Liquidation Risk

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1 Introduction

The turmoil in financial markets in late 1998 accompanied a sharp decrease in market liquidity. Some financial institutions faced unexpectedly high bid-ask spreads when liquidating positions. This paper is an analysis of the effect on key risk measures (such as the likelihood of insolvency, value at risk, and expected tail loss) of bid-ask spreads that are likely to widen just when positions must be liquidated in order to maintain capital ratios, thus triggering additional losses. Our results show that illiquidity causes significant increases in risk measures, especially if spreads are negatively correlated with asset returns.

A potential strategy is to liquidate illiquid assets earlier, keeping a cushion of cash or liquid assets for "rainy days." Our results show that, although this approach is usually effective, it tends to increase expected trading costs, and may fail when asset returns and bid-ask spreads have fat tails.

2 The Model

This section introduces the model. For simplicity, we consider a firm with three assets: cash, a relatively liquid asset, and an illiquid asset.

2.1 Asset Price and Spread Dynamics

Let $S_{0,t}$ denote the value at time t of a position of $S_{0,0}$ invested in cash at time 0. We assume that cash earns a fixed rate of return, r, with no bid-ask spread. Then, $S_{0,t} = S_{0,0} \exp(rt)$.

We assume that the mid-prices of a liquid and an illiquid asset are geometric Brownian

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motions. The mid-price at time t of the liquid asset is

$$S_{1,t} = S_{1,0} \exp\left(\mu_1 t + \sigma_1 B_{1,t}\right),$$

while the mid-price of the illiquid asset at time t is

$$S_{2,t} = S_{2,0} \exp\left(\mu_2 t + \sigma_2 \left(\rho B_{1,t} + \sqrt{1-\rho^2} B_{2,t}\right)\right),$$

where B_1, B_2, B_3, ... are independent standard Brownian motions; μ_i and σ_i determine the instantaneous expected return and volatility of the mid-price S_i; and ρ is the instantaneous correlation between the mid-price increments of the liquid and illiquid asset.

Let $X_{i,t}$ denote the (relative) mid-to-bid spread at time t on asset i. That is, the bid price for the liquid asset is $S_{1,t}(1 - X_{1,t})$ and the bid price for the illiquid asset is $S_{2,t}(1 - X_{2,t})$. We assume that

$$X_{1,t} = X_{1,0} \exp\left(\gamma_1 \left(\rho_1 B_{1,t} + \sqrt{1-\rho_1^2} B_{3,t}\right) - \frac{1}{2}\gamma_1^2 t\right)$$

and

$$X_{2,t} = X_{2,0} \exp\left(\gamma_2 \left(\rho_2 \left(\rho B_{1,t} + \sqrt{1-\rho^2} B_{2,t}\right) + \sqrt{1-\rho_2^2} B_{4,t}\right) - \frac{1}{2}\gamma_2^2 t\right),$$

where γ_i denotes the volatility of the relative bid-ask spread on asset i and ρ_i determines the correlation between the mid-price increment of asset i and the change in the spread on asset i. With $\rho_i < 0$, spreads are expected to widen as prices fall. (This parameterization admits the possibility of negative bid prices, but at typical parameters the likelihood of this over short horizons is negligible.)

This formulation implies no time trend in spreads, nor correlation between spreads across different assets beyond that induced by mid-price movements. Reflecting the idea that asset 1 is more liquid than asset 2, we set initial spread values such that $X_{2,0} > X_{1,0} > 0$.

2.2 The Firm's Liquidation Behavior

At time 0, the firm starts with the following asset and capital structure. It holds $\alpha_{0,0}$ units of cash, $\alpha_{1,0}$ units of the liquid asset, and $\alpha_{2,0}$ units of the illiquid asset. The total portfolio value evaluated at mid-prices is

$$A_0 = \alpha_{0,0} S_{0,0} + \alpha_{1,0} S_{1,0} + \alpha_{2,0} S_{2,0}.$$

The initial value of the liabilities is L_0. Thus, initial capital is

$$K_0 = A_0 - L_0 = \alpha_{0,0} S_{0,0} + \alpha_{1,0} S_{1,0} + \alpha_{2,0} S_{2,0} - L_0.$$

We suppose, for example by a regulatory requirement, that on any given date t, the firm attempts to attain a ratio of capital to total asset value of at least c_r. That is, we liquidate assets as required to achieve $K_t = (A_t - L_t) \geq c_r A_t$. (We assume that raising capital, for example through an infusion of new equity, is not feasible during the short time horizons that we consider.)

