Intermediate Goods and Weak Links: A Theory of Economic Development

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Abstract

Per capita income in the richest countries of the world exceeds that in the poorest countries by more than a factor of 50. What explains these enormous differences? This paper returns to two old ideas in development economics and proposes that linkages and complementarity are a key part of the explanation. First, linkages between firms through intermediate goods deliver a multiplier similar to the one associated with capital accumulation in a neoclassical growth model. Because the intermediate goods share of gross output is about 1/2, this multiplier is substantial. Second, just as a chain is only as strong as its weakest link, problems at any point in a production chain can reduce output substantially if inputs enter production in a complementary fashion. This paper builds a model to quantify these forces and shows that they substantially amplify distortions to the allocation of resources, bringing us closer to understanding large income differences across countries.

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1. Introduction

By the end of the 20th century, per capita income in the United States was more than 50 times higher than per capita income in Ethiopia and Tanzania. Dispersion across the 95th-5th percentiles of countries was more than a factor of 32. What explains these profound differences in incomes across countries?[1]

This paper returns to two old ideas in the development economics literature and proposes that linkages and complementarity are a key part of the explanation. First, intermediate goods provide links between sectors that create a multiplier. Low productivity in electric power generation — for example because of theft, inferior technology, or misallocation — makes electricity more costly, which reduces output in banking and construction. But this in turn makes it harder to finance and build new dams and therefore further hinders electric power generation. This multiplier effect is similar to the multiplier associated with capital accumulation in a neoclassical growth model. In fact, intermediate goods are just another form of capital, albeit one that depreciates fully in production. Because the intermediate goods share of gross output is approximately 1/2, the intermediate goods multiplier is large.

Second, as a result of complementarity, high productivity in a firm requires a high level of performance along a large number of dimensions. Textile producers require raw materials, knitting machines, a healthy and trained labor force, knowledge of how to produce, security, business licenses, transportation networks, electricity, etc. These inputs enter in a complementary fashion, in the sense that problems with any input can substantially reduce overall output. Without electricity or production knowledge or raw materials or security or business licenses, production is likely to be severely curtailed.

The contribution of this paper is to build a model in which these ideas can be made precise. The multiplier that works through intermediate goods turns out to be readily quantified and large: incorporating intermediate goods into our models has a first-order impact on how we think about economic development. The effects of complementarity are more subtle and difficult to quantify — often turning out to be smaller than one might have expected — in part because they must constantly be weighed

[1] Recent work on this topic includes Romer (1994), Klenow and Rodriguez-Clare (1997), Prescott (1998), Hall and Jones (1999), Parente and Prescott (1999), Howitt (2000), Parente, Rogerson and Wright (2000), Acemoglu, Johnson and Robinson (2001), Klenow and Rodriguez-Clare (2005), Manuelli and Seshadri (2005), Caselli and Coleman (2006), Armenter and Lahiri (2006), Erosa, Koreshkova and Restuccia (2006), Marimon and Quadrini (2006), and Restuccia, Yang and Zhu (2006).

against various possibilities for substitution. In the end, however, these two forces substantially multiply the effects of distortions to the allocation of resources. Fifty-fold income differences that are hard to explain in a traditional neoclassical setup appear well within reach when the multipliers associated with intermediate goods and complementarity are taken into account.

The approach taken in this paper can be compared with the recent literature on political economy and institutions; for example, see Acemoglu and Johnson (2005) and Acemoglu and Robinson (2005). This paper is more about mechanics: can we develop a plausible mechanism for getting a big multiplier, so that whatever distortions exist lead to large income differences? The modern institutions approach builds up from political economy. This is crucial in explaining why the allocations in poor countries are inferior — for example, why investment rates in physical and human capital are so low — but the institutions approach ultimately still requires a large multiplier to explain income differences. As just one example, even if a political economy model explains observed differences in investment rates across countries, the model cannot explain 50-fold income differences if it is embedded in a neoclassical framework. The political economy approach explains why resources are misallocated; the approach here takes the extent of misallocation as given and explains how intermediate goods and weak links can amplify the effect of misallocation, potentially leading to large income differences. Clearly, both steps are needed to understand development.

2. Linkages and Complementarity

We begin by discussing briefly the key mechanisms at work in this paper. These mechanisms are conceptually distinct — one can have linkages without complementarity, for example — but they interact in important ways.

2.1. Linkages through Intermediate Goods

The notion that linkages across sectors can be central to economic performance dates back at least to Leontief (1936), which launched the field of input-output economics. Hirschman (1958) emphasized the importance of linkages (and complementarity) to economic development. A large subsequent empirical literature constructed input-

output tables for many different countries and computed sectoral multipliers.

In what may prove to be an ill-advised omission, these insights have not generally be incorporated into modern growth theory. Linkages between sectors through intermediate goods deliver a multiplier very much like the multiplier associated with capital in the neoclassical growth model. More capital leads to more output, which in turn leads to more capital. This virtuous circle shows up mathematically as a geometric series which sums to a multiplier of $\frac{1}{1-\alpha}$ if α is capital's share of overall revenue. Because the capital share is only about 1/3, this multiplier is relatively small: differences in investment rates are too small to explain large income differences, and large total factor productivity residuals are required. This has led a number of authors to broaden the definition of capital, say to include human capital or organizational capital. It is generally recognized that if one can get the capital share up to something like 2/3 — so the multiplier is 3 — large income differences are much easier to explain without appealing to a large residual.[2]

Intermediate goods generate this same kind of multiplier. Problems in the financial services industry can reduce output in a range of sectors, including information technology, plastic manufacturing, and education. But this in turn feeds back and further reduces the output of financial services.

A simple example is quite helpful for understanding how intermediate goods generate a multiplier. Suppose gross output Y_t^G is produced using capital K_t, labor L_t, and intermediate goods X_t:

$$Y_t^G = \bar{A}\left(K_t^{\alpha} L_t^{1-\alpha}\right)^{1-\sigma} X_t^{\sigma}, \tag{1}$$

where σ and α are between zero and one. Gross output can be used for consumption or investment or it can be carried over to the next period and used as an intermediate good. To keep things simple, assume a constant fraction $\bar{s}$ of gross output is used for investment and a constant fraction $\bar{x}$ is used as an intermediate good. Therefore

$$K_{t+1} = \bar{s} Y_t^G + (1-\delta) K_t, \tag{2}$$

[2]Mankiw, Romer and Weil (1992) is an early example of this approach to human capital. Chari, Kehoe and McGrattan (1997) introduced "organizational capital" for the same reason. Howitt (2000) and Klenow and Rodriguez-Clare (2005) use the accumulation of ideas to boost the multiplier. More recently, Manuelli and Seshadri (2005) and Erosa, Koreshkova and Restuccia (2006) have resurrected the human capital story in a more sophisticated fashion. The controversy in each of these stories is over whether or not the additional accumulation raises the multiplier sufficiently. Typically, the problem is that the magnitude of a key parameter is difficult to pin down.

$$X_{t+1} = \bar{x} Y_t^G. \tag{3}$$

Consumption is then given by $C_t = (1-\bar{s}-\bar{x})Y_t^G$, and GDP in this economy is consumption plus investment, or output net of intermediate goods: $Y_t = (1-\bar{x})Y_t^G$. Assume labor is exogenous and constant.

This model features a steady state where all key variables are constant and GDP is given by

$$Y = \left(\bar{A}\bar{m}\right)^{\frac{1}{1-\sigma}} K^{\alpha} L^{1-\alpha}, \tag{4}$$

where $\bar{m} \equiv (1-\bar{x})^{1-\sigma}\bar{x}^{\sigma}$. Moreover, capital in steady state depends on investment, so GDP per worker is

$$y \equiv \frac{Y}{L} = \left(\bar{A}\bar{m}\left(\frac{\bar{s}}{\delta}\right)^{\alpha(1-\sigma)}\right)^{\frac{1}{(1-\alpha)(1-\sigma)}}. \tag{5}$$

A key implication of this result is that a 1% increase in productivity $\bar{A}$ increases GDP by more than 1% because of the multiplier, $\frac{1}{(1-\alpha)(1-\sigma)}$. In the absence of intermediate goods ($\sigma = 0$), this multiplier is just the familiar $\frac{1}{1-\alpha}$: an increase in productivity raises output, which leads to more capital, which leads to more output, and so on. The cumulation of this virtuous circle is $1 + \alpha + \alpha^2 + \ldots = \frac{1}{1-\alpha}$.

In the presence of intermediate goods, there is an additional multiplier: higher output leads to more intermediate goods, which raises output (and capital), and so on. The overall multiplier is therefore $\frac{1}{(1-\alpha)(1-\sigma)}$. Alternatively, let $\beta \equiv \alpha(1-\sigma)+\sigma$ denote the total share of produced inputs. It is easy to show that the multiplier can also be expressed as $\frac{1}{1-\beta}$.

Quantitatively, the addition of intermediate goods has a large effect. For example, consider the multipliers using conventional parameter values, a capital exponent of $\alpha = 1/3$ and an intermediate goods exponent of $\sigma = 1/2$. (Evidence supporting this value will be discussed extensively below.) In this case, the share of produced factors in gross output is $\beta = \alpha(1-\sigma)+\sigma = 1/6 + 1/2 = 2/3$.

In the absence of intermediate goods the multiplier is $\frac{1}{1-\alpha} = 3/2$, and a hypothetical doubling of TFP raises output by a factor of $2^{3/2} = 2.8$. But with intermediate goods, the multiplier is $\frac{1}{(1-\alpha)(1-\sigma)} = \frac{3}{2} \cdot 2 = 3$, and a doubling of TFP raises output by a factor of $2^3 = 8$. If we think of the standard neoclassical factors (like $\bar{s}$ and $\bar{x}$ in the example) as generating a 4-fold difference in incomes across rich and poor countries, then a hypothetical 2-fold difference in TFP leads to an 11.3-fold difference in the model with no

intermediate goods, but to a 32-fold difference once intermediate goods are taken into account, close to what we see in the data.

At a basic level, this simple model captures the main contribution of the paper, and the rest is just elaboration. However, the elaboration turns out to be quite important in answering a key question that may be raised regarding the simple model. First, because the level of TFP is never observed directly but must be measured as a residual, there is a sense in which the calculation above may appear confusing. TFP can be measured using value-added or using gross output, and there is a one-to-one mapping between the two in this simple example. Let $\bar{B} \equiv \bar{A}^{1/(1-\sigma)}$. A 2-fold difference in $\bar{A}$ corresponds to a 4-fold difference in $\bar{B}$ in a value-added representation like equation (4). Does this observational equivalence mean there is no fundamental multiplier after all?

Instinctively, we know the answer to this question must be "no": after all, the injunction from Diamond and Mirrlees (1971) about not taxing intermediate goods is based on this same multiplier. More specifically, this concern is addressed directly below by building a model in which distortions to the allocation of resources at the micro level such as theft or taxation aggregate up into TFP differences at the macro level. These micro-level distortions — which are in principle observable and have a definite magnitude (such as "10% of output gets stolen from the firm") — are amplified by the intermediate goods multiplier. In the quantitative exercises at the end of the paper, one will see clearly that the presence or absence of intermediate goods plays a crucial role in determining the magnitude of income differences for a given set of micro-level distortions.

There is an alternative way to make this point. From Hsieh and Klenow (2009) and others, we know that distortions to the allocation of capital and labor can lead to aggregate TFP differences, and that these TFP differences are multiplied by the capital accumulation multiplier. This paper simply recognizes that production involves intermediate goods as well, and these too can be misallocated. Because intermediate goods are another produced factor of production, this misallocation gets amplified.

