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ABSTRACT

This article provides the causes and symptoms of special repo rates in a competitive market for repurchase agreements. A repo rate is, in effect, an interest rate on loans collateralized by a specific instrument. A "special" is a repo rate significantly below prevailing market riskless interest rates. This article shows that specials can occur when those owning the collateral are inhibited, whether from legal or institutional requirements or from frictional costs, from supplying collateral into repurchase agreements. Specialness increases the equilibrium price for the underlying instrument by the present value of savings in borrowing costs associated with the repo specials.

THIS ARTICLE CHARACTERIZES COMPETITIVE repurchase, or "repo," markets, and the equilibrium relationship between repo rates and the underlying cash market prices. Repurchase markets are often called "financing" markets since they are effectively vehicles for collateralized borrowing that are often used to finance the purchase of the underlying collateral. This article provides causes and effects of "specials," meaning specific repo rates significantly below prevailing market interest rates for loans of similar maturity and credit risk.

The example of United States Treasury instruments is emphasized because of the extensive use of the Treasury repo market as a means of hedging against, or speculating on, changes in U.S. interest rates, and because of the high incidence of specials in Treasury repo rates. Numerous specific instruments, in particular on-the-run[1] issues, are frequently "on special." Specific

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[1] An "on-the-run" is the most recently issued treasury of a given maturity at issue.

Table I

Average Overnight Repo Specialness (Basis Points)

Overnight repo specialness is the difference between the overnight general collateral rate and the corresponding specific collateral rate. Figures shown are averages across reporting banks over the period April 26, 1988 to September 9, 1992 (1001 reporting days in all). No data were available for the period April 9, 1991 to August 23, 1991. In some cases, the reporting banks for the general collateral rate and for the specific collateral rate, respectively, may differ. The data are obtained from anonymous major dealers through The Catalyst Institute. The average specialness shown is the mean over the period, for the indicated instruments, of the daily average (across reporting dealers) of the difference between general and specific collateral rates. The set of reporting dealers varies over instrument and over time; the average number of reporting dealers for a given instrument is indicated. Also indicated is the average, over time, of the daily mean absolute deviation among the dealers, for days on which there is more than one reporting dealer. This statistic gives a sense of how much quote variation there is among dealers, but also reflects likely asynchronous reporting during the day. In rough terms, the mean-absolute deviations are on the order of magnitude of the bid-ask spread in repo markets, which is often (by casual observation and discussions with traders) in the vicinity of 10 basis points. The symbol "na" indicates that the necessary data are not available. The same summary statistics are reported, where available, for each available off-the-run issue, that is, the issue immediately preceding the current issue.

	Maturity									
On-The-Run	3 mo	6 mo	1 yr	2 yr	3 yr	4 yr	5 yr	7 yr	10 yr	30 yr
Average	31	25	33	38	46	26	62	57	87	59
Mean abs. dev.	6	5	6	10	5	na	13	7	7	6
No. banks report	1.9	1.9	1.9	2.8	1.9	1	2.8	1.9	1.9	1.9
Off-The-Run Avg.	na	na	na	na	na	na	30	22	33	20

repo rates are typically compared with the "general collateral rate," the repo rate quoted for Treasury instruments that are not "on special." The repo "specialness" of a given instrument is the difference between the general collateral rate and the specific collateral rate for that instrument. Table I provides some evidence on the average extent of repo specialness across a range of Treasuries, from a new set of data supplied to the author by several anonymous major dealers.

Figure 1 shows the behavior over time of the overnight repo specialness of the on-the-run 10-year Treasury note. The model in this article offers some explanation of the high degree of instability in specialness evident in Figure 1.

Specialness in a specific instrument can come about from the inability, opportunity cost, or transactions cost of supplying that instrument as collateral by certain of its owners. This paper shows that the extent of specialness, for a given total supply of the instrument, is increasing in the demand for short positions and in the degree to which owners of the instrument are inhibited from supplying it as collateral. Specialness is also shown to depend on the liquidity of the instrument: Of two otherwise identical instruments, that with the lower frictional costs of trading is the one more likely to be on special.

Not surprisingly, we find that expectations of specialness in interest rates collateralized by a specific instrument increase the equilibrium price of that

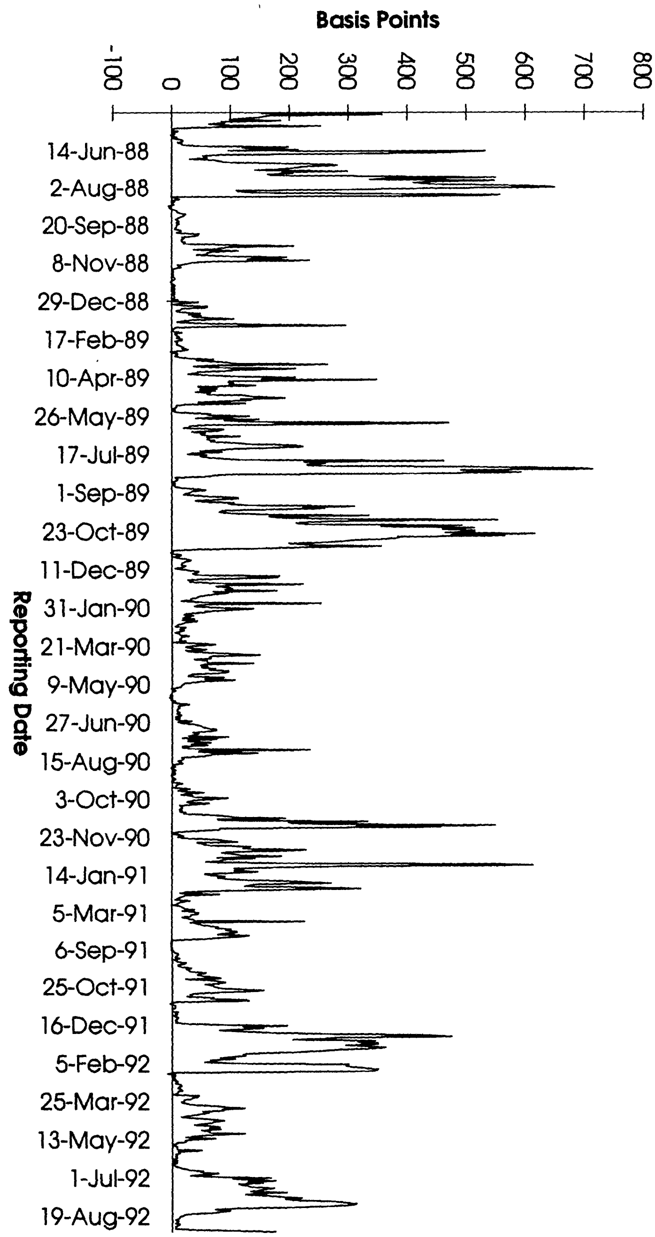

Figure 1. Overnight repo specialness for on-the-run 10-year U.S. Treasury notes. Overnight repo specialness is the difference between the overnight general collateral rate and the corresponding specific collateral rate for the on-the-run 10-year United States Treasury note. Figures shown are day-by-day averages across reporting banks over the period April 26, 1988 to September 9, 1992 (1001 reporting days in all). No data were available for the period April 9, 1991 to August 23, 1991. A negative specialness observation can arise due to the fact that the reporting banks for the general collateral rate and for the specific collateral rate, respectively, may differ. The data are obtained from anonymous major dealers through The Catalyst Institute.

instrument above the price that would prevail without specialness, or above the prices of substitute instruments that are not "on special." The difference between the general collateral rate and the specific collateral rate is effectively an additional dividend yield. The price of an issue "on special" exceeds the price that it would bear if not on special by the present value of the savings in future borrowing costs that can be attributed to the reduced repo rate. For example, consider a modification of the Cox-Ingersoll-Ross (1985) model of the term structure in which the specific collateral overnight repo rate of a given two-year 6.75 percent coupon note is assumed to be 80 percent of the general overnight interest rate, day by day for the month that it is the current issue, and is then assumed to remain "off special" for the remainder of its life. This specialness elevates the price of the note at issue by about 3/32, which is similar to the richness of typical current issues. Specialness may in fact explain a significant portion of the on-the-run effect that is sometimes attributed solely to the superior liquidity[2] of current issues. This has implications, that we consider briefly in Section IV, for estimating the current term structure of zero-coupon interest rates from the current prices of coupon Treasuries.

In an empirical study of the 2-year Treasury note auction, Jegadeesh (1993) tests the conjecture, based on winner's-curse reasoning, that those auctions with a higher ratio of amount bid to amount awarded (the "bid-to-cover" ratio) are typically associated with lower profits for those bidders who are awarded notes. Contrary to his conjecture, Jegadeesh finds a significant positive relationship between the bid-to-cover ratio and the profitability of positions obtained in the auction. Since a natural dealer strategy is to obtain a short position in the When-Issued (WI) market[3] and then cover that position at the auction,[4] an unexpectedly high bid-to-cover ratio is likely to be accompanied by an unexpectedly high number of short WI positions left uncovered at the auction. The model in this article would associate such an event with repo rates more special than expected prior to the auction, resulting in a higher profit to those awarded notes at the auction, other things being equal. This chain of reasoning is consistent with the above-mentioned empirical findings of Jegadeesh. The subsequent empirical findings of Sundaresan (1992) also support this explanation. In general, there are intimate links among the major Treasury markets (WI, auction, secondary cash, and repurchase markets) that make it difficult to analyze any of these markets in isolation of the others.

[2] For models of the price impact of liquidity alone, see Amihud and Mendelson (1991), Vayanos (1995), and Vayanos and Vila (1992).

[3] For U.S. Treasury notes, when-issued (WI) trading begins with the announcement, approximately two weeks before issue, of the size of the issue. The WI market is simply a forward market for delivery on the issue date. The auction at which the Treasury sells the issue occurs approximately one week before the issue date. The secondary market for cash trading of the issue, and the repurchase market, open the day of the issue and continue until the maturity of the note. Bonds and bills are handled in a similar fashion.

