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Asset Pricing with Heterogeneous Consumers

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Empirical difficulties encountered by representative-consumer models are resolved in an economy with heterogeneity in the form of uninsurable, persistent, and heteroscedastic labor income shocks. Given the joint process of arbitrage-free asset prices, dividends, and aggregate income, satisfying a certain joint restriction, it is shown that this process is supported in the equilibrium of an economy with judiciously modeled income heterogeneity. The Euler equations of consumption in a representative-agent economy are replaced by a set of Euler equations that depend not only on the per capita consumption growth but also on the cross-sectional variance of the individual consumers' consumption growth.

I. Introduction and Summary

The primary testable implications of equilibrium in a representative-consumer Lucas (1978)–type exchange economy are the set of Euler

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equations of consumption. The model fares poorly in explaining security prices. Mehra and Prescott (1985) pointed out that the model predicts a mean equity premium that is too low and a mean interest rate that is too high, given the observed low variability of aggregate consumption growth. In formal tests of the conditional Euler equations, Hansen and Singleton (1982), Ferson and Constantinides (1991), and others rejected the model even though no a priori upper bound is imposed on the relative risk aversion coefficient. Thus the poor performance of the model is unmitigated even by unconventionally high values of the relative risk aversion coefficient. The Euler equations are also rejected by the diagnostic tests of Hansen and Jagannathan (1991).

Within the representative-consumer framework, a number of generalizations have been suggested to mitigate the poor empirical performance of the model. They include time-nonseparable preferences (Constantinides 1990; Ferson and Constantinides 1991; Heaton 1993); recursive preferences (Weil 1989; Epstein and Zin 1991); state-nonseparable preferences (Nason 1988; Abel 1990); rare-event declines in aggregate consumption (Rietz 1988); transaction costs (Luttmer 1993; He and Modest 1995); and the combined assumptions of consumer heterogeneity and incomplete consumption insurance (Mehra and Prescott 1985).

Full consumption insurance implies that heterogeneous consumers are able to equalize their marginal rates of substitution state by state and, at least for consumers with von Neumann–Morgenstern preferences, that the equilibrium of a heterogeneous-consumer, full-information economy is isomorphic in its pricing implications to the equilibrium of a representative-consumer, full-information economy (see Wilson 1968; Constantinides 1982). The full consumption insurance hypothesis is suspect given that certain types of insurance, such as unemployment insurance, are conspicuously nonexistent. Direct tests using disaggregated consumption data clearly reject the hypothesis. (See Attanasio and Davis [1993] for most recent tests and a review of the literature. Earlier work includes Cochrane [1991], Mace [1991], Altonji, Hayashi, and Kotlikoff [1992], and Townsend [1992].)

The joint hypothesis of incomplete consumption insurance and consumer heterogeneity offers the prospect of enriching the pricing implications of the representative-consumer model. However, extant research along these lines suggests that the potential enrichment is largely illusory. Building on models by Bewley (1982) and Mankiw (1986), Lucas (1991) and Telmer (1993) calibrated economies in which consumers face uninsurable income risk and borrowing or

short-selling constraints.¹ They concluded that consumers are able to come close to the complete-markets rule of complete risk sharing, even though consumers are allowed to trade in just one security in a frictionless market.

Aiyagari and Gertler (1991) and Heaton and Lucas (1992, 1994a, 1994b) added transaction costs or borrowing costs in economies with uninsurable income risk and concluded that consumers are still able to come close to the complete-markets rule of complete risk sharing, unless the ratio of the net supply of bonds to aggregate income is restricted to an unrealistically low level. These models deliver an unrealistically low mean equity premium or an unrealistically high interest rate, given a realistic ratio of the net supply of bonds to aggregate income.

A common feature of these models, largely responsible for the negative results, is the assumption that the time series of the ratio of each (symmetrically distributed) consumer's labor income to aggregate labor income, I_{it}/I_t , is a stationary Markov process with low persistence. Thus the statistical distribution of I_{iT}/I_T (for consumer i at some future date $T > t$) converges to the statistical distribution of I_{jT}/I_T (for consumer j) as T increases, irrespective of the values of I_{it}/I_t and I_{jt}/I_t . The convergence is faster, the lower the persistence of the Markov process. With low persistence in the Markov process and the availability of at least one market, consumers come close to equalizing their consumption at time T , irrespective of the values of I_{it}/I_t and I_{jt}/I_t . They also come close to equalizing (across consumers) the marginal rates of substitution between any two times. When transaction costs and borrowing costs are introduced in the model, consumers largely circumvent these costs by maintaining an inventory of bonds. To close this loophole, one needs to restrict the net supply of bonds to an unrealistically low level.

In this paper we relax the assumption that the time-series process of each (symmetric) consumer's ratio of labor income to aggregate income is a stationary process. We demonstrate that the joint hypothesis of incomplete consumption insurance and consumer heterogeneity enriches the pricing implications of a representative-consumer model, even without introducing borrowing constraints, short-sale restrictions, borrowing costs, transaction costs, or an unrealistic restriction on the net supply of bonds. For consumers i and j , the statistical distribution of I_{iT}/I_T does not necessarily converge to that of

¹ See also related work by Scheinkman and Weiss (1986), Ben-Zvi and Sussman (1989), Imrohoroglu (1989), Scheinkman (1989), Duffie (1990), Heston (1991), Huggett (1991), Marcet and Singleton (1991), Danthine, Donaldson, and Mehra (1992), Weil (1992), and Mehrling (1994).

I_{jT}/I_T as T increases. Optimal consumption at each date generally differs across consumers. Also, the marginal rate of substitution between any two dates generally differs across consumers. The pricing implications differ substantially from those in a representative-consumer model.