Another issue worth addressing now is vertical integration. Doesn't the extent of vertical integration influence the share of intermediate goods in gross output? If an economy were entirely vertically integrated, would there be no multiplier associated with intermediate goods? As the full model to be presented shortly will show, what matters is the extent to which first-order conditions in the economy get distorted. Consider

an automobile manufacturer that vertically integrates, from the steel production and rubber manufacturing all the way through to the final sale of the automobile. Whether or not there is vertical integration, there are clearly many more first-order conditions that must be satisfied than just the ones involving capital and labor. The steel, rubber, sparkplugs, radios, and all the other parts have to be produced at the right time and delivered to the right place in the right quantity. Theft or corruption could affect any piece of this production chain. Similarly, once the cars are produced, they must be delivered to auto retailers around the country. Whether or not these are owned by the same entity as the producer does not really affect the possibility that some cars or parts may be confiscated by corrupt officials along the way.

To a great extent, the possibility of vertical integration simply raises a measurement issue. With a large amount of vertical integration, statistical authorities would understate the importance of intermediate goods — measured as purchases from other firms. As we will see later, however, there is a wealth of evidence suggesting that the intermediate goods share of gross output is approximately 1/2 across a range of economies at different levels of development. If this is understated, the multiplier associated with intermediate goods would be even larger.

Combining a neoclassical story of capital accumulation with a standard treatment of intermediate goods therefore delivers a very powerful engine for explaining income differences across countries. Related insights pervade the older development literature but have not had a large influence on modern growth theory. The main exception is Ciccone (2002), which appears to be underappreciated.[3]

2.2. The Role of Complementarity

Complementarity and linkages often go together, as in Hirschman (1958). This is in part because complementarity naturally arises when one considers intermediate goods: electricity, transportation, and raw materials are all essential inputs into production. This

[3]Ciccone develops the multiplier formula for intermediate goods and provides some quantitative examples illustrating that the multiplier can be large. The point may be overlooked by readers of his paper because the model also features increasing returns, externalities, and multiple equilibria. Yi (2003) argues that tariffs can multiply up in much the same way when goods get traded multiple times during the stages of production; see also Eaton and Kortum (2002). Interestingly, the intermediate goods multiplier shows up most clearly in the economic fluctuations literature; see Long and Plosser (1983), Basu (1995), Rotemberg and Woodford (1995), Horvath (1998), Dupor (1999), Conley and Dupor (2003), and Gabaix (2005). See also Hulten (1978).

is one reason it is natural to consider the role of complementarity in this paper. The other is the large multiplier suggested by the O-ring story in Kremer (1993): the space shuttle Challenger and its seven-member crew are destroyed because of the failure of a single, inexpensive rubber seal.

In any production process, there are many things that can go wrong that will sharply reduce the value of production. In rich countries, there are enough substitution possibilities that these things do not often go wrong. In poor countries, on the other hand, any one of several problems can doom a project. Obtaining the instruction manual (the "knowledge") for how to produce socks is not especially useful if the import of knitting equipment is restricted, if replacement parts are not readily available, if the electricity supply is erratic, if cotton and polyester threads cannot be obtained, if legal and regulatory requirements cannot be met, if property rights are not secure, or if the market to which these socks will be sold is unknown.

A moment's reflection is enough to convince nearly anyone of complementarity's potential for explaining income differences. This was certainly part of the original appeal of Kremer's paper. For reasons that are not entirely clear, those insights have not had a large influence on growth and development models of the last decade, and part of the goal of this paper is to explore these possibilities more carefully. Hence, complementarity is the second main ingredient in this paper.

2.3. Modeling Complementarity and Substitution

Kremer (1993) offers the basic insight that complementarity can generate a large multiplier by focusing on the extreme case in which all inputs combine in a Leontief fashion. In adding this second ingredient to our model, we choose a more flexible CES formulation that allows the degree of complementarity to be a parameter.[4] As an illustration, suppose

$$Y = \left(\int_0^1 z_i^\eta di \right)^{1/\eta} \tag{6}$$

where z_i denote a firm's purchases of the ith input, and a continuum of intermediate

[4] Kremer does not emphasize that his approach embodies a Leontief technology. Blanchard and Kremer (1997) formalize this interpretation and study a model of chains of production in order to understand the large declines in output in the former Soviet Union after 1989. Grossman and Maggi (2000), motivated in part by Kremer (1993), study trade between countries when production functions across sectors involve different degrees of complementarity. Other related papers include Murphy, Shleifer and Vishny (1989), Milgrom and Roberts (1990), Becker and Murphy (1992) , Rodriguez-Clare (1996), and Rodrik (1996).

inputs are used for production. The elasticity of substitution among these activities is $\frac{1}{1-\eta}$, but this (or its inverse) could easily be called an elasticity of complementarity instead. For intermediate inputs, it is plausible to assume $\eta < 0$, so the elasticity of substitution is less than one. It is difficult to substitute electricity for transportation services or raw materials in production. Complementarity puts extra weight on the activities in which the firm is least successful. This is easy to see in the limiting case where $\eta \to -\infty$; in this case, the CES function converges to the minimum function, so output is equal to the smallest of the z_i.

This intuition can be pushed further by noting that the CES combination in equation (6) is called the *power mean* of the underlying z_i in statistics. The power mean is just a generalized mean. For example, if $\eta = 1$, Y is the arithmetic mean of the z_i. If $\eta = 0$, output is the geometric mean (Cobb-Douglas). If $\eta = -1$, output is the harmonic mean, and if $\eta \to -\infty$, output is the minimum of the z_i. From a standard result in statistics, these means decline as η becomes more negative. Economically, a stronger degree of complementarity puts more weight on the weakest links and reduces output.[5]

Going in the other direction, if $\eta \to +\infty$, output converges to the maximum of the z_i, a "superstar" kind of production function, like that studied by Rosen (1981). More generally, the higher is η, the further up the distribution is the power mean. This case is not usually emphasized in growth models — notice that it implies a *negative* elasticity of substitution — but it turns out to play an important and intuitive role in our model.

3. Setting Up the Model

We now apply this basic discussion of intermediate goods and complementarity to construct a model of economic development.

3.1. The Economic Environment

A continuum of goods indexed on the unit interval by i are produced in this economy using a Cobb-Douglas production function:

$$Y_i = A_i \left(K_i^{\alpha} H_i^{1-\alpha}\right)^{1-\sigma} X_i^{\sigma}, \tag{7}$$

[5]Benabou (1996) studies this approach to complementarity. Interestingly, standard intertemporal preferences with a constant relative risk aversion coefficient greater than one represent a familiar example.

where α and σ are both between zero and one. K_i and H_i are the amounts of physical capital and human capital used to produce good i, and A_i is an exogenously-given productivity level. The novel term in this production specification is X_i, which denotes the quantity of intermediate goods used to produce variety i.

Each of these fundamental goods in the economy can be used for one of two purposes: as a final good (c_i) or as an intermediate input (z_i). Therefore,

$$c_i + z_i = Y_i. \tag{8}$$

The next two equations show how these uses affect the economy. In principle, we could specify a utility function over the continuum of final consumption uses. Instead, it proves more convenient (for modeling capital) to follow the standard trick of aggregating these final uses into a single final good, which will represent GDP in this economy:

$$Y = \left(\int_0^1 c_i^\theta di \right)^{1/\theta}, \quad 0 < \theta < 1. \tag{9}$$

These final goods aggregate up with an elasticity of substition greater than one. Such an aggregator is standard in the literature and there are solid estimates of this elasticity that we will appeal to when it comes time for quantitative analysis.

Whereas final goods combine with an elasticity of substitution greater than one in producing GDP, intermediate inputs combine with an elasticity of substitution less than one. This is the key place where "weak links" enter the model:

$$X = \left(\int_0^1 z_i^\rho di \right)^{1/\rho}, \quad \rho < 0. \tag{10}$$

This aggregate intermediate good is what gets used by the various sectors of the economy.[6] To keep the model simple and tractable, we assume that the same combination of intermediate goods is used to produce each variety (though potentially in a different quantity). Hence, the resource constraint:

$$\int_0^1 X_i \, di \leq X. \tag{11}$$

[6]An issue of timing arises here. To keep the model simple, we make the seemingly strange assumption that intermediate goods are produced and used simultaneously. A better justification goes as follows. Imagine incorporating a lag so that today's final good is used as tomorrow's intermediate input. The steady state of that setup would then deliver the result we have here.

An example illustrating the final and intermediate goods may be helpful here. Varieties that are used as intermediate goods involve substantial complementarity, but when these same varieties combine to produce final consumption, there is more substitutability. For example, computer services are today nearly an essential input into semiconductor design, banking, and health care. But computers are much more substitutable when used for final consumption — for entertainment, we can play computer games or watch television or ride bikes in the park. In order to produce within a firm, there are a number of complementary steps that must be taken. In final consumption (e.g. in utility), however, there appears to be a reasonably high degree of substitution across goods.

Stepping back for a moment, note that the parameter σ measures the importance of linkages in our economy. If $\sigma = 0$, the productivity of physical and human capital in each variety depends only on A_i and is independent of the rest of the economy. To the extent that $\sigma > 0$, low productivity in one sector feeds back into the others. Transportation services may be unproductive in a poor country because of inadequate fuel supplies or repair services, and this low productivity will reduce output throughout the economy.

The remainder of the model is standard. The resource constraints for physical and human capital are

$$\int_0^1 K_i \, di \leq K, \tag{12}$$

and

$$\int_0^1 H_i \, di \leq H \equiv \bar{h}\bar{L}, \tag{13}$$

where $\bar{h}$ is an exogenously-given amount of human capital per worker and $\bar{L} = 1$ is the exogenous number of workers in the economy, both constant. We do not endogenize human capital accumulation in this environment in order to keep the model as simple as possible; this could be added easily, however. Physical capital accumulates in the usual way, and investment consists of units of the aggregate final good:

$$\dot{K} = I - \delta K, \quad K_0 \text{ given.} \tag{14}$$

$$C + I \leq Y. \tag{15}$$

Finally, preferences are standard

$$U = \int_0^\infty e^{-\lambda t} u(C_t) dt, \tag{16}$$

with $u'(C) > 0$ and $u''(C) < 0$. We've dropped time subscripts from this economic environment (except in this final equation) since we will primarily be concerned with the steady state of this model.

4. A Symmetric Allocation of Resources

Before turning to a competitive equilibrium, it is useful to consider a simple "rule of thumb" allocation, analogous to Solow's fixed saving rate. There are two advantages to this approach. First, it is simple, easy to solve for, and allows us to illustrate some of the key points of the model. Second, it serves as a useful benchmark when it comes time to understand why the competitive equilibrium looks the way it does. Our rule of thumb allocation is a symmetric allocation with a constant investment rate:

Definition The *symmetric allocation of resources* in this economy has $K_i = K$, $H_i = H$, $X_i = X$, $I = \bar{s}Y$, and $z_i = \bar{z}Y_i$, where $0 < \bar{s}, \bar{z} < 1$.

Of course, this symmetric allocation is completely unrealistic — even moreso, it will turn out, than one might have guessed. But the advantages mentioned above make this a good place to start.

Under this symmetric allocation, the solution for GDP in the economy at any point in time is given in the following proposition. (Outlines of all proofs are in the Appendix.)

