}\gamma_i}{\sum_{j=1}^N (1-\tau_j)\delta_{Hj}\gamma_j}$	Solution for H_i/H
ω_K	$\delta_{Ki}\log\theta_{Ki}$	Sectoral allocation term for K_i
ω_H	$\delta_{Hi}\log\theta_{Hi}$	Sectoral allocation term for H_i
ω_d	$\sum_{j=1}^N \sigma_{ij}\log(\sigma_{ij}\gamma_i/\gamma_j)$	Sectoral allocation term for d_{ij}
ω_m	$\sum_{j=1}^N \lambda_{ij}\log(\lambda_{ij}\bar{P}\gamma_i/\bar{p}_j)$	Sectoral allocation term for m_{ij}
ω_y	$\omega_{Ki}+\omega_{Hi}+\omega_{di}+\omega_{mi}$	Sum of allocation terms
ω_c	$\log(\beta_i/\gamma_i)$	Consumption allocation term
$\bar{\eta}$	$\log(\eta_i(1-\tau_i))$	Sectoral productivity, adjusted for taxes

There are several points of this proposition that merit discussion. First, and not surprisingly, our N-sector Cobb-Douglas model aggregates up to yield a Cobb-Douglas aggregate production function. More interestingly, aggregate TFP depends on both sectoral TFPs and the underlying tax distortions. This latter point requires digging into the ϵ term, where taxes then enter in two places. Taxes enter directly through $\bar{\eta}$, which is a vector of sectoral productivities, adjusted for tax rates; this is the usual sense in which taxes "directly" affect productivity. Taxes also enter indirectly through the allocation terms, captured by ω. We will return later to the effect of tax distortions.

The second result to note is the presence of the input-output multiplier, reflected by μ. According to the proposition, this vector of multipliers is given by

$$\mu' \equiv \frac{\beta'(I-B)^{-1}}{1-\beta'(I-B)^{-1}\lambda}. \tag{14}$$

Let's break this down piece by piece, since it is one of the essential results of the paper.

The matrix $L \equiv (I-B)^{-1}$ is known as the Leontief inverse. The typical element ℓ_{ij} of this matrix can be interpreted in the following way: (ignoring trade for the moment) a 1% increase in productivity in sector j raises output in sector i by ℓ_{ij}%. This result takes into account all of the indirect effects at work in the model. For example, raising productivity in the electricity sector makes banking more efficient and this in turn raises output in the construction industry. The Leontief inverse incorporates these indirect effects. (Notice that it is the matrix equivalent of $1/1-\sigma$.)

Multiplying this matrix by the vector of value-added weights in β leads to $\beta'(I-B)^{-1} = \sum_{i=1}^{N} \beta_i \ell_{ij}$. That is, we add up the effects of sector j on all of the other sectors in the economy, weighting by their shares of value-added. The typical element of this multiplier matrix then reveals how a change in producitivity in sector j affects overall value-added in the economy.

All of this would be precisely correct if λ_{ij} were zero — that is, in the absence of trade. In the presense of trade, this multiplier gets adjusted by the factor $1/(1-\beta'(I-B)^{-1}\lambda)$. We will discuss this factor in more detail below, but for now it is enough to note that this factor is larger than one: trade strengthens the multiplier rather than attenuating it.

The elasticity of final output with respect to aggregate TFP is $\tilde{\mu} \equiv \mu'\mathbf{1}$. That is, we add up all of the multipliers in μ since an increase in aggregate TFP affects not just sector j but all of the sectors.

A final remark about Proposition 1 concerns the capital exponent in the aggregate production function, $\tilde{\alpha} \equiv \mu'\delta_K$. Recall that δ_K is the vector of capital exponents $\alpha_i(1-\sigma_i-\lambda_i)$. The aggregate exponent is therefore a weighted average of the sectoral capital shares, where the weights depend on the intermediate good shares. This remark will make even more sense after the next proposition.

4. SPECIAL CASES, TO BUILD INTUITION

4.1. The Multiplier in a Special Case

The linear algebra formula is a useful theoretical result and will prove convenient when we apply the model to the rich input-output data that exists. However, analyzing a special case can be helpful in obtaining intuition for how the model works.

Consider the following special case. Suppose all sectors have the same cumulative elasticities of output with respect to domestic and imported intermediate goods, although the composition across sectors is allowed to vary. For example, one sector may use a lot of electricity and steel, while another sector uses a lot of financial services and information technology.