We consider an exchange economy with a single nondurable consumption good serving as the numeraire. There is a fixed number of securities, which are claims to given net dividend processes, and default-free discount bonds of a large range of maturities. The securities are in a fixed positive supply and the bonds are in zero net supply.² Consumers are endowed with heterogeneous income processes consistent with a given aggregate income process, as in Mankiw (1986). We model the individual income processes as nonstationary and heteroscedastic, with the conditional variance of the individual income shocks given by a judiciously chosen function of the state. Consumers have homogeneous preferences represented by a time- and state-separable von Neumann–Morgenstern utility function with a constant subjective discount rate and a constant relative risk aversion coefficient. Essentially, we consider an exchange economy with conventional consumer preferences, given dividend processes and an aggregate income process. Our degree of freedom is in the choice of individual income processes consistent with the given aggregate income process. Even though consumers may trade the securities and bonds in perfect markets, these markets are generally inadequate for full consumption insurance.

The equity premium puzzle, the risk-free rate puzzle, and the empirical rejection of the representative-consumer conditional Euler equations are subsumed by the question whether the observed joint process of dividends, aggregate income, and prices of securities and bonds is consistent with equilibrium in an economy with incomplete consumption insurance and consumer heterogeneity.

The main result of this paper is a proposition demonstrating, by construction, the existence of individual income processes, consistent with the given aggregate income process, such that the equilibrium security and bond prices match the given security and bond price processes. The proposition holds provided that a certain restriction on the joint processes of dividends, aggregate income, and prices holds. We stress that this restriction is not merely technical but has economic content. (We discuss in due course the extant empirical tests that fail to reject it.) Furthermore, the assumed individual income processes, the equilibrium individual consumption processes, and the

² The assumption that the bonds are in zero net supply is an innocuous convention: Any default-free bond in positive net supply is included in the set of "securities."

resulting cross-sectional distributions of individual income and consumption are testable using panel data on individual income and consumption.

The contribution of this paper is theoretical. However, by providing a set of conditions that suffice to support in equilibrium the observed price processes, this paper provides a set of testable hypotheses. For example, the model predicts that a potential source of the equity premium is the covariance of the securities' returns with the cross-sectional variance of individual consumers' consumption growth, a source that is (under typical conditions) irrelevant in an economy with full consumption insurance.

The paper is organized as follows. The economy is defined in Section II. The equilibrium is derived and the main proposition is stated in Section III. The economic interpretation is given in Section IV. The cross-sectional distribution of consumption is discussed in Section V. Concluding remarks are offered in Section VI. Technical aspects of the proof of the main proposition are relegated to Appendix A, and technical aspects of the cross-sectional distribution of consumption and income are relegated to Appendix B.

II. The Economy

We consider an exchange economy with a single nondurable consumption good serving as the numeraire. There are n securities (equities, corporate bonds, and so on) indexed by j . At time t , security j pays a net dividend of d_{jt} and has ex-dividend price P_{jt} . Let $d = (d_1, \dots, d_n)$ and $P = (P_1, \dots, P_n)$ denote the n -dimensional dividend and price processes. Let $D_t = \sum_{j=1}^n d_{jt}$ denote the net aggregate dividend. Each security is in fixed positive supply. We normalize the supply of each security to be one.

There are also default-free discount bonds of all maturities less than or equal to a fixed integer T . The fact that T is finite plays no essential role. The par value of each bond is one unit of the consumption good. Let $B_t = (B_{t,t+T}, \dots, B_{t,t+1})$ denote the T -dimensional bond price process. Define also $\hat{B}_t = (B_{t,t+T-1})$. We assume that the bonds are in zero net supply.

We denote by A the set of consumers.³ In order to take advantage

³ Mankiw and Zeldes (1991), Bertaut (1992), and Blume and Zeldes (1993) documented that only a small fraction of consumers hold any equity at all and that an even smaller fraction of consumers have substantive holdings in equities. We do not attempt to model here the behavior of nonstockholders but focus only on stockholders. The set A of consumers refers only to the subset of stockholders. Then the aggregate labor income of consumers should be understood as the aggregate labor income of consumers in the subset of stockholders.

of the law of large numbers, we assume that there is an infinite number of distinct consumers.

Consumer i is endowed with labor income I_{it} and consumes C_{it} at time t . The aggregate labor income is I_t , and the aggregate consumption is $C_t = I_t + D_t$. We assume that $I_t + D_t > 0$ for all times t .

Underlying our model is an increasing sequence $\{\phi_t: t = 0, 1, 2, \dots\}$ of information sets. We may choose to include in ϕ_t any random variables on which the information available at time t to an econometrician is based. At a minimum, we specify that ϕ_t contains the aggregate labor income history, the securities' dividend histories, and the securities' and discount bonds' price histories.

The information set $\mathcal{F}_t$ available to consumers at time t is the union of the information set ϕ_t and the disaggregated labor income history $\{I_{is}: i \in A, 0 \leq s \leq t\}$. Potentially, it is objectionable to require each consumer to keep track of the labor income history of every consumer in the economy. In our model, however, it turns out that the same equilibrium obtains whether consumers have access to other consumers' income histories or not.

At time t , consumer i holds a portfolio $\theta_{it} = \{\theta_{ijt}: j = 1, \dots, n\}$ of shares of securities. By assumption, consumers enter at period 0 with symmetric endowments in the securities. That is, $\theta_{i,-1} = \theta_{k,-1}$ for all (i, k) . At time t , consumer i also holds a portfolio $b_{it} = \{b_{ijt}: j = n + 1, \dots, n + T\}$ of discount bonds. By assumption, consumers enter period 0 with zero endowments of bonds; that is, $b_{i,-1} = (0, 0, \dots, 0)$. The budget constraints are

$$C_{it} = I_{it} + \theta_{i,t-1} \cdot (P_t + d_t) + b_{i,t-1} \cdot \hat{B}_t - \theta_{it} \cdot P_t - b_{it} \cdot B_t, \quad t = 0, 1, \dots \quad (1)$$

Of course, θ_{it} and b_{it} must be in the information set $\mathcal{F}_t$. We restrict consumers to bounded trading strategies.