Let $\lambda_{i,t}$ denote the number of units of asset i liquidated in period t. We suppose (until later analysis) that the firm liquidates cash first, then the liquid asset, and finally the illiquid asset. Details of the liquidation algorithm are provided in the Appendix. Once this process is completed, the holdings of the three asset types at the end of the period are recorded and carried over to the next period by setting, for each asset type i,

$$\alpha_{i,t+1} = \alpha_{i,t} - \lambda_{i,t}.$$

We assume that liabilities earn the fixed short rate r. (As the liabilities are apparently not default free, we could assign a higher borrowing rate $R > r$, but over short time horizons the effect of this would be similar for typical parameters.) Taking the proceeds from asset sales in period t into account, the value of the liabilities in period $t+1$ is

$$L_{t+1} = \exp(r)\left(L_t - \lambda_{0,t}S_{0,t} - \lambda_{1,t}(1 - X_{1,t})S_{1,t} - \lambda_{2,t}(1 - X_{2,t})S_{2,t}\right).$$

The above asset liquidation process is repeated for ten successive trading days. At the end of the tenth day, the terminal capital, K_{10}, is computed based on current asset holdings and liabilities. The 99%-VaR is the 99% critical value of the distribution of cumulative losses in capital $K_0 - K_{10}$, over the ten-day period. Expected tail loss (ETL) is the expected loss in capital conditional on the event that losses exceed the 99%-VaR. The probability of insolvency is the probability that the firm's capital is eliminated within the 10-day period. It has been noted that VaR is not a coherent risk measure in the sense of Artzner, Delbaen, Eber, and Heath (1999). The expected tail loss, however, is coherent and, although not as commonly reported, is preferred conceptually as a risk measure. The use of a 99% confidence level, rather than some other quantile, is arbitrary but conventional.

3 Basic Results

This section presents the results of our Monte-Carlo analysis, based on 25,000 pseudo-independent ten-day scenarios for each case. Unless otherwise specified, the (annualized) base-case parameters are $r = 0.05$, $\mu_1 = 0.1$, $\mu_2 = 0.2$, $\sigma_1 = \sigma_2 = 0.2$, $\gamma_1 = \gamma_2 = 1$ and $\rho = -0.5$. In order to highlight the effect of liquidity, we have equated the volatility of the assets. We take the target capital ratio c_r to be the typical regulatory ratio of 8%, and assume an initial asset structure of $\alpha_0 = 2$, $\alpha_1 = 8$, $\alpha_2 = 90$, with an initial capital ratio of 9%, implying initial liabilities of $L_0 = 91$. In other words, at time 0, the firm exceeds its regulatory capital requirements by 1%, and holds 90% of its assets in illiquid form.

We study four cases, based on alternative starting values for the mid-bid spread. The base case has no spread. The other three cases assume initial spreads, for the liquid and illiquid assets, respectively, of 0.1% and 0.5%, 0.2% and 1%, and 0.5% and 2.5%. As a point of comparison, Schultz (2001) estimates rount-trip trading costs for corporate bond trades by institutional investors with dealers of approximately 0.27%, indicating tighter spreads than most of our cases. On the other hand, our initial conditions are designed to place the portfolio, in terms of leverage and spreads, in a relatively "distressed" state, given which a seller might anticipate predatory or conservative quotes.