The composition can vary across sectors, but suppose each sector spends 50 percent of its revenue on intermediate goods. What does the multiplier look like in a case like this?

The following proposition provides the answer. In fact, it allows for imported intermediate goods as well (where the overall share spent on these goods is the same in each sector):

PROPOSITION 2 (Multiplier in a special case). *Assume* $\sigma_i \equiv \sum_{j=1}^{N} \sigma_{ij} = \bar{\sigma}$ *and* $\lambda_i \equiv \sum_{j=1}^{N} \lambda_{ij} = \bar{\lambda}$ *for all* i*, where* $\bar{\sigma}$ *and* $\bar{\lambda}$ *are positive scalars whose sum is less than one. Then*

$$\frac{\partial \log Y}{\partial \log A} = \mu'\mathbf{1} = \frac{\beta'(I-B)^{-1}\mathbf{1}}{1-\beta'(I-B)^{-1}\lambda} = \frac{1}{1-(\bar{\sigma}+\bar{\lambda})}.$$

This special case makes two general points about the model. First, the "sparseness" of the input-output matrix B is not especially important. For example, our special case includes a "clock" structure, where every sector uses as an input only the good produced by the sector above it. It also includes the case where every sector uses only its own output. In both of these cases, the input-output matrix is very sparse, with zeros almost everywhere. Yet, the overall multiplier remains equal to one over one minus the intermediate goods share. This special case suggests that if the overall intermediate goods share is about 1/2, we shouldn't be surprised to find a multiplier of about 2. This intuition will be confirmed in the next section when we turn to quantitative results.

The second key point made in this proposition is that the intuition that imports would dilute the multiplier is a red herring. In fact, there is no dilution at all: in the proposition, it is the overall intermediate goods share $\bar{\sigma}+\bar{\lambda}$ that matters for the multiplier, and the composition between domestic and imported goods is completely irrelevant.

Why is this the case? The answer is that we have imposed balanced trade in our (long run) model. Therefore exports are used to "produce" imports. A higher productivity in the domestic computer chip sector raises overall exports, which in turn increases imports, so the virtuous circle is not broken by the presence of trade.[6]

4.2. Symmetry and Taxes

Our second special case allows us to study the distortions associated with taxes. First, we consider a world where the intermediate good shares of production are the same in every sector and there is a symmetric tax at rate $\tau_i = \bar{\tau}$. In this case, GDP in the economy is given by the following proposition:

PROPOSITION 3 (Symmetry and Taxes). *Suppose* $\sigma_{ij} = \hat{\sigma}$, $\lambda_{ij} = \hat{\lambda}$, $\beta_i = 1/N$, *and* $\tau_i = \bar{\tau}$. *Then*

$$\log C = \textit{Constant} + \frac{\bar{\sigma} + \bar{\lambda}}{1 - (\bar{\sigma} + \bar{\lambda})} \log(1 - \bar{\tau}) + \log\left(1 - (1 - \bar{\tau})(\bar{\sigma} + \bar{\lambda})\right), \tag{15}$$

where $\bar{\sigma} \equiv N\hat{\sigma}$, $\bar{\lambda} \equiv N\hat{\lambda}$, *and Constant is a collection of terms that do not depend on* $\bar{\tau}$. *Moreover, (log) consumption is an inverse-U shaped function of the tax rate, with a peak that occurs at* $\bar{\tau} = 0$.

Notice that the effect of a change in the tax rate on GDP depends essentially on $\bar{\sigma} + \bar{\lambda}$. If there are no intermediate goods in this economy, output taxes have no effect. This is because the tax distortions here represent a violation of the Diamond and Mirrlees (1971) dictum of "no taxation of intermediate goods." In our (current) setup, K and H are non-produced factors, so a symmetric tax does not distort the allocation of capital.

[6]This assumption of balanced trade is the key difference that makes the intuition from the Keynesian business cycle model inappropriate. In the business cycle context, an increase in exports leads to a trade surplus and does not increase imports.

The key distortion is between consumption and intermediate goods. A good that gets consumed pays the tax only once when the good is produced; a good that is used as an intermediate pays the tax when it is first produced then when it is used as an intermediate. Since a constant fraction of output is consumed and the rest is used as an intermediate good, this process suffers from the vicious cycle of the multiplier.