Consumers have homogeneous preferences represented by a time- and state-separable von Neumann–Morgenstern utility function with a constant relative risk aversion coefficient, α , and a constant subjective discount rate, ρ , giving utility

$$E \left[(1 - \alpha)^{-1} \sum_{t=0}^{\infty} e^{-\rho t} C_{it}^{1-\alpha} \middle| \mathcal{F}_0 \right], \quad (2)$$

where $\alpha > 0$. (We associate $\alpha = 1$ with logarithmic preferences in the usual fashion.) An optimal strategy for consumer i is a strategy (θ_i, b_i, C_i) maximizing the utility given by (2) subject to the budget constraint (1).

An equilibrium is a security and bond price process (P, B) and optimal strategies $\{(\theta_i, b_i, C_i): i \in A\}$ for the consumers given (P, B)

such that the security and bond markets clear, meaning $\sum_{i \in A} \theta_{ijt} = 1$ and $\sum_{i \in A} b_{ijt} = 0$ for all j and t . Market clearing implies that $\sum_{i \in A} C_{it} = C_t \equiv I_t + D_t$ for all t .

We shall take as given the securities' dividend processes, d , the aggregate labor income process, I , and the parameters α and ρ of the consumers' homogeneous preferences. In the next section we present conditions that imply the existence of an equilibrium supporting given price processes for the securities and bonds, by judicious choice of the consumers' labor income processes.

III. Equilibrium

Since the absence of arbitrage is a necessary condition for the existence of an equilibrium, we assume outright that the given price processes for the securities and bonds are arbitrage-free. Under mild technical conditions that can be deduced from the work of Harrison and Kreps (1979), Kreps (1981), Duffie and Huang (1986), Delbaen and Schachermayer (1992), Clark (1993), Santos and Woodford (1994), or Schachermayer (1994), the absence of arbitrage (given only the econometrician's information) implies the existence for each t of a strictly positive (but not necessarily unique) M_t in the information set ϕ_t , such that, for any time t ,

$$P_{jt} = \frac{1}{M_t} E \left[\sum_{s=t+1}^{\infty} d_{js} M_s \middle| \phi_t \right], \quad j = 1, \dots, n, \quad (3)$$

and

$$B_{t,t+s} = \frac{1}{M_t} E \left[M_{t+s} \middle| \phi_t \right], \quad s = 1, \dots, T. \quad (4)$$

We shall call a process M , satisfying (3) and (4), a pricing kernel.

We assume the existence of a pricing kernel M satisfying the transversality-like condition

$$E[M_t] \rightarrow 0 \quad \text{as} \quad t \rightarrow \infty. \quad (5)$$

Condition (5) is innocuous, since it simply states that the price of every discount bond tends to zero as its maturity tends to infinity. We impose a second condition on M_t that is substantive.

PRICING KERNEL CONDITION.

$$\frac{M_{t+1}}{M_t} \geq e^{-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha}, \quad t = 0, 1, \dots \quad (6)$$

This condition implies an Euler *inequality* of aggregate consumption for every limited liability security, bond, or portfolio of securities and bonds with return R_{t+1} from time t to $t + 1$:

$$E \left[R_{t+1} e^{-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \middle| \phi_t \right] \leq 1, \quad R_{t+1} \geq 0. \quad (7)$$

(For example, the return of security j is $R_{jt} = [P_{j,t+1} + d_{j,t+1}]/P_{jt}$.) Extant diagnostic tests failed to reject equation (7).⁴ The pricing kernel condition remains, however, a hypothesis to be subjected to further empirical scrutiny.

Consumer i has labor income I_{it} at time t defined by

$$I_{it} = \delta_{it} C_t - D_t, \quad (8)$$

where

$$\delta_{it} = \exp \left[\sum_{s=1}^t \left(\eta_{is} y_s - \frac{y_s^2}{2} \right) \right], \quad (9)$$

$$y_t = \sqrt{\frac{2}{\alpha^2 + \alpha}} \left[\log \left(\frac{M_t}{M_{t-1}} \right) + \rho + \alpha \log \left(\frac{C_t}{C_{t-1}} \right) \right]^{1/2}, \quad (10)$$

and the “shocks” $\{\eta_{it}\}$ have the following properties: (a) distinct subsets of $\{\eta_{it}\}$ are independent and (b) for all i and t , η_{it} is standard normal and independent of $\mathcal{F}_{t-1}$ and y_t . Condition (6) guarantees that y_t is a well-defined process.

We want to set up our model so that the law of large numbers can be invoked to obtain $\sum_{i \in A} \delta_{it} = 1$, which would imply by (8) that the aggregate income I_t at time t is indeed equal to its definition, $\sum_{i \in A} I_{it}$. Since, for any standard normal variable η and any constant k , we have $E[\exp[\eta k - (k^2/2)]] = 1$, the law of large numbers implies almost surely, given the independence of y_t and the shocks $\{\eta_{it}\}$, that, for any sequence $\{\delta_{1t}, \delta_{2t}, \dots\}$ of the income ratios $\{\delta_{it}\}$ defined by (8), we have

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \delta_{nt} = 1.$$

⁴ Whereas the Euler *equalities* of aggregate consumption have been rejected by Hansen and Singleton (1982), Ferson and Constantinides (1991), Hansen and Jagannathan (1991), and several others, the Euler *inequalities* of aggregate consumption (7) have not. While addressing a different set of issues than consumer heterogeneity (namely, solvency constraints), Cochrane and Hansen (1992), Luttmer (1993), and He and Modest (1995) performed diagnostic tests of the Euler inequalities (7) and failed to reject them with economically plausible values of the risk aversion coefficient.