Proposition 1 (The Symmetric Allocation, Given Capital): *Given K units of capital, GDP under the symmetric allocation is*

$$Y = \phi(\bar{z})(S_\theta^{1-\sigma} S_\rho^\sigma)^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha}, \tag{17}$$

where

$$S_\rho \equiv \left(\int_0^1 A_i^\rho di \right)^{\frac{1}{\rho}}, \tag{18}$$

$$\phi(\bar{z}) \equiv ((1-\bar{z})^{1-\sigma} \bar{z}^\sigma)^{\frac{1}{1-\sigma}}, \tag{19}$$

and S_θ is defined in a way analogous to S_ρ.

The model delivers a simple expression for GDP. Y is the familiar Cobb-Douglas combination of aggregate physical and human capital with constant returns to scale.

Two novel results also emerge, and both are related to total factor productivity. First, consider the S_θ and S_ρ terms. Each is a CES combination of the underlying sectoral TFPs. Since θ is between zero and one, S_θ is between the geometric mean and the arithmetic mean of the TFPs. But with ρ less than zero, S_ρ ranges from the geometric mean down to the minimum of the underlying A_i, depending on the strength of complementarity. Total factor productivity for the economy as a whole depends on the geometric average of the CES terms, $S_\theta^{1-\sigma} S_\rho^\sigma$. The "substitutes" term gets a weight that equals the share of value-added in gross output, while the "complements" term S_ρ gets a weight that equals the intermediate goods share of gross output, σ. In other words, the importance of "weak links" in production depends on (i) the extent of complementarity and (ii) the relative importance of intermediate goods.

To interpret this result, it is helpful to consider the special case where $\theta = 1$, $\rho \to -\infty$, and $\sigma = 1/2$. In this case, TFP is the product of the average of the A_i and the minimum of the A_i. Aggregate TFP then depends crucially on the smallest level of TFP across the sectors of the economy — that is, on the weakest link. Firms in the United States and Kenya may not differ that much in average efficiency, but if the distribution of Kenyan firms has a substantially worse lower tail, overall economic performance will suffer because of complementarity.

The second property of this solution worth noting is the multiplier associated with intermediate goods. Total factor productivity involves a multiplier, the exponent $\frac{1}{1-\sigma} > 1$. A simple example should make the reason for this transparent. Suppose $Y_t = aX_t^\sigma$ and $X_t = sY_{t-1}$; output depends in part on intermediate goods, and the intermediate goods are themselves produced using output from the previous period. Solving these two equations in steady state gives $Y^* = a^{\frac{1}{1-\sigma}} s^{\sigma/1-\sigma}$, which is a simplified version of what is going on in our model. Notice that if we call X "capital" instead of intermediate goods, the same formulas would apply and this looks like the neoclassical growth model with full depreciation. Intermediate goods are another source of produced inputs in a growth model.

Finally, consider the role of $\phi(\bar{z})$. Differences in the allocation of resources to intermediate uses show up as aggregate TFP differences in this environment. Moreover, this term is a hump-shaped function of $\bar{z}$ which is maximized at $\bar{z} = \sigma$. Not surpris-

ingly, this turns out to be the optimal amount of gross output to spend on intermediate goods. Departures from this optimal amount will reduce TFP.

5. A Competitive Equilibrium with Wedges

The symmetric allocation is useful as a quick guide to how the model works, but it is clearly farfetched. We turn now to a more interesting allocation, the competitive equilibrium in the presence of micro-level distortions.

This approach builds on work by Banerjee and Duflo (2005), Chari, Kehoe and McGrattan (2007), Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), who argue that misallocation at the micro level shows up at the macro level as a reduction in aggregate TFP. Micro-level distortions can be actual formal taxes, but they could also represent theft, product and labor market regulations, protection from competition, or other forms of expropriation. Here, we model misallocation through an expropriation rate: a fraction τ_i of the output of each variety is taken from firms without compensation. This is by no means the only way to model the micro-level distortions — an alternative would be to have the distortions vary for each input as well as each variety. But it allows the main points of the paper to be made in the clearest fashion. The total funds expropriated are then divided evenly among households. If instead the expropriated funds were captured by foreigners, lost through "iceberg" costs, or consumed by a measure-zero despot, the effects on the economy as a whole would be even larger.

5.1. Optimization Problems

Before defining the competitive equilibrium, it is convenient to specify the optimization problems in the economy. Letting the final output good be the numéraire, these problems are described below.

Final Sector Problem: Taking the prices of the consumption varieties $\{p_i\}$ as given, a representative firm in the perfectly competitive final goods market solves at each point in time

$$\max_{\{c_i\}} \left(\int_0^1 c_i^\theta di \right)^{1/\theta} - \int_0^1 p_i c_i \, di.$$

Intermediate Sector Problem: Taking the price of the intermediate varieties $\{p_i\}$ and the price of the aggregate intermediate good q as given, a representative firm in the perfectly competitive intermediate goods market solves at each point in time

$$\max_{\{z_i\}} q \left(\int_0^1 z_i^\rho di \right)^{1/\rho} - \int_0^1 p_i z_i \, di.$$

Variety i's Problem: Taking $p_i, r, w,$ and q as given, and given a variety-specific expropriation rate τ_i, a representative firm in the perfectly competitive variety i market solves at each point in time

$$\max_{\{X_i, K_i, H_i\}} (1-\tau_i) p_i A_i \left(K_i^\alpha H_i^{1-\alpha} \right)^{1-\sigma} X_i^\sigma - (r+\delta) K_i - w H_i - q X_i.$$

Household Problem: Taking the time path of interest rates, wages, and income from expropriation (r_t, w_t, and E_t) as given, and given an initial stock of assets V_0, the representative household solves

$$\max_{\{C_t, V_t\}} \int_0^\infty e^{-\lambda t} u(C_t) dt$$

subject to

$$\dot{V}_t = r_t V_t + w_t H + E_t - C_t,$$

and subject to a no Ponzi-scheme condition.

5.2. Defining the Competitive Equilibrium

Definition A *competitive equilibrium* in this economy consists of time paths for the quantities $Y, X, C, I, K, V, E, \{Y_i, K_i, H_i, X_i\}, \{c_i, z_i\}$ and prices $\{p_i\}, q, w, r$ such that

1. C and V solve the Household Problem.
2. $\{c_i\}$ solve the Final Sector Problem.
3. $\{z_i\}$ solve the Intermediate Sector Problem.
4. K_i, H_i, X_i solve the Variety i Problem for all $i \in [0, 1]$.

5. Markets clear:

   r clears the capital market: $V = K$

   w clears the labor market: $\int_0^1 H_i di = H$

   p_i clears market i: $c_i + z_i = Y_i$ for all $i \in [0,1]$

   q clears the intermediate goods market: $\int_0^1 X_i di = X$.

6. Expropriated funds are rebated to households: $E = \int_0^1 \tau_i p_i Y_i di$.

7. Other aspects of the environment hold:

$$\dot{K} = I - \delta K$$
$$\int_0^1 K_i di = K$$
$$Y_i = A_i \left(K_i^\alpha H_i^{1-\alpha}\right)^{1-\sigma} X_i^\sigma$$
$$Y = \left(\int_0^1 c_i^\theta di\right)^{1/\theta}$$
$$X = \left(\int_0^1 z_i^\rho di\right)^{1/\rho}.$$

Counting loosely, our competitive equilibrium involves 17 endogenous variables and specifies 17 equations to pin them down. The market for final output clears by Walras' Law (so that $C + I = Y$ is redundant).

5.3. Solving for the Competitive Equilibrium

We now discuss the solution of the model, beginning with a result characterizing the aggregate production of GDP at any point in time.

Proposition 2 (The Competitive Equilibrium, Given Capital): *Given K units of capital, GDP in the competitive equilibrium is*

$$Y = \psi(\tau)\left(Q_\theta^{1-\sigma} Q_\rho^\sigma\right)^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha} \tag{20}$$

where

$$Q_\rho \equiv \left(\int_0^1 \left(A_i(1-\tau_i)\right)^{\frac{\rho}{1-\rho}} di\right)^{\frac{1-\rho}{\rho}} \tag{21}$$

and

$$\psi(\tau) \equiv \frac{1-\sigma(1-\tau)}{1-\tau} \cdot \sigma^{\frac{\sigma}{1-\sigma}} \tag{22}$$

where $\tau \equiv E/(Y + qX)$ is the average expropriation rate in the economy, measured relative to gross output, and Q_θ is defined in a way analogous to Q_ρ.

Several insights emerge from this result. Two we can get through quickly, while the third requires more consideration. First, the multiplier associated with intermediate goods appears in exactly the same way as in the symmetric allocation, and for the same reason. This multiplier is a fundamental feature of the economy reflecting the presence of additional produced factors of production. It multiplies any distortion associated with misallocation but is not itself affected by the allocation of resources.

Second, expropriation affects output through TFP. Therefore, this proposition illustrates a very important result found elsewhere: the misallocation of resources at the micro level often shows up as a reduction in TFP at the macro level. This result has been emphasized by Chari, Kehoe and McGrattan (2007), Restuccia and Rogerson (2008), and Hsieh and Klenow (2009) and also plays a key role in Banerjee and Duflo (2005), Caselli and Gennaioli (2005) and Lagos (2006). Importantly, expropriation rates get multiplied by the intermediate goods multiplier. We will discuss the effect of these wedges in more detail below.

Finally, a key difference relative to the previous result on the symmetric allocation is that the curvature parameter determining the productivity aggregates has changed. For example, $\frac{\rho}{1-\rho}$ replaces the original ρ. Notice that if the domain of ρ is $[0, -\infty)$, the range of $\frac{\rho}{1-\rho}$ is $[0, -1)$: there is less complementarity in determining Q_ρ than S_ρ.

This result can be illustrated with an example. Suppose $\rho \to -\infty$. In this case, the symmetric allocation depends on the smallest of the A_i, the pure weak link story. In contrast, the equilibrium allocation depends on the harmonic mean of the (distortion adjusted) productivities, since $\frac{\rho}{1-\rho} \to -1$. Disasterously low productivity in a single variety is fatal in the symmetric allocation, but not in the equilibrium allocation. Why not?

The reason is that the equilibrium allocation is able to strengthen weak links by allocating more resources to activities with low productivity. If the transportation sector has especially low productivity that would otherwise be very costly to the economy, the equilibrium allocation can put extra physical and human capital in that sector to help offset its low productivity and prevent this sector from becoming a bottleneck. Of course, this must be balanced by the desire to give this sector a *low* amount of resources in an effort to substitute away from transportation on the consumption side. This can

be seen in the math: the equilibrium solution for allocating capital is

$$\frac{K_i}{K} = \frac{1-\tau_i}{1-\tau}\left[(1-\sigma(1-\tau))\left(\frac{A_i(1-\tau_i)}{Q_\theta}\right)^{\frac{\theta}{1-\theta}} + \sigma(1-\tau)\left(\frac{A_i(1-\tau_i)}{Q_\rho}\right)^{\frac{\rho}{1-\rho}}\right].$$

Another perspective on the solution is gained by returning to a special case we considered earlier. Suppose $\theta = 1$, $\rho \to -\infty$, and $\sigma = 1/2$, and suppose $\tau_i = 0$. In this case, $Q_\theta \to \max A_i$ while Q_ρ becomes the harmonic mean of the A_i. Total factor productivity is the product of the two. Contrast this with the same example for the symmetric allocation: there, TFP was the product of the arithmetic mean and the minimum. Allocating resources optimally shifts up both of these generalized means. The strengthening of weak links leads the minimum to be replaced by the harmonic mean. Similarly, if consumption goods enter as perfect substitutes, only the good with the highest productivity will be consumed: the arithmetic mean gets replaced by the "max," a superstar effect.