For each case, we analyze four settings, based on the variability of spreads and the degree of correlation between spreads and prices: (1) constant spreads, (2) random spreads that are uncorrelated with asset returns, (3) random spreads moderately negatively correlated with returns, with $\rho_1 = \rho_2 = -0.5$, and (4) random spreads highly negatively correlated with returns ($\rho_1 = \rho_2 = -0.8$). The resulting 99%-VaR, expected tail loss (ETL), and probability of insolvency results for our 10-day period are reported in Table 1. The VaR and expected tail losses show only moderate responses to changes in the degree of illiquidity. Figure 1 compares 10-day insolvency probabilities for the case of large initial mid-bid spreads of 50 and 250 basis points for the relatively liquid and illquid assets, respectively. Also shown is the most adverse of these cases (for large negative spread-return correlation), with a reversal of the order of liquidation, selling least-liquid assets first. Further discussion of liquidation strategy follows in Section 6.

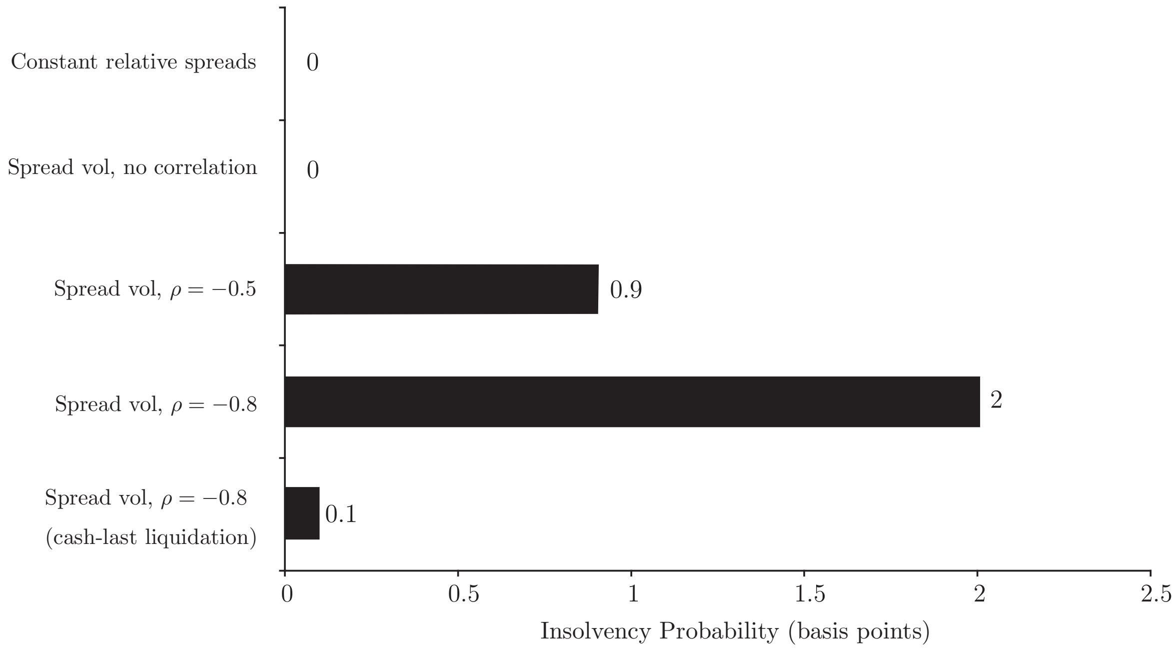

Figure 1: Comparison of Insolvency Probabilities

Ten-day insolvency probabilities, normal returns, 20% return volatility, 0.5% and 2.5% initial spreads, zero or 100% spread volatility, various spread-return correlations, and cash-first liquidation except as noted. Estimates based on 200,000 trials.

It has been widely documented in the literature that asset returns, especially in the short run, are fat tailed. In order to investigate the effect of non-normality on the relevance of spreads for liquidation risk, Table 2 presents the results of similar computations when jumps in prices are allowed. To model jumps in prices, we replaced the normal distribution of the daily increment of each Brownian motion B_i with a mixed normal distribution, with a daily "jump" probability of 0.02 and a kurtosis[1] of 10. The VaR and ETL results are significantly larger. The pattern of results is similar to that in Table 1, with correlation between returns and spreads leading to an increase in VaR and ETL of above 7%. Increasing the degree of negative correlation between returns and spreads leads to a sharp increase in the probability of insolvency of more than 40%, from 0.84% to 1.21%.