[4] See Cammack (1991), Simon (1994), Spindt and Stolz (1991), and Bikhchandandi and Huang (1992, 1993).

One may view repo specialness as a form of "convenience yield" in forward or futures markets. Forward prices are often depressed by an apparent advantage, beyond the dividend cash flow (net of storage costs) paid directly by the asset, of ownership and control of the underlying asset during the time period before delivery of a forward contract. This article provides a theoretical analysis of the convenience yield on treasury notes by embedding the basic institutional features of the repo market into a general equilibrium model.

Section I presents some of the institutional details of repurchase markets for U.S. Treasury instruments. Section II contains the basic ideas of the article along with a simple model of general equilibrium in a one-period setting. Section III extends the simple model by adding features that tie down the degree of specialness as a function of incentives to short and of the inhibitions of those with collateral from supplying it into repurchase agreements. Key among these extra features are variations in the degree of frictions among investors and across securities, and a convention, described in Section II, of staggered settlement between repo and cash markets.

Section IV extends the pricing implications of the model to a multi-period setting, and offers an application of the model to the estimation of the current zero-coupon yield curve from coupon-bond price data. Section V contains concluding remarks.

I. Treasury Note Repurchase Transactions

This section discusses some of the details of repurchase transactions (repos) involving U.S. Treasury notes, and describes the extent to which repo rates are determined by arbitrage. An example involves the purchase of a two-year treasury note with financing by a subsequent repo with another dealer. The numbers used are hypothetical.[5]

A. The Repo Concept

A repo is a single transaction combining a spot market sale with a simultaneous forward agreement to repurchase the underlying instrument at a later date, often the next day. If investor A obtains a repo from investor B, then investor B is said to have done a "reverse repo," a spot market purchase with a simultaneous forward agreement to resell the underlying instrument at the agreed date. An "overnight repo" is for next-day delivery. Any repo that is not overnight is said to be a "term repo."

Under generally accepted accounting practice, a repurchase transaction increases both the assets and liabilities of the borrower and lender. This can have adverse consequences for broker-dealers and others because of minimum capital or liquidity requirements, and has led to some repo transactions being replaced with other forms of agreements that are the same or similar for

[5] For general background on treasury repo markets, see Bollenbacher (1988), Clarke (1993), Lumpkin (1986), Rogg (1991), Russo (1991), and Stigum (1989).

economic purposes. These alternate arrangements include securities lending[6] and "dollar repos." A dollar repo is essentially[7] a simultaneous purchase and sale, with different date of settlement for purchase than for sale. Dollar repos are also apparently used by certain pension funds that are prohibited from engaging directly in repos. Another potential consequence of the adverse implications of repos for the minimum capital requirements of financial institutions is a large measure of quarterly seasonality in repo rates, indicating the impact of quarter-end monitoring[8] of balance sheets.

B. Repo Rates

It is normal to treat a repo as a collateralized loan. The initial sale value of the security (in the immediate cash sale portion of the repo) is the amount of the loan under this analogy; the forward price used in the repo is the payback on the loan. Since the security is held during the repo agreement by the effective lender, it is effectively collateral against default on the loan. Because a repo is effectively a loan, it is settled on the basis of an interest rate, called the "repo rate," rather than directly in terms of a forward price.

For example, consider for sake of simplicity a term repo with the unusually long term of three months. Suppose that the current market price of the underlying instrument is $100 million and the 3-month repo rate is 8 percent. Then the repo transaction is an agreement to sell the instrument for $100 million and to buy it back in three months for $102 million. (This figure is calculated as $100 million multiplied by 1.0 plus three-twelfths of the 8 percent annual interest rate; details on this calculation method are given in the next section.) The forward price implicit in the quoted repo rate is thus $102 million.

C. General and Special Repo Rates

Each Treasury instrument has its own repo rate, for each term. (There are also over-the-counter forward contracts for repurchase agreements.) At any given time, however, a significant number of treasuries have repo rates that are essentially the same and at a level above repo rates quoted on other treasuries. This "highest" repo rate is referred to as the "general collateral rate," and instruments whose repo rate is at or near the general collateral rate are referred to as "general collateral." The overnight general collateral rate is

[6] See Rogg (1991), page 245, and, for tax implications of the difference between repos and securities lending, Connors and Kroop (1993).

[7] The term 'dollar repo' is used in other ways. For example, a dollar repo sometimes refers to a repo involving the return of a different form of collateral. See Lumpkin (1986).

[8] See, for example, The *Wall Street Journal*, September 29, 1992, page C20, where one reads of a "squeeze" in the 10-year Treasury notes, with repo rates as low as 0.1 percent. "Another factor contributing to the squeeze situation is the approach of the end of the quarter, . . . underwriters might be rushing to cover their shorts before the quarter ends tomorrow because they can appear as liabilities in quarter-end financial statements." See Duffie and Singleton (1995) for an analysis of the quarterly seasonality in repo specialness, and the associated impact on swap spreads, which are quoted relative to Treasury yields, possibly depressed at quarter end from repo specialness.

commonly near the federal funds rate, the overnight market clearing interest rate in the federal funds market. As we shall later see, this is a natural state of affairs. Any repo rates that are nontrivially below the general collateral rate are called "special," although the range of instruments that are given special repo rates is often large, and usually includes the current issue of each note and bond as well as many of the more actively traded older issues. The term "special" should not, therefore, be interpreted as "abnormal."

The fact that all treasuries are equally backed by the "full faith and credit" of the government, and are therefore considered default-free, does not mean that they are equally good as collateral. For example, a repo by investor A with dealer B followed by a dramatic decline in the market value of the underlying instrument can expose dealer B to default by investor A. Since longer maturity bonds have more price sensitivity to interest rate changes than short maturity notes, they can therefore involve greater potential repo defaults. This default risk[9] is sometimes mitigated by a "haircut" that is applied to the market value of the instrument for repo purposes. (See Section I.E for details.) Although repo rates could, in principle, vary from instrument to instrument based only on the interest rate risk of the instrument, most variation in U.S. Treasury repo rates is based instead on the demand and supply for particular forms of repo collateral, particularly given the common practice of shorting, typically via reverse repo combined with sale of the purchased instrument. Russo (1991) gives an extensive example of how the need to cover short positions could lead to a high demand for reverse repos, causing the repo market to reach equilibrium, other things being equal, by a reduction in the repo rate. A famous example in which repo rates for 9.25 percent 30-year Treasury bonds plunged well below general collateral rates in 1986 is described by Russo (1991) and by Cornell and Shapiro (1989). There are frequent press accounts[10] of "squeezes" accompanied by significant differences between general and special repo rates.

[9] For more details on default in repo agreements, see Cooke (1994) regarding Wallace Smith Trust Company and European repo market practice, Hogg (1995) regarding Brady bond repos, Singh (1994) regarding Drysdale Government securities, and Sollinger (1994) regarding the use of tri-party repos to mitigate default risk.

[10] See, for example, page C1 of the *Wall Street Journal*, January 8, 1992. Reports of specials in five-year U. S. Treasury notes appear in the *Wall Street Journal* on June 30, 1992 and August 20, 1992 (page C1 in each case). See also the comments of Assistant Secretary of the Treasury Jerome Powell regarding the circumstances surrounding the reissue of 10-year Treasury notes in November, 1992, as reported in The *New York Times*, November 4, 1992, as well as letters from Federal Reserve Bank of New York President Gerald Corrigan, and from Federal Reserve System Chairman Alan Greenspan, to Edward Markey, Chairman of The House Subcommittee on Telecommunications and Finance, dated January 25, 1993 and February 1, 1993, respectively. See The *Wall Street Journal*, April 16, 1993, under "Credit Markets," for a story on specials beginning in early April 1993 on the current 30-year U.S. bond. This "squeeze" which continued well into early May, at repo rates close to zero throughout, was attributed by some to the Treasury's expected (and ultimately realized on May 5) announcement of a reduction in 30-year bond issues. See, for example, *Bridgewater Daily Observations*, written by Dan Bernstein and Ross Waller, May 5, 1993, page 1. This explanation is puzzling, since future scarcity of the 30-year bond would not directly imply reluctance by those holding the current 30-year bond to place their bonds into repurchase agreements. See, also, Siconolfi (1991) regarding the "Solomon squeeze of 1991."

Given the role of shorts in the equilibrium for repurchase agreements, it is not surprising that current-issue notes and bonds are frequently assigned special repo rates. Current issues are those most often used for hedging or speculative investing, because they have the greatest liquidity of treasuries with similar maturities. See, for example, Amihud and Mendelson (1991). As time passes from the auction date for a given instrument, trading tends to become less active as investors who are more likely to hold the instrument for longer time periods gradually acquire an increasing fraction of the issue. Beim (1992) empirically studies this effect. The amount of the instrument that is actively traded in the daily cash and repo markets falls over time, making it more difficult to quickly buy and sell large quantities at a small bid-ask spread. In fact, it is common for traders to roll all or a large portion of their positions into each successive current issue (for empirical evidence on the pattern of repo specialness over the life of a current issue, see Sundaresan (1992)). This tendency for reduced liquidity over time is to some extent a self-fulfilling prophecy, since expectations of lower liquidity will themselves reduce liquidity.