This example illustrates an intuitive way that the model can lead to large income differences across countries. Suppose two countries possess identical economic environments, including the same levels of productivity for each variety. The "rich" country allocates resources as in a competitive equilibrium with no expropriation, but the "poor" country distorts the allocation sufficiently that it looks like the symmetric allocation. In the special case we are considering here, relative TFP between these two countries will be the product of two terms. First is the ratio of average TFP between the two countries, a standard term. But second is the ratio of the maximum TFP in the rich country to the minimum TFP in the poor country. Even if both countries have identical TFP distributions, this misallocation can lead to a large gap driven by the max-min effects associated with superstar and weak link forces. With less extreme parameter values, these forces are still in play, of course, as we will see in the numerical examples later.

5.4. The Steady State

Next, we see that the long-run multiplier in the model depends on the overall share of produced factors — capital as well as intermediate goods. We get the $\frac{1}{1-\alpha}$ effect since capital accumulates in response to a change in productivity or expropriation.

Proposition 3 (The Competitive Equilibrium in Steady State): *Let $y \equiv Y/\bar{L}$. The competitive equilibrium exhibits a steady state in which GDP per worker is given by*

$$y^* = \psi_1(\tau)\left(Q_\theta^{1-\sigma} Q_\rho^\sigma\right)^{\frac{1}{1-\sigma}\frac{1}{1-\alpha}} \left(\frac{\alpha(1-\sigma)}{\lambda+\delta}\right)^{\frac{\alpha}{1-\alpha}} \bar{h}. \tag{23}$$

where $\psi_1(\tau) \equiv \frac{1-\sigma(1-\tau)}{1-\tau} \cdot \sigma^{\frac{\sigma}{1-\sigma}\frac{1}{1-\alpha}}$.

5.5. Symmetric Wedges

A number of useful insights emerge from considering the special case in which the expropriation rates are identical across all varieties.

Proposition 4 (Symmetric Wedges): *Suppose the expropriation rate is identical across sectors: $\tau_i = \bar{\tau}$. Let $z^* \equiv \frac{qX}{Y+qX}$ denote the equilibrium fraction of gross output spent on intermediate goods. Then $z^* = \sigma(1-\bar{\tau})$, and GDP at any given point in time is*

$$Y = (1-z^*)z^{*\frac{\sigma}{1-\sigma}} \left(\tilde{Q}_\theta^{1-\sigma}\tilde{Q}_\rho^\sigma\right)^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha}, \tag{24}$$

where

$$\tilde{Q}_\rho \equiv \left(\int_0^1 A_i^{\frac{\rho}{1-\rho}} di\right)^{\frac{1-\rho}{\rho}}, \tag{25}$$

and $\tilde{Q}_\theta$ is defined analogously. Moreover, GDP per worker in steady state is

$$y^* = \zeta_1 \left(1-\sigma(1-\bar{\tau})\right)(1-\bar{\tau})^{\frac{1}{1-\sigma}\frac{1}{1-\alpha}-1} \left(\tilde{Q}_\theta^{1-\sigma}\tilde{Q}_\rho^\sigma\right)^{\frac{1}{1-\sigma}\frac{1}{1-\alpha}} \bar{h}, \tag{26}$$

where ζ_1 is a collection of terms that do not depend on $\bar{\tau}$.

The first part of this proposition highlights a similarity between the competitive equilibrium with symmetric wedges and the symmetric allocation we studied earlier. The overall effect of expropriation is to distort the allocation of resources between final use and intermediate use. Given capital, GDP is maximized at $\bar{\tau} = 0$.

The second part of the proposition shows explicitly the different effects that symmetric expropriation has on GDP per worker in the steady state. The first term is $1-z^* = 1-\sigma(1-\bar{\tau})$. Notice that this term is an increasing function of the expropriation rate and reflects the fact that expropriation leads to lower spending on intermediate goods and

therefore higher spending on final uses. The second term is the expropriation wedge raised to a power that depends on the overall multiplier in the model. In fact, letting β denote the overall share of produced factors in the sectoral production function (both intermediates and capital), this second term can be written as $(1-\bar{\tau})^{\frac{\beta}{1-\beta}}$. The $\frac{1}{1-\beta}$ term captures the standard multiplier effects of the model. The overall exponent gets reduced by the proportion β because only that fraction of the factors of production are distorted by a symmetric wedge. In particular, the allocation of human capital across sectors is not distorted.

This raises an interesting question: if expropriation is symmetric, why does it distort anything at all? The answer is that it is symmetric across sectors, but not symmetric over time. In particular, goods that are used for final uses suffer expropriation only once, when they are produced. However, a good devoted to intermediate uses suffers expropriation each time production occurs, and it is this that leads to the multiplier effects. This can be viewed as a simple application of the ideas in Diamond and Mirrlees (1971), Chamley (1986), and Judd (1985) regarding the taxation of intermediate goods and capital. From the long-run perspective, capital and intermediate goods are the same: both are produced factors of production. The distortion associated with $\bar{\tau}$ gets multiplied by the production structure of the economy.

This discussion also reminds us that monopoly markups can play the same role in distorting the allocation of resources through "double marginalization," a point emphasized by Epifani and Gancia (2009). For example, suppose every variety i is produced by a firm that charges a markup over marginal cost, which we can conveniently parameterize as $1/(1-\tau_i)$.[7] This wedge also gets multiplied over time through the capital multiplier and the intermediate goods multiplier, just like the expropriation rate, and similar formulas to those we have derived would obtain. To the extent that poor countries have higher markups than rich countries — for example because of pressures that limit competition — these same multiplier effects occur.

[7] Think of this markup as being less than the unconstrained monopoly markup because of regulations, entry threats, and other competitive pressures (all of which may be heterogeneous). This is important because the inelastic demand associated with complementarities could otherwise point toward infinite markups.

5.6. Random Wedges

Symmetric expropriation distorts the allocation of resources in an intertemporal sense, but does not otherwise distort the allocation across the sectors of the economy. As discussed in the introduction, however, one of the key ways in which weak links can be a problem in a country is if resources are misallocated across firms or sectors: electricity may be absolutely essential to production, and problems in that sector can lead to severe disruptions.

To get a sense of how misallocation across firms can matter, we suppose expropriation and productivity levels are distributed log-normally across our continuum of sectors. We then have the following result, which also proves useful when it comes time to examine the model quantitatively:

Proposition 5 (Random Productivity and Wedges): *Let $a_i \equiv logA_i$ and $\omega_i \equiv \log(1-\tau_i)$ be jointly normally distributed so that $a_i \sim N(\mu_a, \nu_a^2)$, $\omega_i \sim N(\mu_\omega, \nu_\omega^2)$, and $Cov(\omega_i, a_i) = \nu_{a\omega}$. Finally, let $\nu^2 \equiv \nu_a^2 + \nu_\omega^2 + 2\nu_{a\omega}$. Then*

$$\log y^* = \underbrace{\log\left(\frac{1-\sigma(1-\tau)}{1-\tau}\right)}_{A} + \frac{1}{1-\sigma}\frac{1}{1-\alpha}\left(\underbrace{(1-\sigma)\log Q_\theta + \sigma \log Q_\rho}_{B}\right) + \zeta_2$$

such that

$$A = \log\left(1 - \sigma \exp\left[\mu_\omega + \frac{1+\rho}{1-\rho}\cdot\frac{\nu_\omega^2}{2} + \frac{\rho}{1-\rho}\nu_{a\omega}\right]\right) - \left(\mu_\omega + \frac{1+\theta}{1-\theta}\cdot\frac{\nu_\omega^2}{2} + \frac{\theta}{1-\theta}\nu_{a\omega}\right)$$

and

$$B = \mu_a + \mu_\omega + \left((1-\sigma)\frac{\theta}{1-\theta} + \sigma\frac{\rho}{1-\rho}\right)\cdot\frac{\nu^2}{2}$$

and ζ_2 is a collection of terms that do not depend on the wedges or productivity. Moreover, given capital, $\frac{\partial \log y}{\partial \nu_\omega^2} < 0$.

GDP per person in steady state depends on two main terms, B and A, which we discuss in turn. Term B involves the CES aggregators, and notice that productivities and expropriation rates enter symmetrically: this term depends basically on the properties of $A_i(1-\tau_i)$, or, in logs, $a_i + \omega_i$. Both the means and the overall variance are subject to the fundamental multiplier of the model.

The variance term also depends on the degrees of substitution and complementarity, essentially on a weighted average of the two effective curvature parameters $\frac{\theta}{1-\theta}$ and $\frac{\rho}{1-\rho}$. This recalls an important result highlighted earlier in the discussion of S_ρ versus Q_ρ, for example. The equilibrium allocation depends on $\frac{\rho}{1-\rho}$ as a curvature parameter rather than just ρ. When $\rho \to -\infty$, to maximize the significance of weak links, $\frac{\rho}{1-\rho}$ falls only as far as -1. As this is the term that multiplies the variance of the distortions in the proposition, problems with weak links cannot get too large in some sense: the economic incentives to overcome a weak link problem ensure that distortions or low productivity cannot be too costly. In contrast, $\frac{\theta}{1-\theta}$ goes to infinity as θ approaches one, so that the superstar effects from distortions can be severe.

Term A involves only the wedges, not the productivities. It captures the offsetting effect associated with the fact that expropriation reduces intermediate use and hence raises final use.

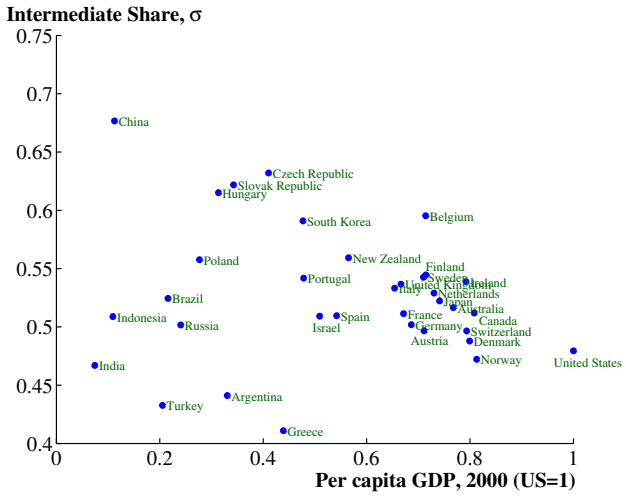
The last part of the proposition makes the important point that variation in the wedges across sectors unambigously reduces GDP at a point in time. Efficiency, of course, requires no expropriation. This result can be contrasted with the effect of variation in productivity. Changes in ν_a^2 have an ambiguous effect. From the standpoint of final uses, a higher variance is a good thing. For example (loosely speaking), if goods were perfect substitutes in consumption, only the good with the highest productivity would be consumed, and a higher variance increases the highest productivity. From the standpoint of intermediate goods, however, the opposite is true.

6. Development Accounting

To what extent can this model with linkages and complementarity help us understand income differences across countries? In this section, we attempt to quantify the mechanisms at work in our theory.

In the analysis that follows, some key parameters — such as the intermediate goods share — are calibrated quite precisely, while others — such as the degree of complementarity or the precise nature of micro-level distortions — are known with much less precision. The robust result that emerges is that intermediate goods can substantially magnify income differences relative to the standard neoclassical growth model, even with conservative choices for parameter values.