4 High Price Volatility

How does the effect of the bid-ask spread on liquidation risk depend on asset price volatility? Intuitively, increasing volatility should lead to more frequent asset sales and therefore to larger spread-induced losses. In order to investigate this issue, we ran additional simulations using an asset price volatility of 40% ($\sigma_i = 0.4$). The results for normal returns, reported in Table 3, show that increasing price volatility leads to a sizable increase in all risk measures, and especially in the probability of insolvency. Although the pattern of results is similar to that in the 20% volatility case of Section 3, it is worth noting that the effect of spreads on liquidation risk is weaker. For small spreads, the increases in VaR and ETL are now only about 1.5%, versus 3% in the base case. Large spreads bring increases of about 10% in these measures, half of the value obtained in Section 3. Moreover, while negative correlation between spreads and returns still leads to an increase in VaR and ETL, this effect is weaker than with low volatility.

These results are driven by early asset sales. When volatility is high, the institution must liquidate assets in greater amounts, and sooner, in order to meet capital requirements. This is similar to the effect of a "stop-loss" strategy for sales. As more assets are sold, the institution's exposure to price fluctuations falls. As a result, VaR rises by less than the increase in asset price volatility would imply. As spreads are introduced, even more assets must be sold in order to meet the capital requirements. The reduction in exposure thus mitigates the increase in VaR caused by larger spreads. The insolvency probability is sensitive, however, to the presence of spreads in the high-volatility case, increasing from 0.14% in the no-spread case to 2.67% for large spreads. With a strong negative correlation between spreads and returns, the insolvency probablity rises further to almost 5%.

As can be seen in Table 4, similar effects come into play with fat tails.

In summary, higher volatility actually reduces the relative impact of spreads on VaR and expected tail loss, but increases their relative effect on the insolvency probability.

[1] To simulate a random variable of zero mean and unit variance with fat tails (excess kurtosis), we proceed as follows. Let Y be the outcome of a Bernouilli trial that takes the value 1 with probability p and the value 0 with probability $1-p$. Let Z denote a standard normal random variable. Then, the random variable $X = \left(\alpha Y + \sqrt{\frac{1-p\alpha^2}{1-p}}(1-Y)\right) Z$ has zero mean, unit variance and a kurtosis of $\frac{3}{1-p}\left(p\alpha^4 - 2p\alpha^2 + 1\right)$. Using this result, one can find values for p and α that achieve the desired degree of kurtosis $\overline{k}$ by setting $\overline{k} = \frac{3}{1-p}\left(p\alpha^4 - 2p\alpha^2 + 1\right)$. Solving, $\alpha = \sqrt{1 + \sqrt{\left(\frac{\overline{k}}{3} - 1\right)\left(\frac{1}{p} - 1\right)}}$.

5 High Spread Volatility

Table 5 reports the effect of spreads on risk measures in a setting of substantial spread volatility of 200% ($\gamma_i = 2$), with a return volatility of 40% ($\sigma_i = 0.4$). At our base-case correlation of -0.5 between returns and spreads, for example, this means that spreads would widen in expectation from 50 basis points to approximately[2] 250 basis points given a sudden reduction in price of 1%.

The percentage increase in VaR caused by spreads is comparable to that in the high-volatility case, whereas the additional percentage increase in VaR caused by *correlation* between spreads and prices is comparable to that in the base case. For example, large spreads lead to an increase in VaR of about 10% (the value reported in Section 4), while correlation leads to an additional increase of almost 6% (the value reported in Section 3).

While a similar pattern of dependence emerges for ETL, the effect of price and spread volatility *compounds* for the probability of insolvency. Both the percentage increase due to spreads and the increase due to correlation are substantially higher here than in Section 4.

6 Cash-Last Liquidation Strategy

We have thus far assumed that the institution liquidates cash first. Only when cash is exhausted does the firm sell its liquid asset position. Coming last in the pecking order, illiquid assets are sold only in extreme cases.