D. The Cash Market Transaction in Detail

Consider, for concreteness, a purchase by Investor A of 6.5 percent Treasury notes of a particular maturity date from Dealer B. Such a transaction is typically negotiated by telephone in a matter of a few seconds. Suppose the sale occurs on a Tuesday that is 80 days since the last coupon date, and 101 days from the next. (Coupons of 3.25 percent of face value are paid roughly every half year.) The transaction price is $101\frac{3}{32}$. Investor A is now obligated to pay to the seller, normally on the next day, the price plus accrued interest to that day, for a total cash payment of

$$\$1{,}000{,}000 \times \left(101\,\frac{3}{32} + \frac{81}{181} \times 3.25\right) = \$102{,}548{,}169.89.$$

While there are some variations from the convention of next-day settlement in the cash market, they require special arrangements. A same-day settlement market exists, in effect, but is much less liquid than the next-day settlement market, probably for the very reason that next-day settlement allows the buyer a period of time to arrange financing of the purchase cost, often through a repurchase agreement. As a convention, then, the quoted cash market price is actually a one-day forward price, net of accrued interest.

Rather than direct payments between buyer and seller, payments would typically be made to and from accounts of the buyer and seller at their respective clearing banks. The cash payment would be made from the buyer's clearing account and into the seller's clearing account, and likewise for the transfer of securities (on a book entry basis). The transfers are made during that day via the Federal funds ("fed funds") wire service. Since the fed funds wire service can handle amounts of at most \$50 million, this particular transfer would be done in three pieces. Each transfer involves a small transactions fee paid to the clearing house of, say, \$10 to \$20.

E. The Repurchase Agreement in Detail

Suppose Investor A finances the cost of these notes by making an overnight repurchase agreement with Dealer C (who could actually be the same Dealer B who sold the notes to Investor A). On the Wednesday morning after the sale, suppose that Dealer C quotes Investor A a repo rate of 4.5 percent for these notes. The new note price quoted by Dealer B for repo purposes is $101\frac{7}{32}$. Suppose Dealer C also requires a haircut of 1 percent. (Haircuts are often not required on short maturity instruments, or after a good business relationship has been established between the parties.) Instructions for the repurchase agreement are given to Investor A's clearing house that morning, for transfer on the same day. The net cash to be received by Investor A from Dealer C (via their respective clearing houses) on Wednesday as part of their repurchase agreement is

$$\$1{,}000{,}000 \times \frac{1}{1.01} \times \left(101\,\frac{7}{32} + \frac{81}{181} \times 3.25\right) = \$101{,}656{,}603.90.$$

Investor A's clearing house will thus have the following list of transactions to perform on Wednesday:

(a) Pay Dealer B \$102,548,169.89 in cash.
(b) Accept \$100 million of the 6.5 percent notes from Dealer B.
(c) Transfer \$100 million of the 6.5 percent notes to Dealer C.
(d) Receive \$101,656,603.90 in cash from Dealer C.

In this example, the accrued interest is included[11] in the base price of the repurchase agreement.

On Thursday morning, Investor A and Dealer C could terminate their overnight repurchase agreement, in which case Investor A pays

$$\$101{,}656{,}603.90 \times \left(1 + \frac{0.045}{360}\right) = \$101{,}669{,}311.00$$

in cash (that day, again via clearing house and fed funds wire) to Dealer C in return for the same \$100 million in notes. This second leg of the repo transaction, like the first, is treated just like a cash market purchase and sale for clearing and fed funds wire purposes.

On the other hand, it would not be unusual for Dealer C and Investor A to renew their repurchase agreement on Thursday morning for another day. In this case, the base price of the note for repo purposes would often remain at the

[11] See Lumpkin (1986), Footnote 4, regarding how the practice of including accrued interest has changed since the failures of Lombard-Wall and Drysdale Government Securities in 1982. For additional details, see Singh (1994).

original price, in this case 101 7/32. If the repo is renewed at a rate of 4.75 percent, the amount payable on Friday is

$$\$101{,}656{,}603.90 \times \left(1 + \frac{0.045}{360} + \frac{0.0475}{360}\right) = \$101{,}682{,}724.00.$$

Note that the interest is not compounding, which gives a slight advantage to the effective borrower (Investor A) should the repo be repeatedly renewed in this fashion for many days. This "open repo" arrangement eliminates the need to repeatedly transfer instruments for cash on daily basis, and pay the associated clearing fees. Eventually, an open repo will be closed or reduced in size, or a significant change in the market price for the underlying instrument will trigger a reset of the base repo price. (Not to reset in this case would expose one side or the other of the repo to a greater-than-necessary default.) Any of these events causes a closing and reopening (if necessary) of the repurchase agreement, and the repo calculation begins anew.

From a Friday to a Monday, the interest portion of the repurchase price is merely 3 times the overnight interest as calculated above, and likewise for business holidays, based on the length of the holiday.

F. Arbitrage and Special Repo Rates

The cost-of-carry formula for forward prices suggests that, for any instrument that can be sold short at its current market price, the overnight repo rate is theoretically the same as the overnight market interest rate. As we shall see, this theoretical relationship need not apply with special repo rates.

Suppose, for example, that a given instrument sells today for P and that the relevant overnight interest rate for borrowing or lending is r. If the instrument can be sold short at the spot price P, then the cost-of-carry formula for the overnight forward price is then $F = P(1 + r)$. This is an arbitrage-based formula, in that if $F > P(1 + r)$ one could commit to deliver forward, purchase the underlying instrument for P, and borrow the purchase cost overnight at the interest rate r. The payback on the loan is $P(1 + r)$. The instrument can be delivered against the forward commitment, for a theoretical risk-free profit of $F - P(1 + r) > 0$. Likewise, if $F < P(1 + r)$, one could buy forward, short the instrument, invest the sale value P at the overnight interest rate, and make an arbitrage profit the next day of $P(1 + r - F > 0$. Thus absence of arbitrage seems to imply that $F = P(1 + r)$. Since the forward price F is calculated from the repo rate R as $F = P(1 + R)$, if the cost-of-carry arguments carry over to the repo market, we would have $R = r$. (An appendix deals with the effect on these calculations of same-day settlement in the repo market versus next-day settlement in the cash market.)

In order to judge by example how the cost-of-carry approach may or may not apply to repo markets, consider the situation on Wednesday morning in the example of the previous section. Suppose the riskless interest rate for overnight borrowing available to market participants is 4.0 percent at the same time that the repo transaction can be made. (For purposes of discussion we can take the overnight interest rate to be near[12] the fed funds rate, since certain banks have access to this rate and can deal freely in repurchase agreements.) Then Dealer C could perform the reverse repo (as indicated above) with Investor A and immediately borrow the $101,656,603.90 paid to Investor A on Wednesday, with a payback of

$$\$101{,}656{,}603.90 \times \left(1 + \frac{0.04}{360}\right) = \$101{,}667{,}899.10$$

on Thursday. Since the receivable on the repo is $101,669,311.00, Dealer C nets a profit of $1,411.90 on Thursday. Barring defaults on any of the transactions, the profit requires no investment and is riskless. The incentive to carry out such "arbitrage" transactions would therefore drive the repo rate down to the lowest borrowing rate available to those in a position to reverse in the securities. For practical purposes, we can take this upper bound on repo rates to be the general collateral rate, which is typically at or near the market clearing fed funds rate for uncollateralized borrowing.[13]

There is no obvious arbitrage argument, however, that works the other way, pushing repo rates on specific issues up to a benchmark interest rate. Suppose, for example, that the 6.5 percent Treasury Notes are "on special" in the repo market, and that the repo rate of 4.5 percent arranged on Wednesday morning between Investor A and Dealer B is actually below the prevailing general borrowing rate of that moment, say 5.0 percent. Owners of these notes are in a position to earn rents from their special ability to offer them as collateral. They can be offered for repo at 4.5 percent, while the proceeds of the repo could be lent at 5.0 percent (or used to reduce other borrowings at higher interest rates) for a net rent of at least $1,391.76. This rent can be viewed as an effective reduction in the costs of financing the Treasury note position below that which would apply at prevailing riskless interest rates. There is no sense in which this is an arbitrage, since the potential scale of such a transaction is limited by the size of the investor's note position. If the position available for repo is reduced to zero, the only way to place additional notes into repurchase agreements (given the standard of same-day delivery in the repo market) is to obtain those notes by reverse repurchase agreements during that day's repo market, hopefully at a higher average repo rate. This is called "trading repo"

[12] Since the fed funds market is for unsecured loans, the fed funds rate is not literally a riskless rate. See, for example, Goodfriend and Whelpley (1986).

[13] The connection between the fed funds rate and the overnight general collateral rate is explored empirically by Griffiths and Winters (1995). They find that these two markets are integrated to the extent that the 2-week settlement cycle in regulatory oversight of fed funds balances causes seasonality in both the fed funds rate and the overnight general collateral rate for government bonds. In particular, both are elevated just prior to the settlement Wednesday of the cycle.

and is a speculative activity, not an arbitrage, since there is no guarantee that a special repo rate will climb toward general rates in the course of a day.

In summary, arbitrage incentives will drive repo rates down to or below the lowest alternative borrowing rates available to those in a position to do reverse repos, but the converse is not true. Below this lowest borrowing rate, usually the general collateral rate, the repo rate on a given issue is determined by general supply and demand conditions in the repo market. Significant short positions (usually obtained through reverse repo and sale) in a given issue can drive repo rates in that issue well below general collateral rates unless offsetting repo collateral supply is readily available. Owners of the issue may be unwilling or legally unable to offer their collateral into repurchase agreements. The repo rate is, in principle, merely that rate at which repo demand and supply are equal.

The market for treasury repurchase agreements is in fact conducted over the counter, so it is somewhat mis-leading to speak here of "the" market-clearing rate. Something along the lines of Diamond's (1982) "coconuts" model, with search costs, may also help to explain the extent of repo specials.