Figure 1: The Intermediate Goods Share of Gross Output



Note: Computed using the OECD Input-Output Database. The numerator is spending on intermediate goods at purchaser prices; the denominator is gross output less net taxes on production.

6.1. Measuring the Intermediate Goods Share

The crucial parameter of the model for explaining large income differences across countries is the intermediate goods share, σ. Fortunately, detailed empirical evidence exists regarding the magnitude of this parameter.

Basu (1995) recommends a value of 0.5 based on the numbers from Jorgenson, Gollop and Fraumeni (1987) for the U.S. economy between 1947 and 1979. Ciccone (2002), citing the extensive analysis in Chenery, Robinson and Syrquin (1986), observes that the intermediate goods share at least sometimes rises with the level of development. However, the numbers cited for South Korea, Taiwan, and Japan in the early 1970s are all substantially higher than conventional U.S. estimates, ranging from 61% to 80%.

For more systematic and recent evidence, there are rich data sets on input-output tables for many countries. For example, the OECD Input-Ouput Database now covers 35 countries (including 9 non-OECD countries) at the level of 48 industries for a year close to 2000; see Yamano and Ahmad (2006). Figure 1 displays the intermediate goods share of gross output using this data. For the United States and India, the share is about 47%. Japan has a share of 52%, and China has the highest share, at 68%. Across 35 countries (mostly OECD, but including Brazil, China, and India as well), the average

intermediate goods share is 52.6%, with a standard deviation of about 6%. Interestingly, there is no apparent correlation between this share and per capita GDP across countries. These numbers are discussed in greater detail in Jones (2007). Given this evidence, we take $\sigma = 1/2$ as a benchmark value.

6.2. Using Factor Shares to Measure Distortions?

An intriguing possibility is that variation in spending on intermediate goods (or capital or labor as in Hsieh and Klenow (2009)) can tell us about distortions to the allocation of resources. With aggregated data on intermediate goods, this approach is interesting, but it turns out not to be especially informative.

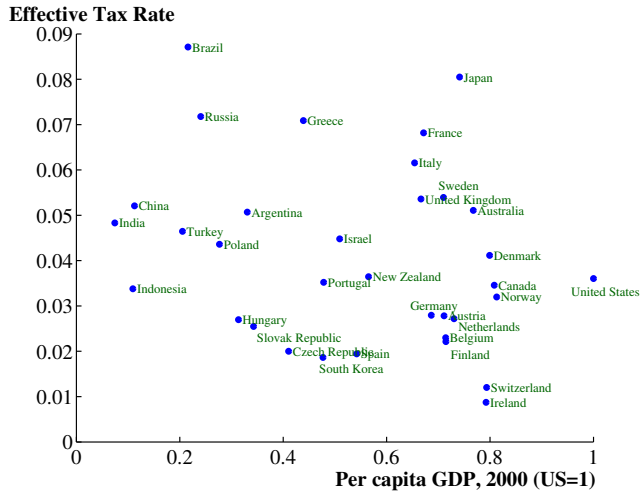
If a country has a high sales tax, its spending on capital will be distorted — recall the first-order condition will be something like $(1-\tau)\alpha Y/K = r$. Similarly, perhaps the intermediate goods share can tell us about the underlying distortions to the allocation of resources. As a useful example, suppose there is a single intermediate good and the production function is $Y = (K^{\alpha}L^{1-\alpha})^{1-\sigma}X^{\sigma}$. Furthermore, suppose sales are taxed at rate τ_Y and purchases of intermediate inputs are taxed at rate τ_X. If p is the price of the intermediate good, the first order condition from profit maximization implies

$$\frac{pX}{Y} = \frac{\sigma(1-\tau_Y)}{1+\tau_X}. \tag{27}$$

In other words, taxes and subsidies can affect the "gross" spending share on intermediates.

At this point, it is helpful to distinguish between taxes and subsidies that are measured according to the System of National Accounts and other underlying distortions that are not measured. Formal sales taxes or value-added taxes are measured. Preferential credit treatment, theft, and markups are not. The OECD Input-Output Database reports measures of the formal taxes net of subsidies, allowing one to compute an "effective tax rate" $(1-\tau_Y)/(1+\tau_X)$ for each country. Of course, this uses aggregated data: subsidies to one firm that are counterbalanced by taxes on another will misleadingly show up in these calculations as a zero tax (this is a key point of Banerjee and Duflo (2005) and Hsieh and Klenow (2009)). But the calculations may still be useful and are shown in Figure 2. The effective tax rates range from about 1% to 9% and are not highly correlated with per capita GDP. One can do this same exercise across the 48 industries

Figure 2: The Effective Tax Rate on Intermediate Goods



Note: The graph shows an estimate of the effective tax rate $1 - \frac{1-\tau_Y}{1+\tau_X}$ from the OECD Input-Output Database. It is computed as $pX/Y \times 1/\sigma$, where pX denotes spending on intermediates at basic prices (i.e. net of taxes), Y is measured as gross output, and σ is the intermediate share from Figure 1.

within each country. Within a country, the standard deviation of the effective tax rates is around 5%, and — disppointingly — there is little correlation between this standard deviation and per capita GDP: the U.S. and China both have a standard deviation across industries in this effective tax measure of around 4%.

What about distortions that are not measured in the national accounts? To the extent that we have not measured and subtracted out distortions in computing σ in Figure 1, one would expect to see variation in these estimates. So perhaps that variation (e.g. deviations from 1/2) reflects distortions. The main difficulty in pursuing this line of reasoning is that we do not know that the true underlying elasticity of production is 1/2; one needs to know the factor exponents in order to carry out such an exercise.

Still, this reasoning does suggest something informative. In particular, notice from Figure 1 that the countries with the highest intermediate goods shares are China, the Czech Republic, the Slovak Republic, and Hungary. From equation (27), we can see that (unmeasured) subsidies to the purchase of intermediate goods — a negative τ_X will raise the intermediate goods share. In other words, even if we saw that all countries spent half of gross output on intermediate goods, this would not tell us that there

were no distortions: this could simply be the result of large output taxes or theft and high subsidies for intermediate purchases offsetting. Given that the countries with high measured intermediate shares are historically associated with central planning, this does not seem implausible.

In the end, this factor share approach to measuring distortions with sectoral aggregates is interesting but not especially informative. To carry out our development accounting exercise, we do need measures of distortions. We discuss alternative approaches below, but ultimately this paper has little to say about the underlying distortions. Instead, we consider a range of possibilities. The point is not to claim that we have accurately quantified the distortions but rather to illustrate that whatever distortions are present get multiplied by linkages and complementarity.

6.3. Other Parameter Values

The baseline parameter values we use are summarized in Table 1. Robustness checks will consider departures from these values. We pick $\alpha = 1/3$ to match the empirical evidence on capital shares; see Gollin (2002), who shows that capital shares across countries have a mean of 1/3 and are uncorrelated with GDP per worker. Rather than modeling differences in human capital we simply assume that across the richest and poorest countries, these differences contribute a factor of 2 to income differences.[8]

For the substitution elasticity, we take as our baseline value an elasticity of substitution in consumption of $\frac{1}{1-\theta} = 6$. This value is consistent with the extensive estimates provided by Broda and Weinstein (2006). Notice that this implies $1/\theta = 1.2$; in a model with monopolistic competition, such an elasticity would deliver markups on the order of 20%. For the complementarity parameter ρ, we assume $\rho = -1$, which delivers an elasticity of substitution among intermediate goods of 1/2, midway between Leontief and Cobb-Douglas. There is very little solid information about this parameter and we will carefully explore robustness to other values in what follows.

6.4. TFP Differences

The place where we have the least amount of information regarding parameter choices is for the productivity differences and expropriation distortions across varieties. The

[8]This factor of 2 is consistent with Klenow and Rodriguez-Clare (1997) and Hall and Jones (1999).

Table 1: Baseline Parameter Values

Parameter	Value	Comment
σ	1/2	Intermediate goods share of gross output
α	1/3	Conventional value for capital share
$\bar{h}^r/\bar{h}^p$	2	Standard contribution from education
θ	1/1.2	Elasticity of substitution is 6
ρ	-1	Elasticity of substitution is 1/2
$\bar{A}^r$	1	Normalization
$\bar{A}^p$	$\{1, 1/2\}$	Illustrative purposes
γ^r	2	Gives a 90/10 ratio of 4.96
γ^p	$\{2, 2.87\}$	Doubles the 90/10 ratio
$\bar{\tau}_0$	0.9	Maximum expropriation rate
ξ	...	To match capital-output ratio factor of 3

Note: Robustness to these baseline values is explored below. Values for ξ vary according to the simulation. In the baseline case, the values are 0.994 and 1.112 for Scenarios 4 and 5 below.

approach we take is to consider a range of alternatives. The precise nature of the distortions or productivity differences is not essential in this paper. Instead, we focus on the extent to which intermediate goods and complementarities magnify the income differences, whatever the distortions may be.

The key source of information about productivity and distortions are the recent studies of plant-level productivity and misallocation, including Hsieh and Klenow (2009) and Bartelsman, Haltiwanger and Scarpetta (2008). Hsieh and Klenow measure plant-level TFP within 4-digit manufacturing sectors for China, India, and the United States, treating each plant as a distinct variety. They find that the 90/10 percentile ratios of plant-level TFP in a value-added production function are about 9 for the United States, 11 for China, and 22 for India. These statistics do not correspond exactly to what we want for our model. We'd like to see the variation across all firms and sectors in the economy. For example, the weak link story involves electricity, transportation, replacement parts, machine tools, etc. — inputs that are taken from different sectors. Moreover, the mapping between their value-added TFP and our gross-output TFP is not en-

tirely clear.[9] Finally, measurement problems may lead Hsieh and Klenow to overstate TFP differences across plants.

Still, these are useful observations to get us started. In particular, the large differences that Hsieh and Klenow observe across plants producing different varieties within a 4-digit industry suggest that the cases we consider below are relatively conservative. For example, variation across all plants in the economy is almost certainly larger than the average variation across plants within a 4-digit industry.

We conduct our quantitative analysis using two broad approaches. The first is to consider several cases inspired loosely by the Hsieh-Klenow evidence, but targeted more toward even larger income differences than those between, say, the U.S. and China. For these exercises, we let productivity and distortions be deterministic functions of variety. Second, we will use the results from Proposition 5 to conduct exercises with log-normally distributed productivity and distortions that makes even closer contact with the Hsieh-Klenow evidence.

To begin, it proves useful to let productivity and expropriation be deterministic functions of the variety index i. For TFP, we assume

$$A_i = \bar{A}e^{-\gamma i} \tag{28}$$

We normalize the order of varieties so that productivity decreases with the index i; moreover, we assume this occurs exponentially.