Thus, if one were to take a sequence of economies with increasing numbers of consumers, we would attain our goal in the limit. We prefer to deal with a particular economy rather than a sequence of economies and therefore place special structure on our set A of agents so that we can rigorously obtain the conclusion that, with probability one, $\sum_{i \in A} \delta_i = 1$. This is done by choosing the set A of consumers in a mathematically careful way, in conjunction with the underlying random variables. There are several alternative constructions that have been proved to work. Rather than going into the technical details here, we merely refer the reader to the results of Green (1989).⁵

The following proposition is our main result.

PROPOSITION 1. Under conditions (5) and (6), there exists an equilibrium with no trade that supports the given price processes of the securities and discount bonds.

The formal proof is given in Appendix A. We give below the gist of the argument. We first calculate the marginal rates of substitution of consumer i with no trade. We then calculate consumer i 's private valuation of security j under these rates of substitution and demonstrate equality of this value with the given price of security j . This demonstrates that no trade is indeed an equilibrium and that this equilibrium supports the given price processes.

Under the no-trade conjecture, the marginal rate of substitution of consumer i between dates t and $t + 1$ is

$$\begin{aligned} e^{-\rho} \left(\frac{C_{i,t+1}}{C_{it}} \right)^{-\alpha} &= e^{-\rho} \left(\frac{I_{i,t+1} + D_{t+1}}{I_{it} + D_t} \right)^{-\alpha} \\ &= e^{-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \exp \left[-\alpha \left(\eta_{i,t+1} y_{t+1} - \frac{y_{t+1}^2}{2} \right) \right]. \end{aligned}$$

Consumer i 's private valuation of security j at time t is thus

$$\hat{P}_{jt}(i) = E \left[(P_{j,t+1} + d_{j,t+1}) e^{-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \right]$$

⁵ Specifically, we take the set A of consumers to be a nonatomic measure space that is determined jointly with the probability space on which the random variables are defined. The sum $\sum_i \delta_i$ is then merely notation for the integral $\int_A \delta_i d\mu(i)$ for the associated measure μ defining weights on subsets of consumers. The typical unit interval "continuum" of agents with the usual equal weighting of intervals of equal lengths will *not* work. Aside from technicalities, we could have relied on the alternative models of a law of large numbers for large economies of Feldman and Gilles (1985), Judd (1985), Uhlig (1990), or Anderson (1991). There is a distinction here from Green's work in that we apply his results t by t , conditional on F_t . Since there is only a countable set of times to consider, these extensions of Green's results are straightforward.

$$\begin{aligned} & \times \exp \left[-\alpha \left(\eta_{i,t+1} y_{t+1} - \frac{y_{t+1}^2}{2} \right) \right] \Big| \mathcal{F}_t \Big] \\ & = E \left[(P_{j,t+1} + d_{j,t+1}) e^{-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} Z_{it} \Big| \mathcal{F}_t \right], \end{aligned}$$

with the law of iterated expectations, where

$$Z_{it} = E \left[\exp \left[-\alpha \left(\eta_{i,t+1} y_{t+1} - \frac{y_{t+1}^2}{2} \right) \right] \Big| \mathcal{F}_t \cup \{y_{t+1}\} \right].$$

Using the independence property *b* above, we calculate that

$$\begin{aligned} Z_{it} &= E \left[\exp \left[-\alpha \left(\eta_{i,t+1} y_{t+1} - \frac{y_{t+1}^2}{2} \right) \right] \Big| \mathcal{F}_t \cup \{y_{t+1}\} \right] \\ &= \exp \left[\frac{\alpha(\alpha + 1)}{2} y_{t+1}^2 \right] \\ &= e^{\rho} \left(\frac{C_{t+1}}{C_t} \right)^{\alpha} \left(\frac{M_{t+1}}{M_t} \right). \end{aligned}$$

Substituting Z_{it} into the expression for $\hat{P}_{jt}(i)$, we obtain

$$\hat{P}_{jt}(i) = E \left[(P_{j,t+1} + d_{j,t+1}) \frac{M_{t+1}}{M_t} \Big| \mathcal{F}_t \right].$$

From the definition of M ,

$$P_{jt} = E \left[(P_{j,t+1} + d_{j,t+1}) \frac{M_{t+1}}{M_t} \Big| \Phi_t \right].$$

Since Φ_t and $\mathcal{F}_t$ differ only by conditioning variables that play no role in computing these conditional expectations, it follows that $\hat{P}_{jt}(i) = P_{jt}$, completing the claim that the private valuation by consumer *i* of security *j* is independent of the consumer's identity, and is equal to the posted price P_{jt} in the market. (For the optimality of no trade, there is also a technical transversality argument left to App. A.) This verifies that no trade is indeed an equilibrium.

The no-trade implication of the equilibrium is counterfactual and merits discussion. The idiosyncratic income processes are modeled as in equations (8)–(10) with the unabashed goal to obtain a tractable equilibrium with a closed-form solution ($C_{it} = \delta_{it} C_t$) to the equilibrium consumption processes. The reader may find it more palatable to think of these idiosyncratic consumption processes as “posttrade”

consumption allocations, and it is this interpretation of the model that is pursued in the next section.

IV. Economic Interpretation

The main economic implication of proposition 1 is summarized by the Euler equation

$$E \left[R_{j,t+1} e^{-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \exp \left[\frac{\alpha(\alpha+1)}{2} y_{t+1}^2 \right] \middle| \phi_t \right] = 1, \quad (11)$$

where $R_{j,t+1} \equiv (P_{j,t+1} + d_{j,t+1})/P_{jt}$. To derive this, we first state the Euler equation of consumption of consumer i for security j :

$$E \left[R_{j,t+1} e^{-\rho} \left(\frac{C_{i,t+1}}{C_{it}} \right)^{-\alpha} \middle| \mathcal{F}_t \right] = 1. \quad (12)$$

We then write consumer i 's consumption as

$$C_{it} = I_{it} + D_t = \delta_{it} C_t, \quad (13)$$

which follows from the no-trade result of proposition 1 and equation (8). We substitute (13) into (12) and simplify by performing a set of calculations similar to those performed in the proof of proposition 1. This yields (11).