(Note for a future version: Physical capital is certainly a produced factor, and it is very much like an intermediate good. Hence the right "number" to plug in for $\bar{\sigma} + \bar{\lambda}$ is probably 2/3: the intermediate goods share plus the capital share. Taking into account the portion of human capital that is produced using output, the share would be even larger; labor could be distorted as well if there were a labor-leisure decision.)

Symmetric taxes affect GDP through the two terms in equation (15). The first term is the direct effect, where taxes enter the model very much like productivity: recall that both $1 - \tau_i$ and A_i are subject to the multiplier effect through the ϵ term in Proposition 1. The second term mitigates this effect somewhat and captures the indirect effect whereby higher taxes raise consumption (by reducing the purchase of intermediate goods).

4.3. Symmetry with Random Taxes

Our final special case allows us to consider variation in taxes across sectors. Suppose everything in the model other than taxes is symmetric, and allow taxes to be a log-normally distributed random variable:

PROPOSITION 4 (Symmetry with Random Taxes). *Suppose* $\sigma_{ij} = \hat{\sigma}$, $\lambda_{ij} = \hat{\lambda}$, *and* $\beta_i = 1/N$. *Assume* $\log(1 - \tau_i) \sim N(\theta, v^2)$. *Then*

$$\text{plim}_{N\to\infty} \log C = \mathcal{C} \equiv \textit{Constant} + \frac{\bar{\sigma} + \bar{\lambda}}{1 - (\bar{\sigma} + \bar{\lambda})} \cdot \theta + \log\left(1 - (\bar{\sigma} + \bar{\lambda})e^{\theta + \frac{1}{2}v^2}\right) - \frac{1}{2}v^2,$$

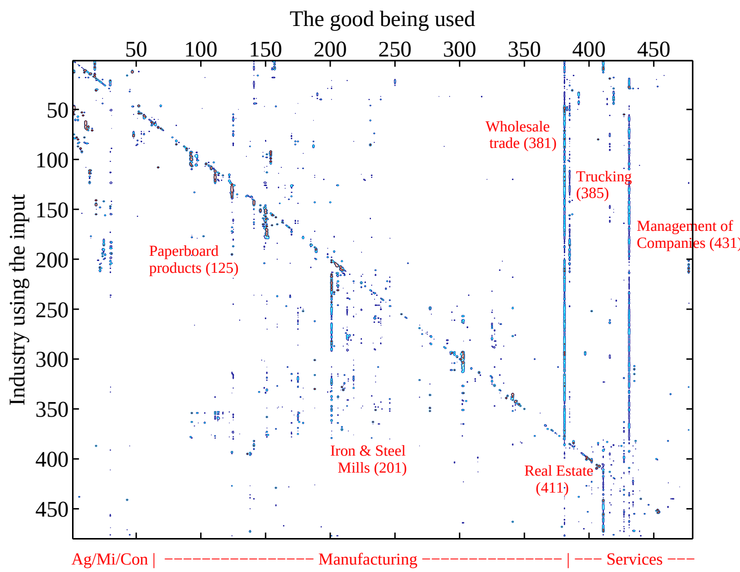
where $\bar{\sigma} \equiv N\hat{\sigma}$, $\bar{\lambda} \equiv N\hat{\lambda}$, *and Constant is a collection of terms that do not depend on* θ *or* v^2. *Moreover, consumption is maximized when there are no taxes; also* $\frac{\partial \mathcal{C}}{\partial v^2} < 0$.

In terms of the mean effect of taxes, this result looks very much like the previous one. Now, however, we have an additional result related to the variance of taxes across sectors. In particular, a higher variance of taxes reduces GDP, even in the absence of intermediate goods, since random taxes will distort the allocation of capital and labor across sectors. However, the variance term itself is subject to a multiplier effect: $\frac{\partial \mathcal{C}}{\partial v^2} = -\frac{1}{2} \cdot \left(\frac{1}{1-(\bar{\sigma}+\bar{\lambda})(1-\bar{\tau})} \right)$, where $\bar{\tau}$ is the average tax rate. A higher variance of taxes is more costly in an economy with intermediate goods. This makes sense: the first best in this economy is to have no taxes. Either a constant tax or a random tax distorts the allocation of resources and reduces GDP. The magnitude of the distortion depends on the Diamond-Mirrlees effect: how important intermediate goods are in production.