This cash-first liquidation strategy raises the concern that, in the most stressful situations, the institution may have only illiquid assets left to sell. An alternative liquidation strategy is to sell illiquid assets first, keeping a "cushion" of cash and liquid assets for "rainy days." This section analyzes the effects of this strategy on VaR, ETL, and insolvency probability. We first consider the low volatility case ($\sigma_i = 0.2$) from Section 3. The results of the simulations for the four spread scenarios are summarized in Table 6. The picture that emerges from these calculations is similar to that of Section 3. Both the sizes of spreads and their correlation with asset returns have a significant impact on VaR and ETL.

Both VaR and ETL are significantly smaller than for the cash-first liquidation strategy. This improvement is accompanied, however, by higher transactions costs. Table 7 contrasts the expected transactions costs for the "cash-first" and "cash-last" liquidation strategies. The cash-last strategy is expected to cost approximately 40% more.

The results of similar computations for the case of fat tails are summarized in Table 8. VaR, ETL, and the probability of insolvency are significantly smaller than in the case in which cash is liquidated first. The percentage decrease is strongest for the probability of insolvency, which falls by almost 20% in the case of large spreads.

Consider, for example, the comparison offered in the high-volatility case between the cash-first and cash-last liquidation strategies, with moderately large spreads (0.2% and 1% mid-to-bid relative prices) and moderately large correlation ($\rho_i = -0.5$). The cash-last strategy increases

[2] The expected spread response is to scale it up by approximately $\exp(Z\rho_i\gamma_i/\sigma_i)$, where Z is the return.

expected liquidation costs by 5 basis points of assets ($0.349 - 0.299$ per initial 100 in assets, from Table 10), and reduces the probability of insolvency by 41 basis points ($0.62\% - 0.21\%$, from Tables 3 and 9). This implies a break-even financial insolvency distress cost of approximately 0.12 basis points of assets, or roughly 1 basis point of initial capital. That is, if the event of insolvency is expected to cost more than 1 basis point of the market value of the portfolio (for example, in terms of franchise value and re-organization fees), then the cash-last strategy is more effective than the cash-first strategy, for this particular case. Obviously, the break-even point of insolvency distress costs depends heavily on the particular regime of volatilities, correlations, and spreads.

Our results suggest an important trade-off between the goals of minimizing expected transaction costs during stressed asset sales and the goal of reducing the probability of insolvency (with the associated costs of overall financial distress).

7 Conclusion

Using a simple model, we analyzed the effect of spreads and their variability on various measures of liquidation risk. If spreads are expected to increase as prices fall, then the effect of market liquidity on liquidation risk can be dramatic, especially with fat-tailed returns. We have not treated the case of "price impact," under which the act of selling itself lowers bid prices, which could be critical if the asset holdings are large relative to the market.

With a goal of minimizing expected transaction costs, cash and liquid assets would be sold first. This liquidation strategy raises the concern, however, that in the most dramatic cases, the institution will have only illiquid assets left to sell, thus triggering large losses. An alternative strategy is to sell illiquid assets first, keeping a "cushion" of cash and liquid assets for "rainy days." Such a strategy, while increasing expected transaction costs, significantly decreases tail losses and, especially, the probability of insolvency. In light of our results, it seems wise for financial institutions to carefully examine their strategies for liquidation during periods of severe stress.

Our analysis has assumed that a given target capital ratio (8% in our case) will be maintained so long as it is possible to do so. Relaxation of this target ratio would presumably increase the probability of insolvency, while reducing expected transactions costs. Optimal liquidation strategies, for given risk-reward objectives, remain an interesting subject for future research.

References

P. Artzner, F. Delbaen, J.-M. Eber, and D. Heath (1999) "Coherent Measures of Risk," *Mathematical Finance* 9: 203-228.

P. Schultz (2001) "Corporate Bond Trading Costs: A Peek Behind the Curtain," *Journal of Finance* 56: 677-698.

Appendix

For the most-liquid-asset-first liquidation strategy, the recipe for liquidation is as follows.