The term y_{t+1}^2 in equation (11) is interpreted as the variance of the cross-sectional distribution of $\log[(C_{i,t+1}/C_{t+1})/(C_{it}/C_t)]$. To see this, note that

$$\begin{aligned} \log \left(\frac{C_{i,t+1}/C_{t+1}}{C_{it}/C_t} \right) &= \log \left(\frac{\delta_{i,t+1}}{\delta_{it}} \right) \\ &= \eta_{i,t+1} y_{t+1} - \frac{y_{t+1}^2}{2} \quad (\text{by eq. [9]}) \\ &\sim N \left(-\frac{y_{t+1}^2}{2}, y_{t+1}^2 \right). \end{aligned} \quad (14)$$

The term y_{t+1}^2 may be estimated from panel data on consumption. Then equation (11) becomes testable. Note also that, since $y_{t+1}^2 \geq 0$, equation (11) implies equation (7), which was discussed earlier as a testable implication of the pricing kernel condition (6).

We consider some special cases of equation (11). If consumers are homogeneous, then $y_{t+1}^2 = 0$ and equation (11) reduces to the familiar Euler equation of consumption in a representative-consumer economy. If consumers are heterogeneous and

$$y_{t+1}^2 = a + b \log\left(\frac{C_{t+1}}{C_t}\right), \quad (15)$$

then equation (11) reduces to

$$E\left[R_{j,t+1} e^{-\hat{\rho}} \left(\frac{C_{t+1}}{C_t}\right)^{-\hat{\alpha}} \middle| \phi_t\right] = 1, \quad (16)$$

where

$$\hat{\rho} = \rho - \frac{\alpha(\alpha + 1)}{2} a \quad (17)$$

and

$$\hat{\alpha} = \alpha - \frac{\alpha(\alpha + 1)}{2} b. \quad (18)$$

We observe that the Euler equation of consumption is isomorphic to the Euler equation of a representative-consumer economy, but with the subjective discount rate and risk aversion coefficient modified as in equations (17) and (18). If an econometrician were to estimate a representative agent's Euler equations without explicitly accounting for consumer heterogeneity, the econometrician would be either overestimating or underestimating the subjective discount rate and the risk aversion coefficient.

If, for example, the variance of the cross-sectional distribution of consumption growth increases in economic downturns ($C_t/C_{t-1} < 1$), then b is negative and $\hat{\alpha} > \alpha$. An econometrician who does not take into account the consumer heterogeneity in estimating equation (16) would be overestimating the risk aversion coefficient. We stress, however, that consumer heterogeneity of the particular form represented by equation (15) is of limited economic interest because representative consumer Euler equations of the form (16) are rejected by the data even when the range of the parameters $\hat{\rho}$ and $\hat{\alpha}$ is unrestricted in the estimation.

We return to the general case and define $R_{F,t+1}$ to be the return from time t to $t + 1$ of a riskless security. Then we may express the expected excess return of security j as

$$E[R_{j,t+1} | \phi_t] - R_{F,t+1} = -\frac{\text{cov}(R_{j,t+1}, H_{t+1} | \phi_t)}{E[H_{t+1} | \phi_t]}, \quad (19)$$

where

$$H_{t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} \exp\left[\frac{\alpha(\alpha + 1)}{2} y_{t+1}^2\right].$$

A security commands a positive (negative) risk premium if its return has negative (positive) covariance with H_{t+1} . Thus an econometrician who is unaware of the existence of uninsurable idiosyncratic income risk may either overestimate or underestimate a security's expected excess return. This finding contradicts a statement in Weil (1992, proposition 3) that if utility has the power form (as modeled in our paper), an econometrician who is unaware of uninsurable idiosyncratic income risk will unambiguously understate the magnitude of the equity premium.

A fundamental problem with the Euler equation implied by a representative-consumer economy is that the aggregate consumption process covaries too little with the securities' returns. As an extreme example, consider the case in which aggregate consumption growth is a constant. The representative consumer's Euler equation then states that all assets' expected excess returns are zero. By contrast, the heterogeneous consumers' Euler equation (19) states that a security commands a positive (negative) risk premium if its return has negative (positive) covariance with y_{t+1}^2 . This suggests a potential source of the equity premium, a source that is assumed away in a representative-consumer economy.

V. The Cross-Sectional Distribution of Consumption

A key feature of our model is that shocks to consumers' income processes are persistent. Lack of persistence renders the pricing implications of the heterogeneous-consumer model similar to those of a homogeneous-consumer model, as demonstrated by Lucas (1991) and Telmer (1993).

In our model we have captured persistence by having idiosyncratic income shocks follow random walks, with the unintended implication that idiosyncratic incomes are nonstationary processes. Indeed, since income shocks are permanent and independently and identically distributed (i.i.d.) across consumers, the cross-sectional distributions of income and consumption would typically diverge in terms of various measures of dispersion such as the cross-sectional variance or the Gini coefficient.

It is an open question whether the cross-sectional distributions of income and consumption diverge or not. We proceed to demonstrate that our main result—that of the existence of idiosyncratic income processes resulting in an equilibrium supporting given price processes—does not hinge on the divergence of the cross-sectional distribution of the income and the consumption processes. We sketch the model and refer the reader to Appendix B for details.

We model consumers as having random lifetimes. At any time a consumer survives to the next period with a fixed probability. Survival is independent across consumers and across time. At death a consumer is replaced by an "heir" to the dynasty, whose idiosyncratic endowment process is initialized at zero and again follows a random walk. A key assumption is that a newborn member of a dynasty does not inherit the exiting member's security holdings. In each period, the exiting consumers' security holdings are redistributed equally among all newborn consumers across dynasties.

It is shown in Appendix B that a slightly modified version of proposition 1 holds. Also under plausible conditions, the cross-sectional distributions of income and consumption are stationary and have finite variance. In particular, one can calibrate the model to yield a cross-sectional distribution of consumption consistent with the distribution reported by Cutler and Katz (1991, table 10) from Consumer Expenditure Survey data.