5. QUANTITATIVE ANALYSIS

We now turn to the rich input-output data that exists, both for the United States and for many other countries. This data allows us to calculate aggregate and sectoral multipliers and to study the effect of sectoral tax distortions on aggregate GDP. First, we use the six-digit level data available from the U.S. Bureau of Economic Analysis for the United States in 1997. Then we turn to the OECD Input-Output Database, which contains data for 48 industries and 35 countries.

FIGURE 1. The U.S. Input-Output Matrix, 1997 (480 Industries)



Note: The plot shows the matrix $[\sigma_{ij} + \lambda_{ij}]$, that is, the matrix of intermediate good shares for 480 industries. A contour plot method is used, showing only those shares greater than 2%, 4%, and 8%. Source: BEA 1997 Input-Output Benchmark data.

5.1. The U.S. Input-Output Data, 480 Industries

Figure 1 shows something very close to the B matrix for the United States, using the 480 commodities in the BEA's 1997 benchmark input-output data. Actually, we plot the matrix of $\sigma_{ij} + \lambda_{ij}$ instead, so that the entries show the overall exponents on intermediate goods used in producing each of the 480 goods. A contour plot method is used, showing only those shares greater than 2%, 4%, and 8%.

Three points stand out in the figure. First, there is a strong diagonal. Second, the matrix is relatively sparse. Finally, there are a few key exceptions to this sparseness: a few key goods are used by a large number of industries

TABLE 2.
Statistics of the U.S. Input-Output Matrix, 1997 (480 Industries)

Properties of the diagonal elements	
Mean:	0.033
75th percentile:	0.045
50th percentile:	0.010
25th percentile:	0.002
Fraction of all elements that are	
equal to zero:	0.510
below 0.1 percent:	0.882
below 0.5 percent:	0.958
below 1.0 percent:	0.979
below 5.0 percent:	0.996
above 10 percent:	0.0013
above 20 percent:	0.0004
above 50 percent:	0.0000
Mean of $\sigma_i + \lambda_i$:	0.564
75th percentile:	0.666
50th percentile:	0.558
25th percentile:	0.477
Aggregate Multipliers	
Domestic, $\beta'(I-B)^{-1}\mathbf{1}$	1.61
Imports, $1/(1-\beta'(I-B)^{-1}\lambda)$	1.03
Overall, $\tilde{\mu}$	1.65
Actual intermediate goods share:	0.434
"As if" intermediate goods share:	0.394

Note: Except where noted, staistics are reported for the overall input-output matrix of $\sigma_{ij} + \lambda_{ij}$.

in a significant way. These include wholesale trade, trucking, management of companies, real estate, paperboard products, and iron and steel mills.

Table 2 reports some basic statistics of the U.S. input-output matrix that help put these visual conclusions in context. Even though the diagonal elements were important visually, the table makes the point that these elements are typically small: the mean of them is only 3.3% and the median is only 1.0%. This is true despite the fact that the typical industry pays a large share of its gross output to intermediate goods: 56.4% at the mean. The industry at the 75th percentile pays out about two-thirds of its revenue to intermediate goods, while even the industry at the 25th percentile pays nearly half. Along these lines, it is worth noting that even though just 0.13% of the elements of the input-output matrix exceed 10 percent, this is still 288 elements over all; similarly, 83 of the entries are greater than 20 percent. As the bottom of the table shows, the overall intermediate goods share for the U.S. economy is about 43.4%: service industries are more important as a share of value-added, and these industries have lower intermediate goods shares.

The last part of the table computes the aggregate multiplier using the 6-digit input-output data. A 1% improvement in TFP in every sector raises overall GDP by 1.65%. This number is the product of a domestic multiplier of 1.61 (that would obtain if no intermediate goods were imported), and an import multiplier of 1.03. Imports are relatively unimportant in the multiplier.

To what extent is the simple $\frac{1}{1-(\bar{\sigma}+\bar{\lambda})}$ formula accurate? The multiplier of 1.65 would result from this formula "if" the intermediate goods share were 0.394. In fact, the intermediate goods share using this 6-digit data is 0.434. This simple aggregate formula appears to give a good approximation to the result found by computing the 480x480 Leontief inverse, although there is a small degree of dilution: applying the formula to the 0.434 share suggests a multiplier that overstates the truth by about ten percent.

5.2. General Purpose Technologies?

To what extent can the input-output structure of the economy help us to understand general purpose technologies? The answer is unclear. To the extent that the adoption of a general purpose technology leads to other changes in the structure of production, this may not be directly apparent in the input-output structure. For example, a common hypothesis is that the adoption of electricity or information technology leads to fundamental changes in the nature and/or organization of production. Perhaps such changes will not be apparent.