G. Fail Penalties and Lower Bounds on Repo Rates

A legal requirement to cover short positions, come what may, could drive repo rates well below zero. That is, if failure to satisfy a repurchase agreement to deliver notes were to be viewed as a formal default, then, in order to avoid default, the short would be willing to offer collateralized loans at *negative* repo rates in order to reverse in notes that are not otherwise available. In fact, failure to deliver collateral under a standard repurchase agreement is not viewed as default, but is instead covered under the terms of a repurchase agreement by requiring the short, in effect, to renew the agreement at a repo rate of zero (See, for example, Rogg (1991)). This limits the extent to which repo rates can become negative, since a sufficiently negative repo rate will induce the short to fail and obtain a more favorable rate of zero. Suppose, for example, that a given trader can only obtain collateral needed to fill a previous repo agreement at a new repo rate of R. If he does this, he will immediately recover the amount P due on the original repo loan, and will be lending the new market value P' of the collateral at R for a payback of $P'(1 + R)$ on the next day. The net present value of this decision not to fail, at a cost-of-capital rate c is

$$V = P - P' + \frac{1}{1 + c} P'(1 + R).$$

The alternative is to fail, and receive P in the next period, for a net present value of

$$V_F = \frac{P}{1 + c}.$$

It follows that a fail is worthwhile if and only if $V_F > V$, or equivalently,

$$R < c\left(1 - \frac{P}{P'}\right).$$

If the value of the collateral has not changed ($P = P'$), a rational fail will occur when the repo rate R is zero or below, regardless of the cost of capital of the trader. For small time periods, P/P' will be close to 1, and fails will occur roughly at zero repo rates. If the new market value P' is significantly lower than P, then R could become somewhat negative before a fail is induced. (As we show in this article, the new price P' of the collateral is itself increasing as R declines relative to the general collateral rate; however, this effect would not normally be large enough to change the conclusion significantly.) This analysis of course neglects credit risk and the reputational cost[14] to the trader of failure to deliver collateral due on a repo. There have also been instances in which significantly negative repo rates have been offered in order to obtain collateral for other purposes. For example, one trader recounted an instance in which a specific repo rate of −21 percent was offered in order to obtain Treasury notes needed to fill a delivery requirement on a futures contract, failure of which would have had severe adverse consequences. A Treasury instrument that is the cheapest to deliver, by a wide margin, against a futures contract with delivery substitutions could also have significantly negative repo rates, given that the value of owning that particular instrument at the futures delivery date may be temporarily inflated by the contractual formula for pricing delivery substitutions.

Also, if one needs to reverse in only a relatively small quantity of securities in order to completely cover a large short position, then one might offer a significantly negative[15] rate on the small remaining amount required so as to avoid a fail on the larger total amount. Presumably, such situations would be isolated and limited by competition among those with collateral, who would all be anxious for loans at a negative interest rate.

II. The Basic Model

We discuss a basic model of equilibrium in cash and repurchase markets, and its implications. For simplicity, we ignore many institutional details such as the opportunity to fail described in the previous subsection. We also ignore, until Section III, the staggered settlement between cash and repo markets.

[14] Conversations with traders indicate that there is not a significant loss of goodwill for failure to deliver; the decision is understood by market participants to be one of relative profitability of the fail. This is not necessarily the case in foreign markets, such as Japan, where failure may have adverse business consequences.

[15] I am grateful to Mark Fisher for this observation.

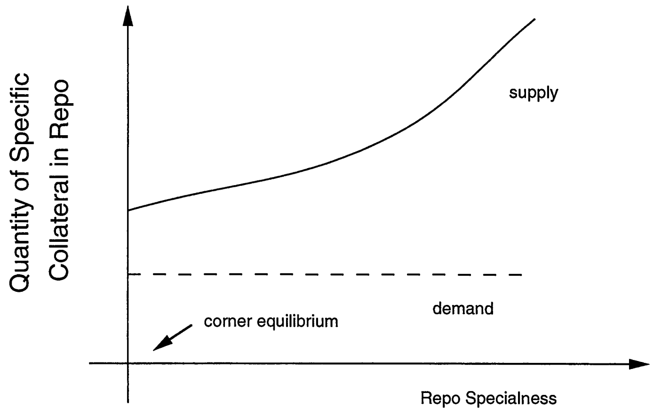

Figure 2. Normal repo rate. The supply of and demand for placement of securities into repurchase agreements, as functions of the repo specialness, are shown in this figure to generate a corner equilibrium, in which there is a larger than sufficient supply to meet the demand at zero specialness. This is a partial equilibrium, fixing investors' holdings of the underlying securities.

A. The Intuitive Model

Figure 2 illustrates a "normal" situation in the repo market of a given instrument. The horizontal axis measures "specialness," the difference between the general collateral rate and the specific collateral rate. Specialness is thus an index of the cost of renting specific collateral. The vertical axis shows the supply and demand of repo collateral from longs and shorts, respectively, given their positions in the cash market for the underlying instrument. Figures 2 thus illustrates a partial equilibrium determination of equilibrium repo specialness, holding fixed investors' positions in the underlying asset market.

Those short in the cash market for the instrument are assumed to obtain their positions by "reversing in" and then selling instruments. (This is almost universally the case in U.S. Treasury markets, as other methods of shorting typically involve greater transactions costs.) We also assume for simplicity that all repo is overnight, rather than term. Until their positions are offset, shorts will therefore continue to reverse in collateral day-by-day in order to meet their requirements to return collateral to its original source. For a given size of short position in the cash market, the quantity of specific collateral that must be reversed in by shorts is therefore inelastic to the specialness of the issue. One should note that an increase in specialness does *not* make it more attractive to offset a short position by buying the underlying asset outright, since the price at which the underlying asset can be purchased in the cash market increases dollar-for-dollar with the extra costs associated with specials in the repo market. This assertion is demonstrated below in Proposition 1.

With simultaneous settlement of the cash and repo markets, specials would be mitigated by the ability of shorts to meet their same-day requirement to deliver collateral by making cash market purchases. With same-day settlement in repo markets, but next-day settlement in cash markets, however, shorts with a given amount of the instrument due to be returned on a given day that was not completely offset by cash market purchases the *previous day* will be forced into reverse repo. Let us suppose, for example, that on a given day that a given firm A needs an additional 100 million dollars (face value) in Treasury notes to meet its repo requirements, after accounting for notes to be received that day in the cash market. Let us also suppose, for simplicity, that there is only one other trader, B, in the market. If repo markets clear only once during the day, as implicitly assumed in our model to follow, then the only source of the 100 million in collateral for firm A is a reverse-repo from firm B for the full 100 million. If, however, repo markets can clear arbitrarily often in a single day, then an arbitrarily small amount of the collateral would meet the needs of the repo market, by virtue of the following[16] series of transactions. Firm A reverses in, say, \$1 million in collateral from firm B, then returns the same collateral back to B, reducing its remaining delivery requirement to \$99 million. Then A again reverses in the same \$1 million from B, and again returns it, reducing its remaining delivery requirement to \$98 million, and so on. By making 100 such rounds of transactions, A can satisfy its delivery requirement for today, leaving a new requirement to deliver \$100 million the next day. Only the original \$1 million in actual notes was ever needed to be placed in repo agreements by B. Of course, if the repo market can clear twice as many times per day, only \$0.5 million of original collateral is needed. In fact, there are limits on clearing frequency and on the ability of shorts and longs to coordinate their trades given the search costs to be expected in a relatively secretive market. This suggests some nontrivial minimum demand for collateral, analogous to theories of the demand for money based on its velocity of circulation. Thus, staggered settlement between cash and repo can exacerbate repo specials.

As for those with long positions in the cash market, some will routinely supply collateral to the repurchase market at any specific collateral rate, because this is their sole available method for financing long positions. Some owners of collateral have effective transactions costs for supplying collateral. Other longs will not supply their collateral to the repo market due to legal and other institutional barriers. Consider the following examples:

(a) Certain insurance companies, pension funds, and mutual funds are precluded from placing certain of their Treasuries into repurchase agreements.
(b) A *tri-party repo*, in which a third party holds the collateral involved in a repurchase agreement involving two other parties, effectively places that collateral out of reach of the repo market for the duration of the tri-party[17] agreement, since the party holding the collateral is normally contractually obligated to maintain possession of the collateral.

[16] This idea, and the significance of the velocity of circulation, were pointed out to me by Mark Fisher.

[17] For details of the tri-party repo market, see Sollinger (1994). Cooke (1994) estimates that 35 percent of U.S. repurchase agreements are of a tri-party nature.

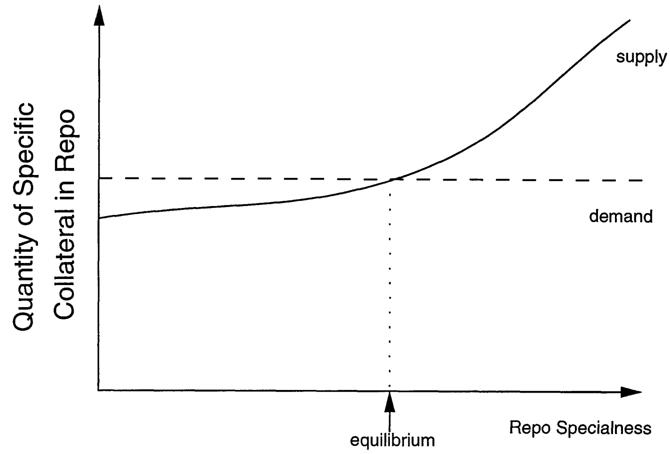

Figure 3. Special repo rate. The supply of and demand for placement of securities into repurchase agreements, as functions of the repo specialness, are shown to cross in this figure at an interior equilibrium, corresponding to positive specialness. This is a partial equilibrium, fixing investors holdings' of the underlying securities.

(c) Certain other types of repo transactions, such as letter repos and certain forms of securities lending, also preclude further use of the collateral (often called "re-hypothecation," including subsequent repo agreements) during the period of the original agreement.
(d) Dealers often enter commitments to deliver portfolios, of a particular mix, of treasuries that are designed for a special purpose, such as the defeasance of a municipal bond. Because of the risk associated with failure to deliver a portfolio with the required mix, it would often be undesirable to place certain elements of the portfolio into repo agreements from which they might not be returned (in the case of a fail).
(e) When treasuries are stripped, the underlying instrument disappears from the pool of collateral available to the repo market.