We consider two hypothetical countries, one "rich" and one "poor." Using the functional form in (28), we then consider three alternatives for how TFP differs between the countries:

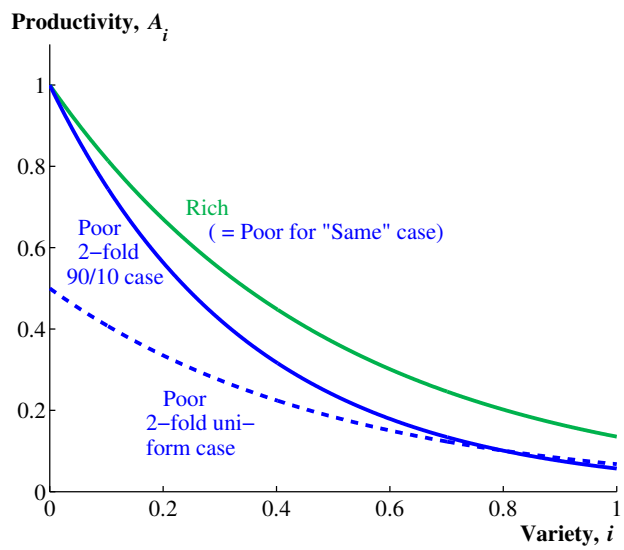
(a) "Same": Rich and poor countries have identical TFP levels for each variety.

(b) "2-fold 90/10": Rich and poor countries have identical TFP levels at variety 0 while differences emerge gradually across varieties, with the poor country having a 90/10 percentile ratio that is twice as large as that in the rich country.

(c) "2-fold uniform": The poor country is half as productive at making each variety: $A_i^{poor} = 1/2 \cdot A_i^{rich}$.

[9] One-good models like that discussed at the beginning of this paper can lead this difference to undo the multiplier. In the main multi-good model, however, the standard deviation of value-added TFP and the standard deviation of gross-output TFP across firms are equal; the intuition is that all the different sectors' TFPs contribute to the productivity implicit in X, which is then symmetric across varieties.

Figure 3: Assumed Productivities



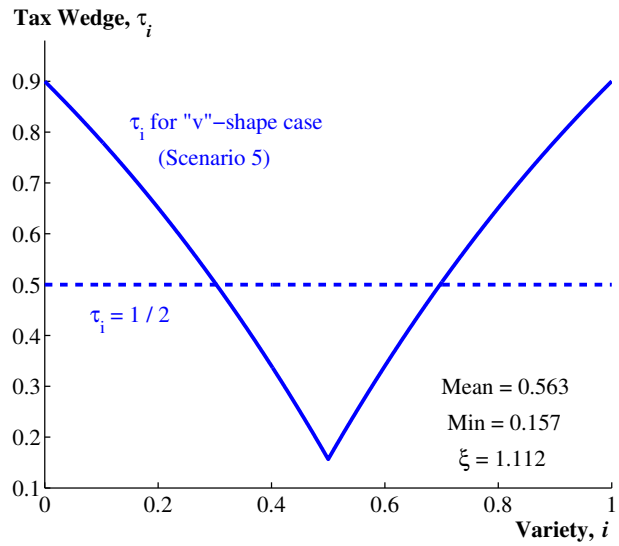
Note: The figure shows the three alternatives we consider for TFP differences. In the "Same" case, the rich and poor countries have identical TFPs. In the "2-fold 90/10" case, they have the same TFP for variety 0 and then differ by a factor of 2 in their 90/10 percentile ratios. Finally, in the "2-fold uniform" case, they differ by a factor of 2 throughout.

Four parameters are needed to formalize these alternatives — $\bar{A}^{rich}$, $\bar{A}^{poor}$, γ^{rich}, and γ^{poor}. We normalize $\bar{A}^{rich} = 1$, and consider two alternatives for $\bar{A}^{poor}$: that it equals one as well and that it takes a value of 1/2. For the decay rate across varieties, we pick $\gamma^{rich} = 2$, so that the 90/10 ratio for our rich country is only 4.96. For the poor country, we will sometimes use this same value, so that both countries have the same distribution of TFP. Alternatively, we will consider $\gamma^{poor} = 2.87$, which leads the poor country to have a 90/10 ratio that is twice as high as the rich country (as in the U.S.-India comparison in Hsieh and Klenow). Future work on productivity differences across all firms could potentially shed better light on these parameter values. Figure 3 shows these different productivity scenarios.

6.5. Expropriation Wedges

Finally, we need to parameterize the distortion measure across the two countries. Once again, we have little information to go on. Instead, we begin by choosing some interesting examples and then judge the model by the extent to which the presence of in-

Figure 4: Assumed Expropriation Wedges in the "Poor" Country



Note: The graph shows the expropriation rates for the two basic alternatives. The simple case has $\tau_i = 1/2$. The "v"-shape case corresponds to Scenario 5 below but looks very similar in other scenarios.

termediate goods amplifies the basic distortions we look at. Later, we will look more closely at the Hsieh-Klenow distortion estimates.

For the "rich" country, we simply assume there are no distortions: $\tau_i^{rich} = 0$. For the "poor" country, we consider two alternatives, shown in Figure 4:

(a) $\tau_i^{poor} = 1/2$: A simple case of a symmetric wedge.
(b) "v"-shaped wedges that distort both "superstar" and "weak link" varieties.

The symmetric wedge is a natural case to consider. The "v"-shaped wedges illustrate the role of complementarity and substitution. In particular, this shape features high expropriation rates at the ends and low rates in the middle. The advantage of this structure is that both superstar and weak link problems appear: resources are allocated away from both the superstars and weakest links and towards mid-productivity sectors.[10]

[10] More specifically, for the "v"-shaped wedge alternative, we assume

$$\tau_i^{poor} = \begin{cases} \bar{\tau}_0 + 1 - e^{\xi i} & i \in [0, 1/2] \\ \tau_{1-i} & i \in (1/2, 1] \end{cases}$$

Two parameters describe the expropriation wedges. We assume the maximum rate $\bar{\tau}_0$ is 90% and choose the decay rate ξ so that the ratio of expropriated funds to gross output is 1/2.

Table 2: Output per Worker in a "Rich" versus "Poor" Country

Scenario	Assumption about TFP across countries	Assumption about Distortions	No Intermediate Goods $\sigma = 0$	Baseline Case $\sigma = 1/2$	*Multiplicative Factor*
1.	Same	$\tau_i = 1/2$	2.8	5.3	*1.9*
2.	2-fold at 90/10	$\tau_i = 1/2$	3.1	12.9	*4.1*
3.	2-fold uniform	$\tau_i = 1/2$	8.0	42.7	*5.3*
4.	Same	"v"-shape	6.3	19.6	*3.1*
5.	2-fold at 90/10	"v"-shape	7.9	53.2	*6.8*
6.	2-fold uniform	"v"-shape	17.8	156.5	*8.8*

Note: The two main columns of the table report income ratios between a "rich" and a "poor" country under various scenarios. The "Multiplicative Factor" column shows the ratio of the two previous columns — that is, the overall multiplier from having $\sigma = 1/2$. Basic parameter values are shown in Table 1. The different assumptions regarding TFP and the expropriation distortions are described in Figures 3 and 4. In solving the model numerically, we evaluable the integrals as summations over 1001 grid points.

6.6. Quantitative Analysis

Table 2 shows the main quantitative results for the model. We report the ratio of output per worker between a rich country and a poor country under various scenarios. The main point of the table is that a model with intermediate goods and complementarity sharply multiplies the income differences that one obtains from a given level of distortions and productivity differences.

The first data column of the table shows the income ratio when the intermediate goods share is zero, shutting off the effects of both the intermediate goods multiplier and complementarity (since complementarity enters only through intermediate goods). In general, one sees that the income differences are substantially larger in the presence of intermediate goods. In fact, the last column of the table quantifies this difference, showing the factor by which income differences increase in the presence of intermediate goods and complementarity. This factor ranges from a low of 1.9 to a high of 8.8.

To understand the source of these multipliers, let's focus in more closely on two of the scenarios. Scenario 3 introduces a uniform 2-fold difference in the A_i between the rich and poor countries to illustrate the intermediate goods multiplier when comple-

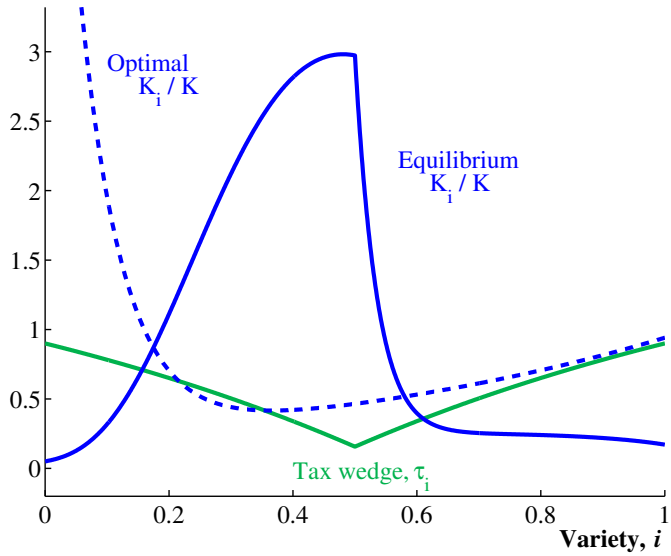
mentarity effects are absent (since the distortion is symmetric and the A_i are the same). This difference could arise from technological differences or could simply reflect theft. Under the theft interpretation, 50% of output gets stolen any time a good is produced.

In the pure neoclassical framework with no intermediate goods, this 2-fold difference in TFP amplifies the basic income difference in Scenario 1 by $2^{1/1-\alpha} = 2^{3/2} \approx 2.8$ to yield a difference of 8.0. In the presence of intermediate goods, however, this multiplier is much stronger: $2^{\frac{1}{1-\sigma}\frac{1}{1-\alpha}} = 2^{2\times 3/2} = 2^3 = 8$, yielding a much larger income difference of 42.7. The intuition is that the "theft tax" gets paid repeatedly when intermediate goods are involved: 1/2 of the steel is stolen from the steel plant, 1/2 of the cars are stolen from the automobile plant, and 1/2 of the pizzas get stolen from the pizza delivery service. In this sense, the steel gets stolen *three* times rather than just once, and this is the intermediate goods multiplier.

The remaining scenarios explore the role of weak link and superstar effects in this environment. For example, consider Scenario 5. This scenario features TFP differences across sectors so that the 90/10 ratio in the poor country is twice as large as that in the rich country. It also features a "v"-shaped pattern of distortions. In a standard neoclassical growth model without intermediate goods, these productivity differences and distortions lead to a 7.9-fold difference in output per worker between the rich and poor country. In the presence of intermediate goods, however, these same distortions lead to an income ratio of 53.2, nearly 7 times larger.

Figure 5 provides some insight into how weak links and superstar effects achieve this multiplier. The lighter (green) line shows the expropriation wedge for each variety, which we have already seen in an earlier figure. The dark (blue) lines reveal the allocation of resources. The solid one plots the equilibrium allocation of K_i/K across varieties, while the dashed line shows the optimal allocation. These differ sharply. The "v"-shaped wedges distort the allocation away from both the highly-productive sectors and the least-productive sectors. At high productivities, the superstar sectors are harmed, which has a large effect on output. At low productivities, the weakest links are not strengthened, which again has a large effect on output. Together with the general multiplier associated with intermediate goods, these forces explain why the model is able to deliver such large income differences.

Figure 5: Expropriation Rates and Allocations in Scenario 5



Note: The "v"-shaped expropriation wedges distort the resource allocation away from both the superstar sectors and the weak link sectors.

6.7. Robustness: Complementarity and Substitution

The size of the weak link and superstar effects in this framework depends on the elasticities of substitution for intermediate goods and consumption goods. In the baseline case explored so far, we set these elasticities at 1/2 ($\rho = -1$) and 6 ($\theta = 0.833$). Neither of these parameters is especially well pinned down in the literature; this is particularly true of the degree of complementarity among inputs. Hence, checking the robustness of the results along this dimension is important.