On the other hand, perhaps they will. Sectoral multipliers — the μ_i terms in the model — should represent something like a first-order or local derivative of output with respect to a particular productivity level. If the production function really is Cobb-Douglas, then this local derivative could extend more broadly. Or if we had input-output tables from 1970 or 1900, perhaps tracing the path of the local derivatives would be informative. (In fact, the BEA does provide benchmark input-output tables every five years, going back to 1967 at least. So it should be possible to explore the GPT nature of information technologies in more detail.)

Table 3 provides some detail on the sectoral multipliers, μ_i. In particular, we find the sectors that have the largest excess multiplier — that is, the sectors where $\mu_i - \beta_i$ is the largest. (Recall that a 1% increase in productivity in sector i raises output by $\beta\%$ directly because of this sector's role in final consumption. So the net multiplier effect from the input-output structure of the economy subtracts this off.) Important sectors according to this measure include real estate, wholesale trade, management of companies, and advertising. Just below that are telecommunications, oil and gas extraction, power generation, banking, trucking, and legal services. All seem like sectors that are generally important in the production of a wide range of goods in the economy. In this sense, the sectoral multipliers may indeed be telling us something about general purpose technologies.

For comparison, we also report the multipliers for some industries related to information technology at the bottom of the table. These multipliers are small for two reasons. First, the share of these industries in value-added is small, so the direct effect is already small. Second, and perhaps more

TABLE 3.
U.S. Input-Output Multipliers, 1997 (480 Industries)

Industry	Excess Multiplier $\mu_i - \beta_i$	Multiplier μ_i	V.A. Share β_i	— IntGood Shares — Domestic σ_i	Import λ_i
Real estate	0.043	0.094	0.051	0.306	0.003
Wholesale trade	0.034	0.091	0.057	0.356	0.009
Management of companies	0.029	0.056	0.027	0.291	0.004
Advertising services	0.020	0.032	0.011	0.446	0.012
Telecommunications	0.018	0.036	0.018	0.394	0.013
Oil and gas extraction	0.014	0.018	0.004	0.579	0.016
Power generation/supply	0.013	0.030	0.017	0.355	0.010
Banking (depository)	0.013	0.042	0.029	0.271	0.003
Truck transportation	0.012	0.022	0.010	0.501	0.011
Legal services	0.011	0.024	0.013	0.276	0.003
Information Technology Industries					
Computer manufacturing	...	0.001	0.001	0.845	0.016
Computer storage devices	...	0.001	0.001	0.619	0.054
Other computer equipment	...	0.001	0.001	0.662	0.074
Semiconductors	...	0.011	0.006	0.351	0.030
Software publishers	...	0.005	0.005	0.305	0.022
Custom programming	...	0.008	0.008	0.294	0.024

importantly, many of the products of information technology (including software) are capital goods, not intermediate goods . To compute the true multipliers associated with these sectors, one would need to know how much capital each sector produces and where that capital is used. These calculations are possible, using the capital flow table provided by the BEA. However, I haven't yet had a chance to do these calculations.[7]

5.3. The OECD Input-Output Data, 48 Industries

The 2006 edition of the OECD Input-Output Database contains input-output data for 35 countries and 48 industries, typically for the year 2000. In addition to covering OECD countries, the data also include some poor and middle-income countries, such as China, India, Argentina, Brazil, and Russia.

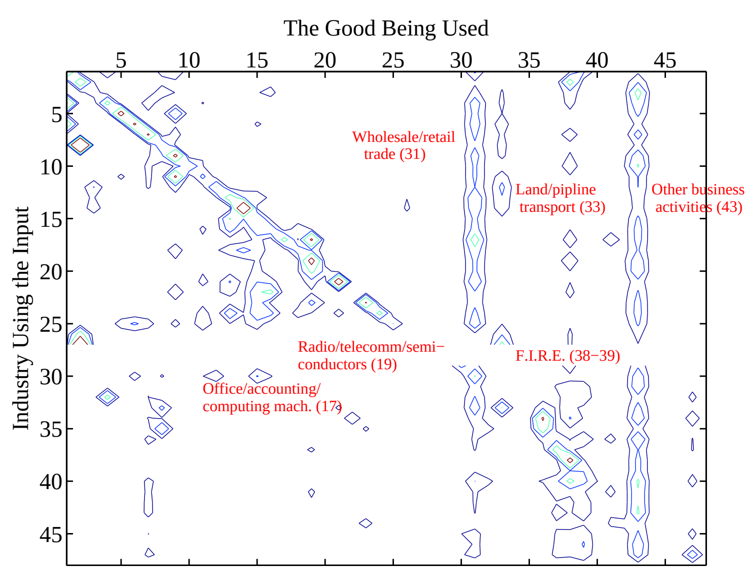
Figure 2 shows the input-output matrix for the United States at this higher level of aggregation. The pattern at the more detailed level of aggregation of a sparse matrix with a strong diagonal and just a few goods that are used widely is repeated at this higher level of aggregation.