VI. Concluding Remarks

In this paper we selected the period utility function for its convenient homotheticity properties in characterizing the equilibrium of a multiperiod, multiagent economy in a tractable way. The incidental property of convex marginal utility makes necessary the pricing kernel condition (6).⁶ It remains a challenge, at least to us, to explore tractable multiperiod economies of consumer heterogeneity that are free from the assumption of convex marginal utility and therefore from the associated bound imposed on the pricing kernel.

Appendix A

Optimality and Transversality

This Appendix shows that the Euler equation characterizes optimality. We adapt arguments from Duffie and Skiadas (1994), simplified as in Duffie (1992, chap. 4).

We call a process h a budget-feasible deviation for consumer i from the candidate optimum consumption process C_i if there exists a budget-feasible strategy of the form $(\theta, b, C_i + h)$. Our objective is to show, for any such h , that total infinite-horizon utility $U(C_i + h)$ is less than or equal to $U(C_i)$. Let $u(\cdot)$ denote the period utility. Concavity of u implies that $u(C_{it} + h_t) \leq u(C_{it})$

⁶ Kimball (1990) motivates the assumption of convex marginal utility in the context of precautionary savings. Mankiw (1986) and Weil (1992) have also identified the role of convex marginal utility in heterogeneous-consumer economies.

+ $u'(C_{it})h_t$ and therefore that

$$\begin{aligned} U(C_i + h) - U(C_i) &= E \left[\sum_{t=0}^{\infty} e^{-\rho t} [u(C_{it} + h_t) - u(C_{it})] \right] \\ &\leq \Delta(h) \equiv E \left[\sum_{t=0}^{\infty} e^{-\rho t} u'(C_{it}) h_t \right]. \end{aligned} \quad (\text{A1})$$

It is therefore enough to show that $\Delta(h) = 0$ for any budget-feasible deviation h . For convenience, let $\pi_t = e^{-\rho t} u'(C_{it})$ denote the “shadow price” process for consumption. The stochastic Euler equation, already demonstrated, is written as

$$E[(P_{t+1} + d_{t+1})\pi_{t+1} | \mathcal{F}_t] = \pi_t P_t \quad \text{almost surely.} \quad (\text{A2})$$

Likewise, for any t and $\tau > t$,

$$E[B_{t+1,\tau}\pi_{t+1} | \mathcal{F}_t] = B_{t,\tau}\pi_t \quad \text{almost surely.} \quad (\text{A3})$$

In order to obtain the budget-feasible consumption process $C_i + h$, the consumer must deviate from the candidate no-trade portfolio trading strategy by some risky-asset strategy φ and some bond trading strategy b , satisfying $\varphi_{-1} = 0$, $b_{-1} = 0$, and, for all t ,

$$h_t = \varphi_{t-1} \cdot (P_t + d_t) + b_{t-1} \cdot \hat{B}_t - \varphi_t \cdot P_t - b_t \cdot B_t.$$

From this, for any t , we have

$$\pi_t h_t = V_t - E[V_{t+1} | \mathcal{F}_t] \quad \text{almost surely,} \quad (\text{A4})$$

where $V_t = \pi_t[\varphi_{t-1} \cdot (P_t + d_t) + b_{t-1} \cdot \hat{B}_t]$, from (A2) and (A3). Adding (A4) from $t = 0$ to $t = T$ and using iterated expectations leaves

$$E \left[\sum_{t=0}^T \pi_t h_t \right] = V_0 - E[V_{T+1}]. \quad (\text{A5})$$

Next we show that $E[V_t] \rightarrow 0$ as $t \rightarrow \infty$. For this, we use the fact that the trading strategy is bounded, so that no more than some number k of any security or bond is ever held long or short, which implies, with the law of iterated expectations, that

$$\begin{aligned} |E[V_t]| &\leq k\bar{\theta} \cdot |E[\pi_t(P_t + d_t)]| + k\bar{b} \cdot E[\pi_t \hat{B}_t] \\ &= k\bar{\theta} \cdot |E[M_t(P_t + d_t)]| + k\bar{b} \cdot E[M_t \hat{B}_t] \\ &= k\bar{\theta} \cdot E \left[\sum_{s=t}^{\infty} M_s d_s \right] + k\bar{b} \cdot E[(M_t, M_{t+1}, \dots, M_{t+\bar{T}-1})], \end{aligned}$$

which converges to zero with t since $M_0 P_0 = E[\sum_{t=1}^{\infty} M_t d_t]$ is finite and since $E[M_t] \rightarrow 0$ by assumption. Indeed, then we have shown that $E[V_t] \rightarrow 0$. With this and the fact that $V_0 = 0$, we have $\Delta(h) = E[\sum_{t=0}^{\infty} \pi_t h_t] = 0$.

This confirms the assertion that any budget-feasible deviation strategy does not increase utility.

Appendix B

Cross-Sectional Distributions in a Dynasties Model

The model is modified from that of the text by assuming that, at any time t , any consumer will survive to the next period with probability q . At death, the consumer is replaced with another. Survival is independent across consumers and across time. This can be modeled more precisely as follows. We assume that there is a space A of "dynasties" of consumers. The k th consumer of dynasty $a \in A$ lives from the stopping time $\tau(a, k - 1)$ to the stopping time $\tau(a, k)$, where $\tau(a, k) = \sum_{j=0}^k t_{aj}$ and $\{t_{aj}: a \in A, 1 \leq j < \infty\}$ is a family of geometrically distributed random variables (lifetimes), each of parameter q , that is independent in both a and j and Φ -independent.

In order to account for the probability of death, we modify condition (6) as follows.