Table 3 does this. The first two columns replicate the results from the previous table while the remaining columns consider alternative elasticities, ranging from a Cobb-Douglas case ($\rho \approx 0$ and $\theta \approx 0$) to a Leontief case. The bottom line is that the results hold up quite well. Income differences are amplified by a factor that ranges from a low of 1.4 to a high of 10.8, while the bulk of the factors are around 4 or 5. Another interesting result in the table is that the difference between the Cobb-Douglas case and the Leontief case is not nearly as large as the weak link intuition might suggest. As indicated in our simple examples earlier, the ability of the economy to substitute capital

Table 3: Output per Worker Ratios: Robustness

Scenario	No Intermediate Goods $\sigma = 0$	Baseline Case $\sigma = 1/2$	Cobb-Douglas	*Robustness Results* $\theta = 0.9$ (EofS=10)	$\theta = 1/2$ (EofS=2)	"Leontief" $\rho = -100$
Ratio of output per worker:						
1.	2.8	5.3	5.3	5.3	5.3	5.3
2.	3.1	12.9	19.6	12.3	17.5	14.4
3.	8.0	42.7	42.7	42.7	42.7	42.7
4.	6.3	19.6	8.8	24.2	11.9	22.4
5.	7.9	53.2	32.2	62.3	40.7	72.9
6.	17.8	156.5	70.2	193.3	95.1	178.9
Multiplicative factor relative to $\sigma = 0$:						
1.	1.0	1.9	1.9	1.9	1.9	1.9
2.	1.0	4.1	6.2	3.9	5.6	4.6
3.	1.0	5.3	5.3	5.3	5.3	5.3
4.	1.0	3.1	1.4	3.8	1.9	3.5
5.	1.0	6.8	4.1	7.9	5.2	9.2
6.	1.0	8.8	3.9	10.8	5.3	10.0

Note: The "Scenarios" are those described in detail in Table 2. The next two columns repeat the main results from the previous table. The remaining columns illustrate the robustness of the results by varying one or two parameters at at time, with others left at their baseline values. For the "Cobb-Douglas" case, we set $\rho = -0.01$ and $\theta = 0.01$. The bottom half of the table reports the extent to which intermediate goods and complementarity magnify income differences; that is, these are the ratios of entries in the top part of the table to the entries in the first column.

and labor for low productivity in a weak link sector seems to mitigate the worst of these problems.

6.8. Using the Hsieh-Klenow Statistics

The distortions and productivity patterns in the previous examples are loosely based on facts, in large part because the precise nature of the facts are unknown. This section pursues an alternative approach, which is to use the precise statistics calculated by Hsieh and Klenow (2009) for China and India. This has the advantage that hard numbers are behind these examples. But it has the disadvantage that the numbers only loosely correspond to the theoretical concepts in the model. Neither approach is perfect, but until the relevant micro data is studied carefully, perhaps these two imperfect approaches will prove useful.

This section is based on Proposition 5 above, which characterizes output per worker in the presence of log-normally distributed productivities and distortions. Hsieh and Klenow report the standard deviation of the log of productivity ("TFPQ") and a measure of distortions ("TFPR") for the United States, China, and India across firms within a 4-digit industry. As mentioned above, we'd rather have these measures across all firms in the economy, not just within industry, and we'd rather see the calculations using gross output instead of value-added. Moreover, the distortion measure potentially includes distortions *between* capital and labor as well as the overall τ_i distortion measure in our theory. So for these and other reasons, these numbers do not correspond exactly to what we'd like.

To implement this quantitative exercise, we assume the U.S. economy is undistorted and use a combination of the data for India and China to characterize our "poor" economy. Empirically, the ratios of GDP per worker in the United States to China and India in the year 2000 were 8.9 and 13.4, respectively.

Table 4 reports the results under a range of different assumptions using the Hsieh-Klenow evidence. Overall, the exercises with $\sigma = 1/2$ come much closer to matching the empirical income ratios, and the multiplicative factors induced by intermediate goods and complementarity are substantial, ranging from about 1.5 to 3.5. (Note that because we are targeting smaller income ratios — something like 10 rather than 40 or 50, these factors are naturally smaller: just as 2^3 is disproportionately smaller than 3^3.)

Table 4: Output per Worker Ratios using the Hsieh-Klenow Statistics

Scenario	Description	"Average" TFP in Poor Country	No Intermediate Goods $\sigma = 0$	Baseline Case $\sigma = 1/2$	*Multiplicative Factor*
7.	Baseline	0.404	2.4	15.0	*6.2*
8.	Identical TFPs	1.000	2.0	2.4	*1.2*
9.	$\nu_a^{rich} = \nu_a^{poor} = 0.84$	0.750	3.1	5.7	*1.8*
10.	$\nu_a^{rich} = \nu_a^{poor} = .5$	0.750	3.0	5.5	*1.9*
11.	$\nu_a^{rich} = .5$, $\nu_a^{poor} = .75$	0.559	3.0	9.6	*3.2*
12.	Same as 11, but $\nu_{aw} = 0$	0.559	2.7	9.1	*3.4*

Note: The two main columns of the table report income ratios between a "rich" and a "poor" (e.g. China or India) country based on Proposition 5. For comparison, the income ratios between the U.S. and China/India were 8.9 and 13.4 in 2000. The "Multiplicative Factor" column shows the ratio of the two previous columns — that is, the overall multiplier from having $\sigma = 1/2$. Baseline parameter values are $\theta = 2/3$ (the value Hsieh and Klenow assume in calculating their statistics), $\rho = -1$, $\bar{h}^{rich}/\bar{h}^{poor} = 1.52$ (corresponding to a 6 year educational attainment differential and a 7% return to education), $\nu_a^{rich} = 0.84$ (U.S.), $\nu_a^{poor} = 1.23$ (India), $\nu_w^{rich} = 0$, $\nu_w^{poor} = .67 - .49 = .18$, $\mu_a^{rich} = 0$, $\mu_w^{rich} = 0$, $\mu_w^{poor} = \log(1 - 1/4)$, $\nu_{aw}^{rich} = 0$ and $\nu_{aw}^{poor} = -.647 \cdot \nu_a^{poor}\nu_w^{poor}$. Most of these numbers are taken from Hsieh and Klenow (2009), Tables 1 - 2. The main exception is the value for μ_w, on which Hsieh and Klenow are silent; we assume a baseline expropriation rate in the poor country of 1/4. The correlation coefficient 0.647 is based on Table 4 of the NBER WP version, Hsieh and Klenow (2007), not reported in the published version. Scenario 7 is most faithful to the Hsieh-Klenow statistics; the other scenarios consider alternatives, especially ones with smaller variations in productivity since these seem particularly large and suggestive of measurement error (for example, the 95-5 ratio of productivities in India, with a standard deviation of 1.23 in logs, is roughly $exp(4*1.23) \approx 137$.) The column labeled "Average TFP in Poor Country" reports $exp(\mu_a^{poor})$. In Scenarios 7, 11, and 12, this value is chosen so that TFP at the 99th percentile is the same in the rich and poor countries (otherwise, the higher variance in the poor country would lead firms at the top of the distribution to be substantially more productive than those in the rich country).

The strength of this section is the reliance on the Hsieh-Klenow data. The weaknesses (and hence the value of the previous exercises) are two-fold. First, the Hsieh-Klenow distortions are calculated assuming $\theta = 2/3$, so one cannot use their statistics to consider other elasticities. Second, and more importantly, the log-normal approach here assumes a linear relationship between productivity and wedges. For example, Hsieh and Klenow find that on average China and India seem to have higher distortions on more productive firms (although there is a lot of noise in this relationship; the standard error is even larger than the regression coefficient). In the log-normal approach, this means that low productivity firms are distorted the least, so weak link problems are minimized.

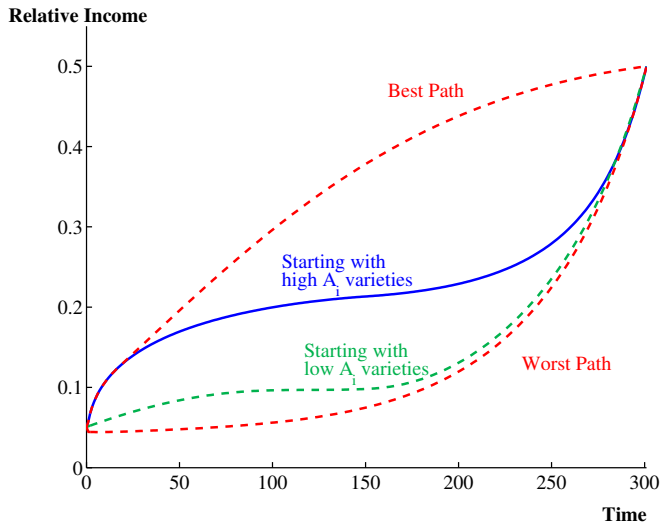
A simple calculation helps illustrate this, as well as the more general point that weak link problems tend not to be too large in these calculations because of the response of other inputs. Consider Scenario 7, which uses "high" values for the variances, maximizing the potential role of distortions and complementarity. Set the covariance term to zero, which since it is negative will also maximize the role of complementarity. Now consider the last piece of "term *B*" in Proposition 5: the contribution from complementarity to the rich/poor income ratio turns out to be $-\frac{3}{8}(\nu^2_{rich} - \nu^2_{poor}) = -\frac{3}{8}\cdot(0.84^2 - 1.23^2 - 0.18^2) = -\frac{3}{8}\cdot(-.84) = .315$ log points, or a factor of $exp(.315) = 1.37$. Going all the way to the Leontief case would raise this by at most double (in logs), leading to a factor of 1.88 ("at most" because an offsetting force comes into play from term *A* when $\rho \neq 1$). The "v"-shaped wedge example of the previous section allows us to consider cases where distortions occur at both ends of the distribution, leading to a more significant role for weak link problems.

7. The Cumulative Effect of Reforms

The model possesses two key features that seem desirable in any theory designed to explain large differences in incomes across countries. First, relatively small and plausible differences in underlying parameters can yield large differences in incomes. That is, the model generates a large multiplier.

The second feature is one we explore now. Despite this large multiplier, reforms that eliminate the expropriation wedges may have relatively small effects on output. If a chain has a number of weak links, fixing one or two of them will not change the overall

Figure 6: The Cumulative Effect of Reforms



Note: The economy is initially characterized as in Scenario 3, where we approximate the continuum of varieties with a 300-point grid. Each period, one of the expropriation wedges is eliminated. The plot shows the sequence of steady states that result, depending on the nature of the reform process.

strength of the chain.

This principle is clearly true in the extreme Leontief case, but it holds more generally as well. To see this, consider a simple exercise. Suppose the economy is characterized by Scenario 4 above: it has the "v"-shaped wedges but is otherwise identical to the richest country in the world, apart from a 2-fold difference in human capital. A sequence of reforms then eliminates the distortions one at a time. As was shown in Table 2, our poor country begins with an income about 1/20th of that in the rich country. For this exercise, we approximate the continuum of varieties with a 300-point grid, so after 300 reforms, the poor country will have an income of 1/2 the rich country, due only to the human capital difference. The question is this: what does the transition path look like as the economy undergoes these reforms?

Figure 6 shows the sequence of steady states that results from several different paths of reform. The "best path" solves for the reform in each period that increases output by the most (an approach advocated by Hausmann, Rodrik and Velasco (2005)). The "worst path" finds the reform that increases output by the least. The other two paths

start with either $i = 0$ (the most productive sector) or $i = 1$ (the least productive sector) and proceed sequentially across the varieties.