One of the nice features of the OECD data is that we can consider the question of how much the input-output structure of an economy differs across countries. The general and perhaps surprising answer that one obtains is "not much." Figure 3 shows the input-output matrix for two countries, Japan and China, as an example.

The matrix for Japan looks very much like the matrix for the United States. This is true more generally, especially for the richer countries in the data set. But it is even true for the poorer countries. The input-output

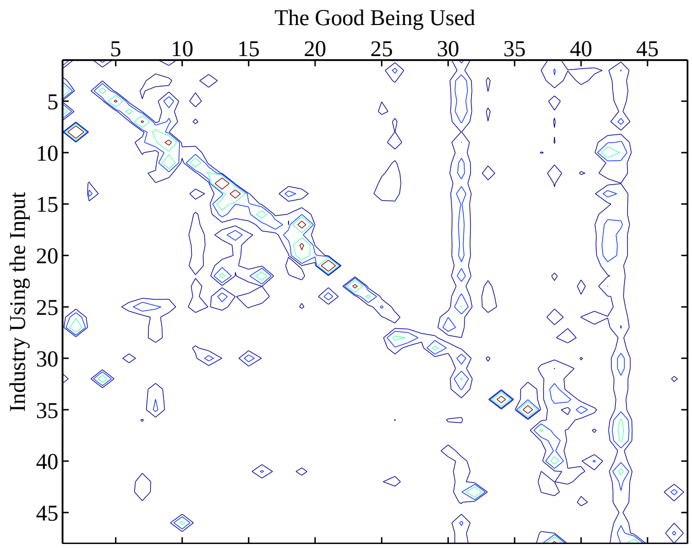
[7]U.S. BEA has a "capital flow" matrix which essentially decomposes private investment into a CxI use table. (Some aggregation issues). If we really want to pursue the GPT and industry multiplier logic, then incorporating the capital flow matrix is an obvious next step. (Question: rK/Y versus pI/Y — if we just treat capital as an intermediate good, we will get too low a share.)

FIGURE 2. The U.S. Input-Output Matrix, 2000 (48 Industries)

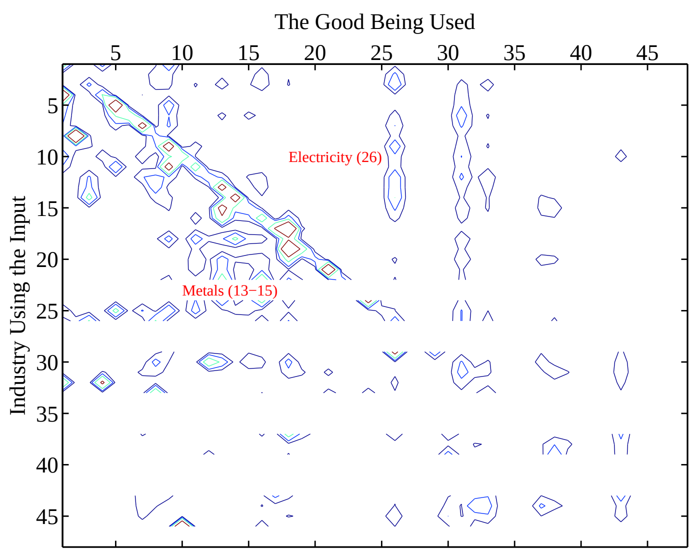


Note: See notes to Figure 1. Source: OECD 2006 database.

FIGURE 3. Input-Output Matrix in Japan and China (48 Industries)



(a) Japan



(b) China

matrix for China is perhaps the most different from the United States, but the overall structure is still similar. Electricity shows up as being noticeably more important, and other business activities (which include advertising, accounting, and legal services) as somewhat less important. These are the main differences.