PRICING KERNEL CONDITION.

$$\frac{M_t}{M_{t+1}} \geq qe^{-\rho} \left(\frac{D_t + I_t}{D_{t+1} + I_{t+1}} \right)^{-\alpha}, \quad t = 1, 2, \dots \quad (\text{B1})$$

The k th consumer in dynasty $a \in A$ has the labor income process I_k^a defined by

$$\begin{aligned} I_{kt}^a &= (D_t + I_t) \exp \left[\sum_{s=\tau(a, k-1)+1}^t \left(\eta_{as} y_s - \frac{y_s^2}{2} \right) \right] - D_t, \\ &\quad \tau(a, k - 1) \leq t < \tau(a, k) \\ &= 0 \quad \text{otherwise,} \end{aligned} \quad (\text{B2})$$

where $\{\eta_{at}: a \in A, 1 \leq t < \infty\}$ is i.i.d. standard normal jointly in a and in t and is independent of Φ , and

$$y_t = \sqrt{\frac{2}{\alpha^2 + \alpha}} \left[\log \left(\frac{M_t}{M_{t-1}} \right) + \rho - \log q + \alpha \delta_{t-1, t} \right]^{1/2}, \quad (\text{B3})$$

where

$$\delta_{t-1, t} = \log(D_t + I_t) - \log(D_{t-1} + I_{t-1}) \quad (\text{B4})$$

is the growth rate of aggregate consumption.

As in the main model, we can choose our set A of consumers so that the law of large numbers can be applied, equating the total labor income I_t available at time t to the sum of the individual labor incomes across all consumers alive at a given time t , in that, for arbitrary k and a ,

$$\sum_{l=1}^{\infty} \sum_{a \in A} I_{at}^l = E[I_{at}^k | \tau(a, k - 1) \leq t < \tau(a, k), \phi_t] = I_t \quad \text{almost surely.} \quad (\text{B5})$$

This calculation uses the fact that the fraction of consumers that have been alive at time t for any given number m of periods is a constant $f_{t, m}$ almost

surely, and within that cohort, which is itself nonatomic, the idiosyncratic component of labor income is i.i.d. with mean zero. As in note 5, the notation $\Sigma_{a \in A}$ is to be interpreted as an integral over the space of dynasties.

All consumers have the information filtration $\mathcal{F} = \{\mathcal{F}_t: t = 0, 1, 2, \dots\}$ with

$$\mathcal{F}_t = \phi_t \cup \{(I_s, \eta_{as}, 1_{\tau_{ak} \leq s}): a \in A, 0 \leq s \leq t\},$$

meaning that, in addition to ϕ_t , consumers are informed of all births, deaths, and endowments as they occur.

We endow the k th consumer of dynasty a at his or her "birth date" $\tau(a, k - 1)$ with no bonds and with the security portfolio $\bar{\theta}$. The k th consumer in dynasty a can adopt a strategy (θ, b, C) in the form of a bounded $\mathbb{R}^n$ -valued $\mathcal{F}$ -adapted security trading process θ , a bounded $\mathcal{F}$ -adapted bond trading process b valued in $\mathbb{R}^T$, and a consumption process C that is subject to the analogue of (1) given by

$$\begin{aligned} C_t &= I_{kt}^a + \theta_{t-1} \cdot (P_t + d_t) + b_{t-1} \cdot \hat{B}_t - \theta_t \\ &\quad \cdot P_t - b_t \cdot B_t, \quad \tau(a, k - 1) \leq t < \tau(a, k), \\ &= 0 \quad \text{otherwise.} \end{aligned} \quad (\text{B6})$$

where $\theta_{\tau(a, k-1)} = \bar{\theta}$ and $b_{\tau(a, k-1)-1} = 0$. At death, a consumer's portfolio of bonds and securities is confiscated and used for purposes of endowing new agents at birth. (The model would work identically if each agent's portfolio were bequeathed to the successor agent of that dynasty, provided that there is no utility for bequests.)

A budget-feasible strategy (θ, b, C) for the k th consumer in dynasty a is optimal if there is no other budget-feasible strategy (θ', b', C') with $U(C') > U(C)$. Because of our informational assumptions and the fact that C_{kt}^a is constrained to be zero except during the lifetime of the k th consumer in dynasty a , maximization by that consumer of $U(C)$ is equivalent to maximization by that consumer (state by state, at birth) of

$$U_k^a(C) \equiv \frac{1}{1 - \alpha} E \left[\sum_{t=\tau(a, k-1)}^{\tau(a, k)-1} e^{-\rho[t - \tau(a, k-1)]} C_t^{1-\alpha} \middle| \mathcal{F}_{\tau(a, k-1)} \right]. \quad (\text{B7})$$

An equilibrium is a price process (P, B) and a collection $\{(\theta_k^a, b_k^a, C_k^a): a \in A, 1 \leq k < \infty\}$ of optimal strategies for the respective consumers such that, at any time t , markets clear:

$$\sum_{k=1}^{\infty} \sum_{a \in A} (\theta_{kt}^a, b_{kt}^a, C_{kt}^a) = (\bar{\theta}, \bar{b}, \bar{C}_t) \quad \text{almost surely.} \quad (\text{B8})$$

PROPOSITION B1. Let any pricing kernel M satisfying (B1) be fixed with $E[M_t] \rightarrow 0$. Suppose that I_k^a is defined by (B1) for each a and k . Then there exists an equilibrium with no trade and the prices implied by M .

The Euler equation (11) is modified to

$$E \left[R_{i,t+1} e^{-\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha}} \exp \left[\frac{\alpha(\alpha + 1)}{2} y_{t+1}^2 \right] \middle| \phi_t \right] = 1, \quad (\text{B9})$$

where $\hat{p} = p - \log q$. The economic interpretation of the Euler equation and the testable restrictions on securities' rates of return remain unchanged.

The cross-sectional consumption distribution at time t has a cumulative distribution function F_t defined by letting $F_t(r)$ denote the fraction of the consumer population alive at time t whose ratio of individual to per capita consumption,

$$s_{at} = \frac{C_{k(a,t),t}^a}{\bar{C}_t}, \quad (\text{B10})$$

is less than or equal to r .