Some long position holders, such as certain dealers "trading repo," are able to supply repo collateral with little frictional costs, but act with discretion, sometimes conjecturing that providing only a portion of their inventory into specific collateral repurchase agreements at the current specific repo rate may leave opportunities for lower specific collateral borrowing rates later in the day. (In a noncompetitive model, which we will not examine here, some might even withhold collateral monopolistically or oligopolistically in order to profit from "squeezing" shorts.) In many cases, however, given the accounting identity that the total of long positions exceeds the total of short positions by the size of the issue, the supply of collateral is so plentiful relative to the demands of shorts to reverse in collateral that the equilibrium interest rate in the specific collateral repo market is at the general collateral rate, the corner solution shown in Figure 2.

In other cases, as illustrated in Figure 3, there is a large demand for repo collateral by shorts, relative to the willingness to cheaply supply collateral into repurchase agreements by longs, possibly pushing the equilibrium specific collateral rate to special low levels. The specialness could be as great as the transaction cost for supplying repo collateral of a "marginal" long investor. The intuition embedded in Figures 2 and 3 is formalized in the models to follow in this section and in Section III.

B. A Formal Model of Repo Markets

Consider a market with four possible investments:

(a) An asset, called "specific collateral," whose value in the next period is X, a nondegenerate random variable. The current price for buying specific collateral is P'. Specific collateral can only be sold short by reverse repo (see (d) below) combined with simultaneous sale of the reversed-in specific collateral.
(b) A second asset, called "general collateral," whose value in the next period is also X. The current price of general collateral is P.
(c) A riskless one-period bond paying one unit of account in the next period. The current interest rate for borrowing or lending (selling or buying this bond) is r.
(d) Reverse repurchase of the specific collateral. This investment means buying the asset for P' and agreeing to resell at the next period for $P'(1 + R)$. That is, the repo rate (collateralized lending rate) is R. Repurchase is the negative of this activity, and can only be undertaken by placing into repurchase agreements specific collateral on hand at a price P', and agreeing to repurchase at the price $P'(1 + R)$.

In this model, the market clears once; there is no possibility for profitable repo trading at different repo rates.

For any given agent with an endowment of q units of the specific collateral, a feasible transaction portfolio is some 4-tuple (a, b, c, d) representing net trades in these four investments, respectively, such that

$$aP' + bP + \frac{c}{1 + r} + \frac{d}{1 + R} = 0, \tag{1}$$

the usual notion of budget feasibility, and also satisfying

$$d < 0 \Rightarrow (a + q)P' \geq -\frac{d}{1 + R} \tag{2a}$$

$$a + q < 0 \Rightarrow (a + q)P' = -\frac{d}{1 + R}, \tag{2b}$$

representing the constraint (2a) that only those with specific collateral may use collateralized borrowing, and the additional requirement (2b) that one may short specific collateral only by reversing it in, forcing one to lend at the specific collateral rate. We assume for simplicity that only specific collateral is endowed; this is not crucial for the results. The set of budget-feasible transactions portfolios given endowment q is denoted $B(q)$. This definition of the budget set implicitly assumes simultaneous repo and cash market settlement. Sections III.C and III.D consider the impact of staggered settlement between repo and cash markets.

By adding analogues of equations (2a) and (2b) for general collateral, we could also think of r as the general collateral rate, that is, the interest rate on loans collateralized by the general collateral asset. Essentially all of our conclusions follow with this interpretation, and indeed we briefly extend our model to this case at the end of Section IV.A in order to show that the more liquid of the two forms of collateral will be the asset more likely on special in repo markets.

C. The Agents

For simplicity, we consider a market with three types of agents, described as follows:

(L) An investor called the "Long" is initially endowed with $q_L > 0$ units of specific collateral. This investor has transactions cost coefficients λ for collateralized borrowing, τ_a for trading specific collateral, and τ_b for trading general collateral. The Long's problem is therefore

$$\max_{(a,b,c,d)\in B(q_L)} U_L[(a + q_L + b - x_L)X - \tau_a|a| - \tau_b|b| + c + d - \lambda|d|], \quad (3)$$

where U_L is a utility function defined on any random variable of the form $k + KX$, for constants k and K. For example, Section III considers examples with the special case of expected utility. In equation (3), $x_L \geq 0$ is a previous commitment to deliver collateral in the next period, which gives the Long an incentive to actually undertake a long position. The Long's previous decision to obtain q_L units of the collateral could have arisen from a desire to hedge a commitment to supply collateral, or could have been based on a speculative motive, such as relatively optimistic beliefs regarding the probability distribution of X.

(T) A "Trader," who has no commitments, endowments, or transactions costs, has the problem

$$\max_{(a,b,c,d)\in B(0)} U_T[(a + b)X + c + d], \quad (4)$$

where U_T is a utility function.

(S) A "Short" hedges a commitment to accept $x_S \geq 0$ units of the specific or general collateral in the next period. For simplicity, this investor has neither endowments nor transaction costs. The Short's problem is therefore

$$\max_{(a,b,c,d)\in B(0)} U_S[(a + b + x_S)X + c + d], \quad (5)$$

where U_S is a utility function. This investor has an incentive to hedge the commitment to accept specific collateral by taking a short position in specific or general collateral.

We adopt from this point the assumption that U_L, U_T, and U_S are strictly increasing, in the sense that if $Z \geq Z'$ and $Z \neq Z'$, then $U_L(Z) > U_L(Z')$.

D. Equilibria

An equilibrium for the market described above is a collection (P', P, r, R) of prices and interest rates such that there exist solutions to equations (3), (4), and (5), respectively, that sum to zero (market clearing).

PROPOSITION 1. *For any equilibrium, $P' = P(1 + r)/(1 + R)$.*

Proof: Suppose not, and $P' > P(1 + r)/(1 + R)$. Consider the Short or the Trader's opportunity to short-sell an arbitrary number, say n units, of specific collateral, buying an equal number of units of general collateral to cover the position, for zero risk. The reverse repo requirement to loan (or reduce borrowing by) nP' at the specific rate R can be funded by borrowing an equal amount at the rate r, and generates the net additional payoff per unit in the next period of

$$(P' - P)(1 + r) - P'(r - R) = P'(1 + R) - P(1 + r).$$

But this is positive by assumption, generating an increase in utility, contradicting the equilibrium. (Even if positive amounts of specific collateral were previously owned, this would be an arbitrage for a sufficiently large number n.)

Conversely, suppose $P' < P(1 + r)/(1 + R)$. Consider the Short or Trader's opportunity to buy an additional unit of the specific collateral and sell one unit of the general collateral to cover the position. The ability to borrow P' at the specific repo rate R and invest the proceeds at the rate r, generates the net additional payoff per unit in the next period of

$$-(P' - P)(1 + r) + P'(r - R) = -P'(1 + R) + P(1 + r).$$

This is positive by assumption, generating an increase in utility, contradicting the equilibrium. Q.E.D.

PROPOSITION 2. *In any equilibrium, $R \leq r$.*

Proof: If $R > r$, the Short or the Trader can reverse in the specific collateral and lend at R, borrowing the cost at r. The associated portfolio is of the form $(0, 0, c, d)$, with $c(1 + r)^{-1} = -d(1 + R) < 0$. With $R > r$, this implies $c + d > 0$, an arbitrage. Q.E.D.

If $R < r$, there need not be an opportunity for arbitrage. For example, in order to borrow at R, one must obtain the specific collateral. In order to do this, one must pay P' to get the collateral, and then must either take the risk of an uncovered position, or short the general collateral at a price of P for a net profit of zero, according to Proposition 1. This asymmetry stems from the unhindered ability to buy the asset at P' versus the ability to short it only by reverse repo and sale.

We let s denote the repo specialness. That is, $s = r - R$.

PROPOSITION 3. *In any equilibrium,* $0 \leq s \leq \lambda(1 + R)$.

Proof: We already have the inequality $R \leq r$ from Proposition 2, so $s \geq 0$. For the other inequality, suppose not, and $s > \lambda(1 + R)$. The Long will note that borrowing collateralized by specific collateral is cheaper, even after transactions costs, than uncollateralized borrowing, since $s > \lambda(1 + R)$ is equivalent to $(1 + \lambda)(1 + R) < 1 + r$. In fact, every agent holding specific collateral will find it advantageous to place all of it in repo, borrowing at R and lending at the higher rate r, even after transactions costs. All shorts must hold exactly their short positions by lending at the collateralized repo rate R. This implies, since the total amount q_L of specific collateral is positive, that the repurchase market cannot clear, a contradiction. Q.E.D.

It will be useful to exploit the constrained efficiency of equilibrium allocations of the four investments. An allocation is defined to be *constrained Pareto optimal*, given an equilibrium (P', P, r, R), if there is no market-clearing allocation satisfying the repurchase market constraint (2) for each agent such that each agent has higher utility, and at least one agent's utility is strictly higher. (This constrained notion of optimality depends on the equilibrium prices, as it must since the constraint (2) itself is price-dependent.)

PROPOSITION 4. *Any equilibrium allocation is constrained Pareto optimal.*

Proof: Given an equilibrium (P', P, r, R), suppose not. Then there is a constrained Pareto dominant allocation, in the sense of the definition, at which all of the agents weakly exceed the budget constraint (1), and at least one strictly exceeds it. Adding up equation (1) across the agents produces a contradiction of market clearing, à la Arrow (1953). Q.E.D.

In any equilibrium, it follows from constrained Pareto optimality and the existence of transactions costs that the Long does not substitute specific collateral with general collateral, selling one and buying the other, since doing so unnecessarily increases transactions costs. A *natural equilibrium* is defined by the fact that the Long is actually long in the specific collateral. For example, because of constrained Pareto optimality, any equilibrium is natural if $U_L = U_T = U_S = U$, a strictly concave, strictly monotone utility function.