The first column of Table 4 makes these comparisons more systematically. It shows the fraction of elements in the input-output matrix that differ by more than 0.02 from the corresponding elements in the U.S. input-output matrix. Just over 16 percent of the elements exceed this difference in China's input-output matrix, while the corresponding number for Japan is about 9 percent. For this level of the cutoff, the average across the 35 countries is 11 percent. If we lower the cutoff to 0.01, the typical country has differences of this magnitude in just over 20 percent of the cells. If we raise the cutoff to 0.05, the average across countries is 3.9 percent of cells.

Figure 4 shows the aggregate multipliers, $\tilde{\mu}$ for the 35 countries in our sample. The average value for the multiplier in this sample is about 1.9. It ranges from a high of 2.53 in China to lows of 1.51 in Greece and 1.59 in India. Interestingly, China and India are two of the poorest countries in the sample, and they have widely different multipliers. The multiplier for the United States using this data works out to be 1.77, slightly higher than what we found in the 6-digit data.

Table 4 shows these multipliers in more detail, including the contribution from imported intermediate goods as well as the aggregate intermediate goods share and the "as if" share that corresponds to the multiplier computed using the Leontief inverse. The simple approximation of "one over one minus the intermediate goods share" does a very good job of approximating the true multiplier.

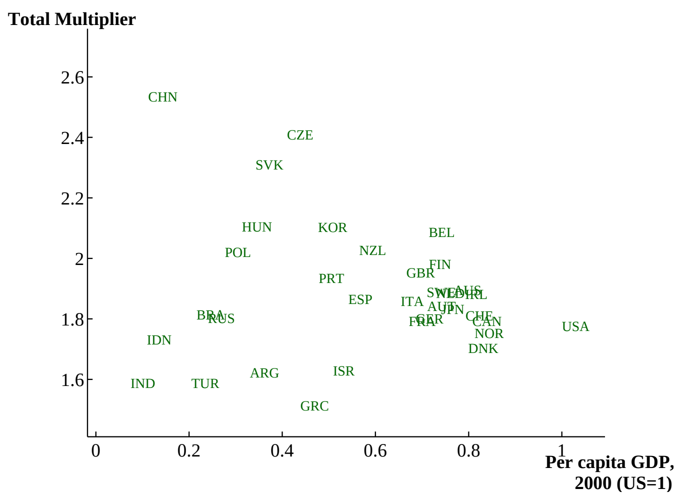
zzz The general similarity of these matrices across countries suggests that C-D not too bad...

TABLE 4.
The Multiplier across a Range of Countries (48 Industries)

Country	Fraction > .02 Different	Multipliers: Domestic	Multipliers: Import	Multipliers: Total	Overall Interm. Share	"As If" Interm. Share
China	0.161	2.21	1.14	2.53	0.63	0.61
Czech Republic	0.115	1.75	1.38	2.41	0.62	0.58
Slovak Republic	0.114	1.68	1.38	2.31	0.61	0.57
Hungary	0.107	1.53	1.38	2.10	0.60	0.52
Korea	0.109	1.72	1.22	2.10	0.58	0.52
Belgium	0.104	1.60	1.30	2.09	0.57	0.52
New Zealand	0.114	1.77	1.15	2.03	0.54	0.51
Poland	0.120	1.73	1.17	2.02	0.53	0.50
Finland	0.101	1.63	1.21	1.98	0.53	0.50
United Kingdom	0.096	1.72	1.14	1.95	0.51	0.49
Portugal	0.112	1.63	1.18	1.93	0.52	0.48
Australia	0.104	1.71	1.11	1.89	0.49	0.47
Sweden	0.096	1.57	1.21	1.89	0.51	0.47
Netherlands	0.096	1.54	1.22	1.89	0.51	0.47
Ireland	0.135	1.35	1.39	1.88	0.53	0.47
Spain	0.099	1.59	1.17	1.87	0.50	0.46
Italy	0.094	1.62	1.15	1.86	0.50	0.46
Austria	0.085	1.51	1.22	1.84	0.48	0.46
Taiwan	0.104	1.53	1.20	1.83	0.52	0.45
Japan	0.092	1.75	1.05	1.83	0.48	0.45
Brazil	0.109	1.69	1.07	1.81	0.48	0.45
Switzerland	0.151	1.54	1.17	1.81	0.49	0.45
Russia	0.242	1.63	1.11	1.80	0.47	0.45
Germany	0.104	1.58	1.14	1.80	0.49	0.44
France	0.104	1.63	1.10	1.79	0.48	0.44
Canada	0.087	1.52	1.18	1.79	0.48	0.44
United States	0.000	1.68	1.05	1.77	0.46	0.44
Norway	0.098	1.53	1.15	1.75	0.46	0.43
Indonesia	0.133	1.52	1.14	1.73	0.49	0.42
Denmark	0.098	1.48	1.15	1.70	0.43	0.41
Israel	0.106	1.49	1.10	1.63	0.41	0.39
Argentina	0.096	1.53	1.06	1.62	0.42	0.38
Turkey	0.114	1.43	1.11	1.59	0.41	0.37
India	0.153	1.49	1.07	1.59	0.44	0.37
Greece	0.114	1.37	1.10	1.51	0.38	0.34
Average	0.110	1.61	1.17	1.88	0.50	0.46