Because of our independence assumptions, the law of large numbers implies that the age distribution converges over time to a steady-state cross-sectional age distribution that is the same as the probability distribution of the lifetime of a given consumer. This allows us to show that the cross-sectional consumption distribution converges, as a function of the history $y^t = (y_t, y_{t-1}, \dots, y_0, y_{-1}, y_{-2}, \dots)$ of "aggregate shocks" to a time-invariant function of y^t . (The "prehistory" $y_{-1}, y_{-2}, \dots$ is arbitrary and plays no role other than notational.)

In order to show this, we fix a dynasty a and compare the cross-sectional consumption distribution at time t with the probability distribution of the idiosyncratic consumption shock of the current generation of dynasty a ,

$$s_{at} \equiv \exp \left[\sum_{s=\tau(a,k(a,t)-1)}^t y_s \eta_{as} - \frac{y_s^2}{2} \right]. \quad (\text{B11})$$

Given the history of "aggregate shocks," these two distributions converge. This can be shown as follows. For a given infinite sequence $z = (z_0, z_1, \dots)$ of real numbers, a given integer $T \geq 0$, and a given number r , let $\pi_T(r|z)$ denote the probability that $\sum_{k=0}^T z_k \epsilon_k - (z_k^2/2)$ is less than or equal to r , where $\{\epsilon_k\}$ is i.i.d. standard normal. Given our assumptions, the probability that $s_{at} \leq r$, conditional on y , is almost surely

$$\Pi(r|t, y^t) \equiv \sum_{T=0}^{t-1} \pi_T(r|y^t) Q(t, T), \quad (\text{B12})$$

where $Q(t, T)$ denotes the probability that $t - \tau(a, k(a, t) - 1)$ is T , which does not depend on a or y^t , and converges exponentially fast with t to $(1 - q)q^T$. Since $\pi_T(r|z)$ is bounded above by one and below by zero, this implies that, as t goes to infinity, $\Pi(r|t, z)$ converges for each fixed z to

$$\Pi(r|z) \equiv (1 - q) \sum_{T=0}^{\infty} \pi_T(r|z) q^T.$$

Likewise, the probability that $s_{at} \leq r$ given y^t converges to $\Pi(r|y^t)$ almost surely.⁷ Since y^t itself depends on t (which is going to infinity), this statement

⁷ We could also consider the unconditional distribution of s_{at} . Under the assumption that y is a stationary process, $\log s_{at}$ defines a regenerative process, as defined by Asmussen (1987, p. 125). By Asmussen's proposition 1.1 (p. 126), s_{at} is therefore also

should be read as follows: For any r , the difference between $\Pi(r|y^t)$ and the conditional probability that $s_{at} \leq r$ given y^t goes to zero as t goes to infinity, almost surely.

By the law of large numbers, since $\{s_{at}: a \in A\}$ is i.i.d. given y^t , the cross-sectional distribution of consumption converges as well to $\Pi(\cdot|y^t)$. That is, for any r , the difference between $\Pi(r|y^t)$ and the fraction of current consumers whose consumption is no larger than a fraction r of average consumption goes to zero as t goes to infinity, almost surely.

Thus, as opposed to the model in the text, the asymptotic cross-sectional consumption distribution for the model in this Appendix exists for each state of the world. Of course, if y is itself an "exploding" process, the cross-sectional consumption distribution can have a "fat tail." Under purely technical conditions, however, this cross-sectional distribution can have a finite mean and variance, state by state. In order to see this, let us first compute, for each integer $T \geq 1$,

$$\begin{aligned} m_T^2(y^t) &\equiv E \left[\exp \left(\sum_{s=0}^{T-1} \epsilon_s y_{t-T} - \frac{y_{t-T}^2}{2} \right) \middle| y^t \right] \\ &= \exp \left(\sum_{s=0}^{T-1} y_{t-T}^2 \right) \\ &= \exp \left\{ \frac{2}{\alpha + \alpha^2} \left[(\rho - \log q) T + \log \left(\frac{M_t}{M_{t-T}} \right) + \alpha \log \left(\frac{C_t}{C_{t-T}} \right) \right] \right\}, \end{aligned}$$

using telescopic cancellation. Under the asymptotic cross-sectional distribution function $\Pi(\cdot|y^t)$, the cross-sectional second moment of consumption ratios with a given infinite history y^t is then

$$\begin{aligned} E[s_{at}^2|y^t] &= (1 - q) \sum_{T=0}^{\infty} q^T m_T^2(y^t) \\ &= (1 - q) \sum_{T=0}^{\infty} \exp(\zeta_t), \end{aligned}$$

where

$$\zeta_t = \frac{\alpha^2 + \alpha - 2}{\alpha^2 + \alpha} t \log q + \frac{2}{\alpha^2 + \alpha} \left[\log \left(\frac{M_t}{M_{t-T}} \right) + t\rho + \alpha \log \left(\frac{C_t}{C_{t-T}} \right) \right]. \quad (\text{B13})$$

a regenerative process. Since the survival probability is constant and is in $(0, 1)$, the renewal is aperiodic of finite expected lifetime, so Asmussen's corollary 1.5.5 (p. 128) implies that the probability distribution of s_{at} converges (in total variation norm). This is useful, e.g., if one is using asymptotic econometric methods that would exploit the stationarity of the probability distribution of cross-sectional consumption. It shows, for instance, that the fraction of consumers whose consumption ratio is in some interval is a random variable with a converging probability distribution. This fact can also be ascertained directly from the arguments above.

Whether or not $E[s_{at}^2|y^*]$ is finite (or in which states it is finite) depends on the rates of growth of M_t and C_t relative to $\log q$. Naturally, the greater the magnitude y_t^2 of the idiosyncratic consumption shocks in our model, other things equal, the greater the death probability necessary to induce finite cross-sectional moments or to reduce the cross-sectional consumption variance to a given level.

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