PROPOSITION 5. *In any natural equilibrium,*

$$s \leq \frac{(\tau_a + \tau_b)(1 + R)}{(1 + r)P}. \tag{6}$$

Proof: From Proposition 3, we have $0 \leq s \leq (1 + R)\lambda$. Because of Proposition 1, inequality (6) is then equivalent to $(1 + r)(P' - P) \leq \tau_a + \tau_b$. This inequality holds; the Long would otherwise substitute specific collateral with general collateral at least to the point of holding no specific collateral, contrary to the definition of natural equilibrium. Q.E.D.

III. Analysis and Discussion of The Model

This section analyzes various aspects of specials by examples or variations of the model presented in the previous section. Throughout, we suppose that the asset payoff X is binomially distributed with equally likely outcomes $1 + \epsilon$ and $1 - \epsilon$, for some $\epsilon \in (0, 1)$. The Long and the Trader have risk-neutral utilities given by $U_L(Y) = U_T(Y) = E(Y)$. The Short, who has a hedging motive, has utility $U_S(Y) = E[\log(Y)]$.

Since there is linear homogeneity throughout in prices, there is a degree of freedom in prices that can be eliminated by fixing any price or borrowing rate, without effect on relative prices, specialness, or equilibrium allocations.

A. Indeterminate Repo Specialness Example

We fix some $r > -1$. Consider any specific collateral repo rate R such that $0 \leq s = \lambda(1 + R)$. Since the Long is risk-neutral, if $s = \lambda(1 + R)$ the Long will be indifferent to supplying repo collateral; if $s < \lambda(1 + R)$, the Long will supply none. Since the Trader is risk-neutral and has no transactions costs, the unique prices for the general and specific collateral respectively are

$$P = \frac{E(X)}{1 + r} \qquad P' = \frac{E(X)}{1 + R}. \tag{7}$$

Without loss of generality because of the arbitrage relationship between (P, r) and (P', R) given by Proposition 1, the Short can limit his other portfolio to one consisting of only specific collateral and borrowing or lending through the specific collateral repo market, solving the problem

$$\max_{a,d} E(\log[(x_S + a)X - d]) \quad \text{subject to } aP' + \frac{d}{1 + R} = 0. \tag{8}$$

The unique solution to this problem, by solving explicitly the first order necessary and sufficient conditions for a maximum, is

$$a = -d = -x_S. \tag{9}$$

At the given prices, the Long (being risk-neutral) is satisfied with no net trade provided τ_a and τ_b are large enough that equation (6) is satisfied, so that the Long will not be tempted to replace the more expensive specific collateral (whose repo borrowing advantages are of no use to the long) with the cheaper general collateral. The Trader is indifferent to minus the trade of the Short, and will thus clear the market. Thus we conclude as follows.

PROPOSITION 6. *For any general collateral rate $r > -1$ and specific collateral rate $R > -1$ such that $0 \leq (r - R) \leq \lambda(1 + R)$, the prices P and P' given by (7) form an equilibrium for the example considered, provided the transactions costs τ_a and τ_b are large enough that equation (6) is satisfied.*

This example shows that equilibrium specialness can easily occur, although it does not determine the degree of specialness. We will eventually see an example with a determinate equilibrium.

B. Determining a Marginal Repo Entrant with a Continuum of Types

We may view Figure 3 as an equilibrium in which the repo transaction cost coefficient of the Long indifferent to entering the repo market is equal to the repo specialness. In order to formalize this marginal characterization of specialness, we introduce a continuum of types of long investors. That is, we replace the single long agent in the original model with a continuum of long agents whose types are distinguished by the transaction cost coefficient λ. The density of Longs' transaction cost coefficients is given on $[0, \infty)$.

In a given equilibrium (P', P, r, R) (whose definition is the obvious extension of that given in Section II), for each given transactions cost coefficient λ we let $F(\lambda)$ denote the total quantity of specific collateral held by all agents holding long specific collateral positions whose repo transactions cost coefficients are less than or equal to λ. If the total quantity of specific collateral held short is S, then there is a sufficient quantity of collateral held by longs with repo transactions costs coefficient less than or equal to λ, where $S = F(\lambda)$.

PROPOSITION 7. *If $s > 0$ then $S = F(s/(1 + R))$.*

Proof: It suffices to consider the decision of a Long of type λ about whether to use collateralized borrowing. A Long of type $\lambda < (r - R)/(1 + R)$ places all specific collateral into repo. A Long of type $\lambda > (r - R)/(1 + R)$ places none into repo. Since the types of longs have a density, F is continuous. Thus $F[s/(1 + R)]$ is the total collateral placed into repo by longs. In equilibrium, this is equal to the total short position S. Q.E.D.

C. Determinate Specialness via Staggered Settlement

In order to give an example in which the equilibrium special repo rate is determined in the manner of Proposition 7, and naturally corresponds to the intuition that repo specials are increasing in the demand for short positions and in the transactions costs of longs with collateral, we will adapt the example model of Section III.*B*, making the following modifications. The model becomes much more restrictive, although some of this is due to reasons of tractability.

(a) Uniform Distribution of Longs. There is a continuum of long agents of total "mass" equal to one. Each Long is as described above, with the transaction cost coefficient λ of the Longs uniformly distributed on the interval $[0, \bar{\lambda}]$, for some $\bar{\lambda} \in (0, 1]$. The density of longs on this interval is thus $1/\bar{\lambda}$, and $\bar{\lambda}$ indexes the degree of transactions costs. The Longs share equally as an endowment the total supply q_L of specific collateral. That is, any fraction ϕ of longs initially has ϕq_L units of specific collateral.

(b) Staggered Settlement. We introduce the requirement that the repo market must clear before the cash market. In order for the Short to reverse in specific collateral, it must therefore come from the Long's original supply, and not from a simultaneous repo and cash market trade with the Trader. Otherwise, with simultaneous repo and cash market clearing, the Short could sell specific collateral to the Trader and simultaneously reverse in the same collateral from the Trader in order to obtain the collateral he is selling (that is, in order to meet requirement (2b)). This would allow the Short to create an arbitrarily large short position without ever drawing from the supply of collateral originally held by Longs. Since the repo market is in practice on a same-day-settlement basis, whereas the cash market has a next-day settlement standard, such trading would in practice be impossible. One should not confuse this limited total supply of reverse repo on a given day with the issue of "trading repo," which allows an arbitrarily large volume of reverse repo on a single day, or with the possibility of creating an arbitrarily large total short interest[18] over many days. In order to introduce staggered settlement in a formal way, we simply replace the budget constraint ($2a$) with

$$d < 0 \Rightarrow qP' \geq -\frac{d}{1+R}. \tag{2a'}$$

That is, borrowing at the specific collateral rate requires previous ownership of specific collateral. This effectively precludes the Trader from using specific collateralized borrowing since he has not been endowed with that asset. This staggered settlement requirement is extended in the following subsection. Based on the discussion of the importance of the velocity of circulation given at the beginning of Section II, one should not take the interpretation of staggered settlement given here literally, in that multiple rounds of clearing during a given trading day relaxes the impact of staggered settlement. Here, there is implicitly a single round of clearing on a given day.

(c) Superior "Liquidity" of Specific Collateral. We will effectively make specific collateral more liquid than general collateral by including a general-collateral transactions cost coefficient $\gamma_b > 0$ for the Trader. That is, the new problem for the Trader is:

$$\max_{(a,b,c,d)\in B(0)} \quad U_T[(a+b)X + c + d - \gamma_b|b|],$$

[18] Consider, for example, the possible doubling of the short interest each day by having shorts reverse in each day the total supply of long positions, with subsequent sale of those positions back to longs for settlement the next day, and so on.

with the term γ_b implying that the Long will not trade the general collateral at any price consistent with Proposition 1, since the same effective trade can be accomplished more cheaply with specific collateral. Since the Short has no transactions costs, the arbitrage relationship implied by Proposition 1 will apply in this example as well.

Now, in order for the Short to obtain a given amount of specific collateral in repo and for markets to clear, the Longs must supply it. The amount that they supply is strictly monotonic, as in the proof of Proposition 7, in the repo specialness. At any specialness $s \in [0, \bar{\lambda}]$, the total repo collateral supplied by longs is $q_L s/\bar{\lambda}$.

We can without loss of generality take $R = 0$, given linear homogeneity in prices. This means that $r = s$. Let

$$P' = \frac{E(X)}{1 + r}, \qquad P = \frac{E(X)}{(1 + r)^2}. \tag{10}$$

At the prices given by equation (10), the demand for specific collateral by the Short can be computed from the first order conditions for (8) as $-D(r)$, where

$$D(r) = \frac{x_S}{2} \frac{2(1 + \epsilon)(1 - \epsilon)(1 + r)^2 - 2(1 + r)}{(1 + \epsilon)(1 - \epsilon)(1 + r)^2 - 2(1 + r) + 1}. \tag{11}$$

This demand $-D(r)$ is negative and strictly increasing in r, which is obvious since the Short has a standard demand function for two strictly desirable goods (specific collateral and collateralized borrowing) at a relative price $1/(1 + r)$ for specific collateral that is decreasing in r. Because the repo supplied by Longs is equal to the absolute magnitude of the Short's specific collateral position, and because $R = 0$ implies that $s = r$, we have the equilibrium characterization

$$\frac{q_L s}{\bar{\lambda}} = D(s). \tag{12}$$

The left hand side is strictly increasing in r and ranges from zero at $s = 0$ to q_L at $s = \bar{\lambda}$ (which is the maximum level of specialness for an interior solution). The right hand side of equation (12) is strictly decreasing in s and ranges from x_S at $s = 0$ down to $D(\bar{\lambda})$ at $s = \bar{\lambda}$. Thus, under mild parameter restrictions given by the following proposition, equation (12) has a unique[19] interior solution, and we have the following characterization of equilibrium.