Note: The first column reports the fraction of entries in a country's input-output matrix that differ from those in the U.S. matrix by more than 0.02.

FIGURE 4. The Multiplier across a Range of Countries (48 Industries)



The figure plots the value of $\tilde{\mu}$ computed for each country against 2000 per capita GDP from the Penn World Tables.

5.4. Extensions

Tax multipliers. The multipliers I've calculated so far are exactly correct for TFP changes, but only approximately correct for tax changes. They include the direct tax effect, but not the indirect effects. This needs to be done in the future...

6. CONCLUSIONS

The simple example provided in Section 2 of this paper suggests that intermediate goods may provide a very substantial multiplier in models of growth and development. With an intermediate goods share of 1/2, the simple $1/(1-\sigma)$ formula suggests a multiplier of 2. Recall that this is powerful enough to turn 11-fold differences in incomes across countries into 32 fold differences.

The question considered in the main part of the paper is whether this simple formula holds up when one considers the detailed input-output structure of modern economies. The answer is that it does: the average multiplier in the 35 countries for which we have data, for example, is 1.88, ranging from a low of about 1.6 in India to a high of about 2.5 in China. The input-output multiplier, then, may be an important part of a theory of economic development.

There are numerous directions for additional research suggested by this analysis. Sectoral multipliers and the multipliers on idiosynchratic tax distortions have barely been explored. Do some sectors, like electricity or information technology, have a particularly significant role that can be detected in the input-output tables? Would distortions to the allocation of resources in these sectors have large negative effects on GDP? How different are the input-output structures across economies? Why do China and India have such different structures, while the rich countries, especially, seem much more similar? How have these input-output structures changed over time?

APPENDIX: PROOFS

Proposition 1: Solving for Y and C.

Proof. To be provided. ∎

Proposition 2: The Multiplier in a Special Case.

Proof. In matrix notation, the assumption that all sectors have a cumulative domestic intermediate goods share of $\bar{\sigma}$ is simply $B\mathbf{1} = \bar{\sigma}\mathbf{1}$. This implies the following:

$$\begin{aligned}(I-B)\mathbf{1} &= (1-\bar{\sigma})\mathbf{1} \\ \mathbf{1} &= (I-B)^{-1}\mathbf{1}\cdot(1-\bar{\sigma}) \\ 1=\beta'\mathbf{1} &= \beta'(I-B)^{-1}\mathbf{1}\cdot(1-\bar{\sigma}) \\ \Rightarrow \quad & \beta'(I-B)^{-1}\mathbf{1} = \frac{1}{1-\bar{\sigma}}.\end{aligned}$$

Similarly, $\beta'(I-B)^{-1}\lambda = \frac{\bar{\lambda}}{1-\bar{\sigma}}$. Therefore

$$\mu'\mathbf{1} = \frac{\beta'(I-B)^{-1}\mathbf{1}}{1-\beta'(I-B)^{-1}\lambda} = \frac{1}{1-(\bar{\sigma}+\bar{\lambda})}.$$

▮

Proposition 3: Symmetric and Taxes.

Proof. The key step in solving the model is to use the same general result as in the previous proposition: if a matrix X has rows that sum to the same value, $\bar{x}$, then $(I-X)^{-1}\mathbf{1} = \mathbf{1}\cdot\frac{1}{1-\bar{x}}$. In this case, this result is used in computing $\gamma = (I-\bar{B})^{-1}\beta$, where $\beta_i = 1/N$. Everything else follows from careful calculation. ▮

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