The key point of the figure is that three of the four paths considered feature long, flat regions — substantial periods of reform that have relatively small impacts on incomes. For example, for the "worst path" reforms, the first doubling of incomes does not occur until nearly 60% of the sectors are reformed; the second doubling occurs much more rapidly, by the 80% reform point. Valuable reforms can have small impacts until other complementary reforms are undertaken, at least unless the sequence of reforms is chosen quite carefully.

Of course, it should also be recognized that some reforms could affect distortions in multiple sectors simultaneously. One example of this is considered next.

7.1. Multinationals and Trade

Multinational firms and international trade may help to solve these problems if they are allowed to operate. For example, multinationals may bring with them knowledge of how to produce, access to transportation and foreign markets, and the appropriate capital equipment. Indeed many of the examples we know of where multinationals produce successfully in poor countries effectively give the multinational control on as many dimensions as possible: consider the maquiladoras of Mexico and the special economic zones in China and India. Countries may specialize in goods they can produce with high productivity and, to the extent possible, import the goods and services that suffer most from weak links.[11]

On the other hand, domestic weak links may still be a problem. A lack of contract enforcement may make intermediate inputs hard to obtain. Knowledge of which intermediate goods to buy and how to best use them in production may be missing. Weak property rights may lead to expropriation. Inadequate energy supplies and local transportation networks may reduce productivity. The right goods must be imported, and these goods must be distributed using local resources and nontradable inputs, as in Burstein, Neves and Rebelo (2003).

Incorporating international trade into this framework is a natural direction for fu-

[11] Nunn (2007) provides evidence along these lines, suggesting that countries that are able to enforce contracts successfully specialize in goods where contract enforcement is critical. See also Grossman and Maggi (2000) and Waugh (2007).

ture research. Of course, to the extent that these channels are shut down in the poorest countries of the world, the closed economy benchmark considered here illustrates the range of income differences that could exist. Moreover, the multipliers in this framework may help explain why rapid catch-up growth in response to opening the economy to multinationals and trade is possible.

8. Conclusion

In an effort to understand the large income differences across countries, this paper explores two related channels for amplifying the effects of distortions. First, the presence of intermediate goods leads to a multiplier that depends on the share of intermediate goods in gross output. This amplification force echoes the familiar multiplier associated with capital accumulation and is relatively easy to quantify. By raising the effective share of produced inputs in total output from 1/3 to 2/3, the addition of intermediate goods delivers a substantial multiplier. Distortions to the transportation sector reduce the output of many other activities, including truck manufacturing and highway construction. This in turn further reduces output in the transportation sector and in the rest of the economy. This vicious cycle is the source of the multiplier associated with intermediate goods.

The second amplification force — the "O"-ring effects of complementarity — is more difficult to quantify in practice. Simple examples suggest that this force can be quite powerful. For example, in comparing the symmetric allocation and the optimal allocation in the case in which final goods are perfect substitutes while intermediate goods are Leontief, income in the rich country depends on the maximum productivity (a superstar effect), while income in the poor country depends on the weakest link. However, in the main calibration exercises explored in this paper, the effects of complementarity tended to be much smaller. The reason is that market forces are often quite effective at offsetting weak links. One way to see this formally is in the switch in the curvature parameter from ρ in the symmetric allocation to $\frac{\rho}{1-\rho}$ in the equilibrium allocation for the TFP aggregator functions (S_ρ versus Q_ρ). Even in the extreme case of $\rho = -\infty$, the equilibrium allocation depends on the harmonic mean of expropriation-adjusted productivities, not on the minimum. In this case, the Cobb-Douglas production structure for varieties lets other inputs like capital and labor substitute for low pro-

ductivity to alleviate severe bottlenecks. The more general lesson seems to be that one must carefully consider various substitution possibilities when seeking to quantify the effects of weak links.

An important channel for future research concerns the role of intermediate goods. For example, it may be useful to pursue plant level studies like Hsieh and Klenow (2009) and Bartelsman, Haltiwanger and Scarpetta (2008) using gross output instead of value added in order to measure the effect of misallocating intermediate goods. On a different vein, the present model simplifies considerably by focusing on a single intermediate input. The input-output matrix in this model is very special. This is a good place to start. However, it is possible that the rich input-output structure in modern economies delivers a multiplier smaller than $\frac{1}{1-\sigma}$ because of "zeros" in the matrix. In work in progress, Jones (2007) explores this issue. The preliminary results are encouraging. For example, if the share of intermediate goods in each sector is σ but the composition of this share varies arbitrarily, the aggregate multiplier is still $\frac{1}{1-\sigma}$. More generally, I plan to use actual input-output tables for both OECD and developing countries to compute the associated multipliers. I believe this will confirm the central role played by intermediate goods in amplifying distortions.

A Appendix: Proofs of the Propositions

This appendix contains outlines of the proofs of the propositions reported in the paper.

Proof of Proposition 1. The Symmetric Allocation, Given Capital

Follows directly from the fact that $Y_i = A_i m$, where $m = (K^\alpha H^{1-\alpha})^{1-\sigma} X^\sigma$ is constant across activities. QED.

Proof of Proposition 2. The Competitive Equilibrium, Given Capital

1. The first order conditions from the Variety i Problem are

$$(1-\tau_i)p_i\alpha(1-\sigma)\frac{Y_i}{K_i} = r+\delta$$

$$(1-\tau_i)p_i(1-\alpha)(1-\sigma)\frac{Y_i}{H_i} = w$$

$$(1-\tau_i)p_i\sigma\frac{Y_i}{X_i} = q.$$

Substituting these conditions back into the production function yields an equation that characterizes the price of good i:

$$p_i = \frac{mc}{A_i(1-\tau_i)\epsilon}, \tag{29}$$

where $mc \equiv ((r+\delta)^\alpha w^{1-\alpha})^{1-\sigma}q^\sigma$ is a key piece of the marginal cost and $\epsilon \equiv (\alpha^\alpha(1-\alpha)^{1-\alpha})^{1-\sigma}(1-\sigma)^{1-\sigma}\sigma^\sigma$.

2. Integrating the Variety i first order conditions above gives

$$(r+\delta)K = \alpha(1-\sigma)\int(1-\tau_i)p_iY_idi \tag{30}$$

$$wH = (1-\alpha)(1-\sigma)\int(1-\tau_i)p_iY_idi \tag{31}$$

$$qX = \sigma\int(1-\tau_i)p_iY_idi \tag{32}$$

where the limits of the integration are understood to be 0 to 1. Note that

$$\int p_ic_idi = Y, \quad \int p_iz_idi = qX, \quad \int p_iY_idi = Y + qX.$$

Define $\tau \equiv \frac{E}{Y+qX}$ to be expropriation revenues as a share of gross output. Then

$$\int(1-\tau_i)p_iY_idi = (1-\tau)(Y+qX).$$

Substituting this expression into (30), (31), and (32) gives

$$(r+\delta)K = \alpha(1-\sigma)\frac{1-\tau}{1-\sigma(1-\tau)}Y \tag{33}$$

$$wH = (1-\alpha)(1-\sigma)\frac{1-\tau}{1-\sigma(1-\tau)}Y \tag{34}$$

$$qX = \frac{\sigma(1-\tau)}{1-\sigma(1-\tau)}Y. \tag{35}$$

These expressions allow us to solve for mc (see the definition under (29)) as

$$mc = \frac{1-\tau}{1-\sigma(1-\tau)} \cdot \epsilon \cdot \frac{Y}{(K^{\alpha}H^{1-\alpha})^{1-\sigma}X^{\sigma}}. \tag{36}$$

3. Next, consider the first-order conditions from the Final Good and Intermediate Good Problems. For each of these problems, take the first order condition and then integrate it back into the firm's production function. For the final good, this gives

$$\left(\int p_i^{-\frac{\theta}{1-\theta}} di\right)^{-\frac{1-\theta}{\theta}} = 1 \tag{37}$$

and for the intermediate good

$$\left(\int p_i^{-\frac{\rho}{1-\rho}} di\right)^{-\frac{1-\rho}{\rho}} = q \tag{38}$$

Now substitute (29) into (37) to get

$$mc = \epsilon Q_{\theta} \tag{39}$$

where

$$Q_{\theta} \equiv \left(\int_0^1 (A_i(1-\tau_i))^{\frac{\theta}{1-\theta}} di\right)^{\frac{1-\theta}{\theta}}. \tag{40}$$

Combining (29) with this expression, we can solve (38) to find

$$q = \frac{Q_{\theta}}{Q_{\rho}} \tag{41}$$

where Q_{ρ} is defined analogously to Q_{θ}. Combining (36), (39), (35), and (41) yields the main result in the proposition.

4. Finally, we need to solve for τ. From the first-order conditions for the Final Goods Problem and the Intermediate Goods Problem we get

$$p_i Y_i = p_i c_i + p_i z_i = p_i^{-\frac{\theta}{1-\theta}} Y + (p_i/q)^{-\frac{\rho}{1-\rho}} (qX).$$

Multiplying this expression by τ_i, integrating, and then using (29), (39), and (41)

leads to the solution for τ:

$$\tau = (1-\sigma(1-\tau))T_\theta + \sigma(1-\tau)T_\rho \tag{42}$$

where $T_\rho \equiv \int_0^1 \tau_i \left(\frac{A_i(1-\tau_i)}{Q_\rho}\right)^{\frac{\rho}{1-\rho}} di$. That is, T_ρ is a weighted average of the sector-specific expropriation rates, where the weights depend on ρ; T_θ is defined analogously. QED.

Proof of Proposition 3. The Competitive Equilibrium in Steady State

Straightforward using (33) and the Euler equation from the Household Problem. QED.

Proof of Proposition 4. Symmetric Wedges

Straightforward evaluation given earlier results. QED.

Proof of Proposition 5. Random Productivity and Wedges

1. Define $Q(\eta) \equiv \left(\int (A_i(1-\tau_i))^\eta di\right)^{1/\eta}$. Define $m_i \equiv \eta(a_i+\omega_i)$. Then $m_i \sim N(\eta(\mu_m+\mu_a), \eta^2\nu^2)$. Therefore,

$$\begin{aligned} Q(\eta) &= (E(e^{m_i}))^{1/\eta} \\ &= e^{\mu_a+\mu_\omega+\frac{1}{2}\eta\nu^2}. \end{aligned} \tag{43}$$

Let $\bar{Q} \equiv [Q(\frac{\theta}{1-\theta})]^{1-\sigma}[Q(\frac{\rho}{1-\rho})]^\sigma$. Then

$$\log \bar{Q} = \mu_a + \mu_w + \frac{1}{2}\cdot\left((1-\sigma)\frac{\theta}{1-\theta} + \sigma\frac{\rho}{1-\rho}\right)\nu^2$$

which is term B of the proposition.

2. To get term A, we need to solve for τ. From equation (42), one can obtain

$$1-\tau = \frac{1-T_\theta}{1-\sigma(T_\theta - T_\rho)}.$$

Evaluating the integrals in T_θ and T_ρ as above gives

$$T_\theta = 1 - \exp\{\mu_\omega + \frac{1}{2} \cdot \frac{1+\theta}{1-\theta}\nu_\omega^2 + \frac{\theta}{1-\theta}\nu_{a\omega}\}$$

and T_ρ is the analogous expression. These expressions deliver term A. QED.

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