PROPOSITION 8. *Suppose $D(\bar{\lambda}) < q_L$. Let s^* be the unique solution to $D(s) = q_L s/\bar{\lambda}$. Let $r = s^*$, let $R = 0$, and let P and P' be given by (10). Suppose the proportional general-collateral transaction cost coefficients γ_b and τ_b are strictly greater than $s^* E(X)/(1 + s^*)$. Then (P', P, r, R) is an equilibrium. Moreover, under these conditions, in any equilibrium the specialness is s^*,*

[19] Relation (12) is a cubic equation with an explicit (but messy) solution that we have no need to compute here.

which is strictly increasing in the position x_S to be hedged by the Short and (with $x_S > 0$) is strictly increasing in the repo transactions cost index $\bar{\lambda}$ of Longs.

Proof: At the prices given, the following trades are market clearing:

(i) The Short finds it optimal to reverse in $D(s^*)$ units of specific collateral from the Longs, and sell the same amount of specific collateral to the Trader.

(ii) The trader will borrow $x_S P'$ from the Longs at the general collateral rate $r = s^*$ to cover the cost of its specific collateral purchases from the Short. (The Trader, being risk-neutral, is happy to do this since the specific collateral is priced at its expected discounted payoff.)

(iii) The Longs with transactions cost coefficient $\lambda < s^*$ will earn an interest rate spread of $r - R$ on their combined repo and lending operation with Short and Trader respectively, less the repo transactions costs they incur. The Longs with $\lambda > s^*$ do nothing. Since Longs are risk-neutral, buying specific collateral has at best a zero utility effect, regardless of transactions costs, and given the assumption on τ_a, general collateral trades have at best a zero utility effect.

No agent has a strict incentive to deviate from these trades, so (P', P, r, R) is therefore an equilibrium.

The specialness is unique by the following reasoning. Since there are no specific-collateral transactions costs to the Trader who is risk-neutral and can borrow or lend at r, we must have $P' = \mathrm{E}(X)/(1 + r)$. Since the Short can otherwise arbitrage, we have $P = \mathrm{E}(X)/(1 + r)^2$, as in the proof of Proposition 1. The demand by the Short for a short position cannot be met by general collateral in equilibrium, since the Long and the Trader have transactions cost coefficients τ_b and γ_b that would make the purchase of general collateral strictly sub-optimal at the given price P and financing rate r. Thus, the Short will only obtain his short position by reverse repo and sale of specific collateral. The amount of repo trade is determined by equating the supply and demand for repo as analyzed before the statement of the proposition, which is done uniquely at the specialness s^*. Any other specialness will cause the longs to supply too much or too little repo collateral to meet the needs of the short. Q.E.D.

One could work out a more complicated formula for the repo specialness by extending the model to allow for multiple rounds of clearing in the repo markets, under given assumptions about the velocity of circulation of repo collateral.

We can also address which of two essentially identical instruments will be on special in repo markets, relative to the other. Other things being equal, one would expect the more liquid of two instruments to be the more special in repo.

This can be verified by adjusting the model just described so that general and specific collateral are symmetric in all regards, including the supply among Longs and transactions costs of the Longs for both types of collateral, with the sole exception of the transactions costs τ_a, τ_b and γ_a and γ_b. It is then easily seen that if $(\tau_a, \gamma_a) < (\tau_b, \gamma_b)$, then the specific collateral can be special relative to the general collateral but not the converse, and if $(\tau_b, \gamma_b) < (\tau_a, \gamma_a)$ then the general collateral can be special relative to the specific but not the converse. In other words, the asset with greater liquidity is the more likely to be relatively special. This is consistent with the current-issue effect in repo-specials and cash-yields that is well documented. For empirical evidence, see Table I, as well as Sundaresan (1992), Cornell (1993), and Beim (1992). Of course, we have only modeled liquidity here by exogenous transactions costs.

D. Multi-Period Staggered Settlement and Determinate Specialness

We could also obtain a much simpler and more dramatic characterization of specials simply by endowing the Short with a position of $-q_S < 0$ in specific collateral before the start of trade. Given the requirement that this short position must be met by reverse repo before trade in cash markets can begin (extending the staggered settlement requirement of the previous subsection), we obtain the unique equilibrium specialness $s^* = \min(1, \bar{\lambda} q_S / q_L)$, which has all of the right intuition built in. Formally, this is accomplished in the model of the previous subsection, modified only by replacing (2*b*) with the staggered-settlement analogue:

$$q < 0 \Rightarrow qP' = -\frac{d}{1 + R}. \tag{2b'}$$

One notes that the transactions cost conditions on τ_b and γ_b assumed in Proposition 8 are then completely unnecessary.

This makes for a rather stark one-period model, however, and as discussed earlier, with multiple rounds of clearing per day the impact of staggered settlement on specials is reduced, in that a small supply of collateral can be recirculated so as to create the effect of a large supply of repo collateral, in the sense described at the beginning of Section II.

The whole story becomes much more interesting in a multi-period model. Let us assume for simplicity a single round of clearing per day, as we have implicitly assumed throughout the formal modeling. Each day t, as in actual markets, each given trader has a beginning specific collateral position of q_t, which is the original position q_{t-1} of the day before, plus any cash trades a_{t-1} made during the previous day for settlement on the current day. This new position q_t for the day may be long, in which case no repo transactions are required, or may be short, in which case one *must* reverse in at least the entire size of the short position in order to avoid a fail. (Not even buying one's way out of the short position would avoid the requirement to reverse in collateral, since the new collateral purchased at day t would not be available until day $t + 1$.) As in our more stark one-period model, the current repo market equilibrium may involve a significant degree of specialness, *even* if the cash market transactions cost coefficient γ_b and τ_b are zero. This follows basically from the standard no-substitutions clause in repo agreements that we are assuming here; general collateral is not an acceptable substitute for meeting previously established repo commitments; only the specific collateral will do. This could create dramatic specials in principle, although all traders would know of the dangers of taking short positions and being stuck by surprise in a "natural squeeze," which would be generated either by an unexpectedly large aggregate short interest in the market, or an unexpectedly small amount of the collateral falling into the hands of those with low shadow prices (λ) for repo transactions. (We could include institutional reasons that effectively preclude repo for certain investors, some of which are mentioned in Section I.) Again, the opportunity to service many repo shorts with a small amount of collateral and multiple rounds of clearing would reduce the impact of staggered settlement described above.

The situation between potential longs and potential shorts is somewhat asymmetric, since a "marginal" investor considering the purchase of a specific instrument would have less to gain from the special than would the marginal short have to lose. (This is assuming that the cost or shadow price of repo transactions of the marginal long is greater than that of the marginal short, which is reasonable in the author's opinion.)

IV. Extensions and Applications

This section describes an extension of the valuation effects of specials to a multi-period setting, as well as an application to the measurement of the current term structure of interest rates using price data for bonds that may be on special.

A. Multiperiod Cash-Repo Equilibrium Relationship

The basic relationship between repo rate specialness and cash market prices for specific collateral represented by Proposition 1 can be extended to a multi-period setting quite easily. (It is more challenging, however, to extend a model for the special rate itself to a reasonably general multi-period setting.)

As we have shown, the specialness $s = r - R$ may be treated for pricing purposes as an additional dividend yield on the underlying instrument. We could adopt a discrete-time or a continuous-time setting in which r and s are processes satisfying the usual technical conditions for a short rate and a continuous dividend yield, represented as a fraction of price.

We proceed under the assumption that there is at least one agent with no transactions costs. In that case, under mild technical conditions explained by Harrison and Kreps (1979), the absence of arbitrage for that agent implies the existence of an equivalent martingale measure Q, a probability measure under which the expected discounted gains from any trading strategy are zero. Here, we will take "discounted" to mean discounted at time t by the value $\delta_t = (1 + r_1)(1 + r_2) \cdots (1 + r_{t-1})$ of one dollar rolled-over each period s at the short rate r_s. (The continuous-time counterpart is $\delta_t = \exp(\int_0^t r_s \, ds)$.)

Under risk-neutral probabilities, the expected rate of price growth on the underlying instrument is $r - s = R$, the specific collateral rate. This implies that the arbitrage-free price p_t of the underlying at day t is given in terms of its price p_T on a later day T, by

$$p_t = E_t^Q\left[\frac{1}{1+R_t}\frac{1}{1+R_{t+1}}\cdots\frac{1}{1+R_{T-1}}p_T\right], \tag{13}$$

assuming no coupon payments on the bond between t and T, where E_t^Q denotes conditional expectation on day t with respect to an equivalent martingale measure Q. Relation (13) is a testable restriction on the joint behavior of bond prices and overnight repo rates. Analogous restrictions apply to term repo rates, although term repo markets are somewhat less liquid than overnight markets.

Figure 4 shows the estimated percentage impact of special repo rates on the price of a 6.75 percent 2-year Treasury note at issue, using the interest rate model of Cox, Ingersoll, and Ross (1985).[20] The special repo rate is assumed, on each day until a given date, to be a constant fraction of the general collateral rate for that day, whatever it may turn out to be. For example, if the two-year note is to remain at a special repo rate that is 80 percent of the general collateral rate (20 percent discount) for 1 month, the note is estimated to trade 0.10 percent "rich" (or about \$4/32 per hundred dollars of face value) at issue. This price richness at issue should decline approximately linearly through the current issue month to zero at the end of that month (under the assumption that it stays off special after its first month). Assuming that there is also a liquidity premium associated with the current issue (see Beim (1992)), this would argue for an empirical test of the special repo rate effect on cash market prices under which the current issue has a yield depression (relative to that implied by a well synthesized zero-coupon yield curve) that is roughly a constant plus a constant multiple of remaining time on-the-run. (This assumes that market expectations are that off-the-run issues are not on special on repo, which is typically but not always the case.)

B. Zero-Coupon Yield Curve Splining

It is common practice to use the prices of coupon treasury instruments as data in estimating the current zero-coupon yield curve, for example, by some form of spline.[21] One estimates the discount function or forward rates at various maturities that "explain" the prices of coupon instruments via the usual present value formula. The fit is inexact, by design, so as to achieve a degree of smoothness in the estimated curve. From discussions with the author, certain major broker-dealers avoid the use of on-the-run treasuries as data in this procedure as their prices are known to be "contaminated" by the on-the-run price richness noted earlier. Other major dealers fit only the on-

[20] Barone and Risa (1994) explore the implications of the model used to prepare Figure 4 for the effect of repo specialness on options on floating rate notes.

[21] See, for example, Coleman, Fisher, and Ibbotson (1992) or Fisher, Nychka, and Zervos (1994).

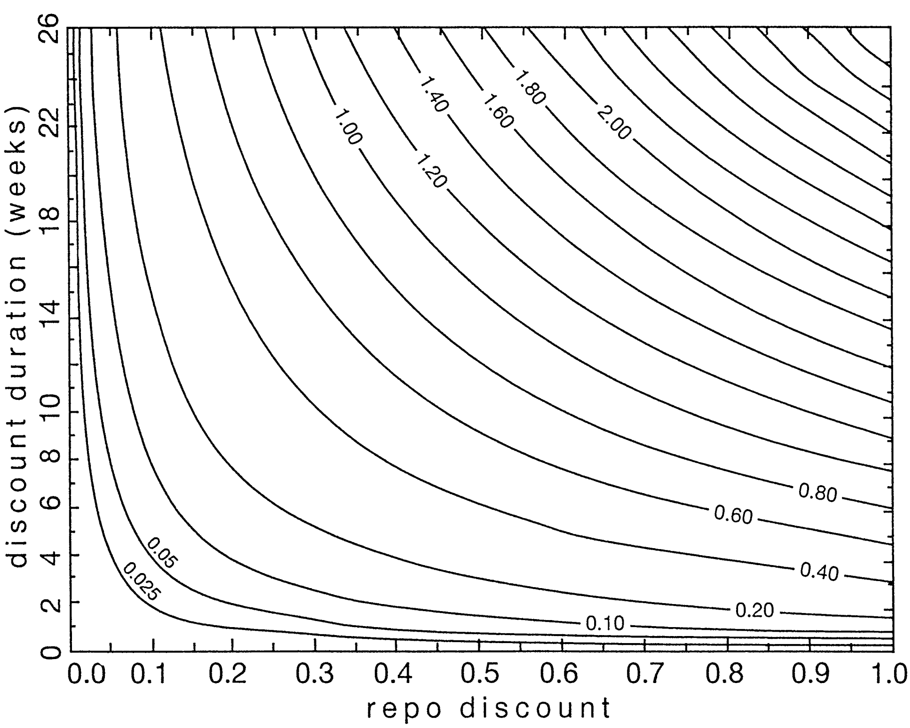

Figure 4. Percentage Impact of Repo Specialness on Note Price. The calculations for this figure are based on the interest rate model of Cox, Ingersoll, and Ross (1985), in which the short rate process r is given by $dr_t = (a + br_t)dt + c\sqrt{r_t}dB_t$, where B is a standard Brownian motion under an equivalent martingale measure. We take $R_t = kr_t$, for some constant $k \in [0, 1]$. With the continuous-time version of equation (13), $p_t = f(r_t, t)$, where f solves the partial differential equation, given via the change of variables $g(x, u) = kf(x, u/k)$, by $g_u(x,u)+g_x(x, u)(a/k + (b/k)\, x) + 1/2g_{xx}(x, u)c^2x/k-xg(x, u)=0$, with the obvious boundary condition. The solution shown for a 2-year Treasury note includes the effects of coupons and reversion to $R_t = r_t$ (no specialness) after the indicated passage of time. The calculation is done with the Crank-Nicholson finite-difference algorithm, using the code shown in Duffie (1992). The parameters used for the CIR model are as estimated from U.S. Treasury bond data by Pearson and Sun (1994).

the-run maturities, again avoiding the discrepancy associated with on-the-run "richness."

A natural application of the repo pricing relationship given in this paper would be to "decontaminate" on-the-run price data, removing the effect of repo specials. Assuming that most of the on-the-run effect is due to repo specialness, one might then use all issues (except flower and callable issues, which require other correction) as data for "splining." Off-the-run issues are also influenced by specials, as indicated in Table I, and might also be worth the trouble of "decontamination."

From Proposition 1, given a particular treasury issue, in order to obtain the price of an otherwise identical instrument that is not on special, we would factor down the quoted market price (including accrued interest) by $(1 + R)/$

$(1 + r)$, where R is the specific term repo rate and r is the general collateral term repo rate for the same term. As the term of the repo grows, this correction factor theoretically converges to a constant that reflects expectations of specialness over the life of the instrument. In practice, it is uncommon for specific term repo rates to be reliably quoted (with liquidity) for more than 3 months, which might be satisfactory for this purpose.

V. Concluding Remarks

This section gives a nontechnical summary of the assumptions and results of the model. The basic assumptions motivating the model are:

(a) Reverse repo is always the cost effective method of shorting.
(b) Institutional factors and transactions costs limit the supply of repo collateral from certain holders of the underlying instrument.
(c) There are also effective transactions costs or "clientele" effects limiting the extent to which those cash-longs not supplying repo collateral are likely to substitute their cash positions with "cheaper" alternative instruments.

For a determinate model of specials, we can include the additional assumption:

(d) There is staggered settlement; that is, repo markets must clear before cash markets.

The following conclusions can be drawn on the basis of this model:

(1) When an issue goes special, there is a transfer of wealth from old shorts to those old longs who choose to supply repo collateral or to immediately sell in the cash market.
(2) The current-issue effect on yields may be significantly driven by special repo rates, and not only by low cash-market bid-ask spreads.
(3) Specialness in repo rates is a threshold phenomenon. "Normally," at least for many old issues, the supply curve for repo collateral is sufficiently large relative to the demand curve to drive specialness close to zero, a corner solution.
(4) Repo specialness is increasing in the size of short-hedging and short-speculative demands in the cash market, relative to the issue size.
(5) Specialness can occur in a competitive model. It can also occur from monopolistic behavior by cash-longs who hold collateral away from repo markets.
(6) Specialness can also be measured as the proportional shadow cost of those longs who are indifferent to supplying repo collateral. This shadow cost can reflect various transactions costs and also legal or institutional barriers to the use of repo transactions.
(7) Specialness is not itself a form of transactions costs. Those entering or continuing short positions at times of special repo rates do so at cash-market prices and repo rates that, in combination, need not involve any

frictions. Shorts continuing their positions have recognized sunk costs. Large dealers and traders can open and close special cash-repo positions at transactions costs typical of, or below, those in nonspecial instruments.

(8) The observed cash market price of any specific instrument, ignoring other institutional factors, is the same as the price of an instrument with (a) no specialness, (b) the same coupons and principal, and (c) an additional dividend yield equal to the difference between the general collateral rate process and the specific repo rate process. Under risk-neutral probabilities, the expected rate of return on such an instrument at any time is the specific repo rate. With this in mind, standard (arbitrage-based) asset pricing theory applies as well to special as to nonspecial instruments.

(9) Other things being equal, one would expect the more liquid of two instruments to be the more special in repo.

One of the general conclusions that one might draw is that repo rates are sensitive to even detailed institutional features of the market, such as settlement timing in both the cash and repo market, small frictional costs of trading, as well as the methods for obtaining information regarding the holdings of various investors of a given security. Recently contemplated changes[22] in the auction procedure used by the Treasury for sale of governments securities might also influence the behavior of repo rates.

Appendix

Asynchronous Settlement Effects in Repo

The cost-of-carry analysis in Section I neglected the effect of the usual standard of same-day settlement of the first leg of a repo transaction versus next-day settlement in the cash market. The more careful analysis below leads to the same conclusion that $R \leq r$, and that $R = r$ when the cost-of-carry formula applies.

In order to see this, recall that the quoted cash market price today is actually the forward price for next day delivery. In order to create a synthetic same-day spot market transaction, one can reverse in the repo market at the repo base price P_R and repo rate R, simultaneously buy in the cash market at the cash market price P_C, and also borrow $(P_R(1 + R) - P_C)/(1 + r)$ at the riskless rate r. The net cash outflow today is

$$P^* = P_R - \frac{P_R(1 + R) - P_C}{(1 + r)}.$$

[22] See Back and Zender (1993) and Bikhchandani and Huang (1989).

If the repo base price P_R is equal to the current cash market price P_C (the usual convention), we have the effective spot price

$$P^* = P_C\left(\frac{1 + r - R}{1 + r}\right).$$

One receives the instrument today from the first leg of the repo transaction. The next day, there is no net cash flow, and the instruments collected on the settlement of the cash market trade are given back on the second leg of the repo transaction. Thus P^* is the effective spot market price for same-day settlement.

The cost-of-carry formula, when it applies, equates the effective forward price P_C (no typo here!) with $P^*(1 + r)$. Whether or not the repo base price P_R is equal to the cash price P_C, some simple algebra leads from $P^*(1 + r) = P_C$ to $r = R$. The same calculations apply even if the prices are reported net of accrued interest.

This is not to say, however, that the cost-of-carry relationship $P_C = P^*(1 + r)$ should in fact apply in practice, as pointed out in Section I. The missing link in the arbitrage is the ability to short sell in the cash market at exactly the cash price P_C. Only those already owning the securities can take an unambiguous advantage of special repo rates, but they can only do so to the extent of their holdings. The same analysis, however, does show that the absence of arbitrage implies that $R \leq r$.

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