1. If

$$\alpha_{1,t}S_{1,t} + \alpha_{2,t}S_{2,t} - (L_t - \alpha_{0,t}S_{0,t}) \geq c_r(\alpha_{1,t}S_{1,t} + \alpha_{2,t}S_{2,t}),$$

   then the firm's cash holdings are sufficient to meet the capital requirement. In this case, the firm's cash is reduced by $\lambda_{0,t}$ so as to satisfy the capital requirement. Solving,

$$\lambda_{0,t} = \frac{L_t - (1 - c_r)(\alpha_{0,t}S_{0,t} + \alpha_{1,t}S_{1,t} + \alpha_{2,t}S_{2,t})}{S_{0,t}c_r}.$$

   By assumption, none of the liquid or illiquid asset holdings are to be sold in this case. That is, $\lambda_{1,t} = \lambda_{2,t} = 0$.

2. Whenever

$$\alpha_{1,t}S_{1,t} + \alpha_{2,t}S_{2,t} - (L_t - \alpha_{0,t}S_{0,t}) < c_r(\alpha_{1,t}S_{1,t} + \alpha_{2,t}S_{2,t}),$$

   the $\alpha_{0,t}$ units of cash available are not sufficient to meet the capital requirement. Some of the liquid asset is therefore liquidated. If

$$\alpha_{2,t}S_{2,t} - (L_t - \alpha_{0,t}S_{0,t} - \alpha_{1,t}(1 - X_{1,t})S_{1,t}) \geq c_r\alpha_{2,t}S_{2,t},$$

   then the current holdings of the liquid asset and cash together are sufficient to meet the capital requirement. In this case, cash is reduced first. That is, $\lambda_{0,t} = \alpha_{0,t}$. The number of units of the liquid asset to be sold is based on the bid price, $S_{1,t}(1 - X_{1,t})$. Thus,

$$\frac{(\alpha_{1,t} - \lambda_{1,t})S_{1,t} + \alpha_{2,t}S_{2,t} - (L_t - \alpha_{0,t}S_{0,t} - \lambda_{1,t}S_{1,t}(1 - X_{1,t}))}{(\alpha_{1,t} - \lambda_{1,t})S_{1,t} + \alpha_{2,t}S_{2,t}} = c_r,$$

   yielding

$$\lambda_{1,t} = \frac{(L_t - \alpha_{0,t}S_{0,t}) - (1 - c_r)(\alpha_{1,t}S_{1,t} + \alpha_{2,t}S_{2,t})}{S_{1,t}(c_r - X_{1,t})}.$$

   Since none of the illiquid assets must be sold in this case, we have $\lambda_{2,t} = 0$.

3. Finally, if

$$\alpha_{2,t}S_{2,t} - (L_t - \alpha_{0,t}S_{0,t} - \alpha_{1,t}S_{1,t}(1 - X_{1,t})) < c_r\alpha_{2,t}S_{2,t},$$

   then current holdings of cash and liquid asset are not sufficient to meet the regulatory capital requirement, and some of the illiquid asset holdings must be sold as well. In this case, all cash and liquid asset positions are liquidated ($\lambda_{0,t} = \alpha_{0,t}$ and $\lambda_{1,t} = \alpha_{1,t}$), and we find that

$$\lambda_{2,t} = \min\left(\frac{(L_t - \alpha_{0,t}S_{0,t} - \alpha_{1,t}S_{1,t}(1 - X_{1,t})) - (1 - c_r)\alpha_{2,t}S_{2,t}}{S_{2,t}(c_r - X_{2,t})}, \alpha_2\right).$$

   If $\lambda_2 = \alpha_2$, then the firm is effectively insolvent.

a. VaR

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	6.204	6.407	6.614	7.344
Variable Spreads, $\rho_1 = \rho_2 = 0$	6.204	6.398	6.595	7.380
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	6.204	6.434	6.672	7.659
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	6.204	6.459	6.736	7.850

b. ETL

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	6.635	6.825	7.030	7.740
Variable Spreads, $\rho_1 = \rho_2 = 0$	6.635	6.827	7.036	7.796
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	6.635	6.868	7.125	8.087
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	6.635	6.895	7.186	8.273

c. Insolvency Probability (in %)

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	0	0	0	0
Variable Spreads, $\rho_1 = \rho_2 = 0$	0	0	0	0
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	0	0	0	0.009
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	0	0	0	0.020

Table 1: 99%-Value at Risk, expected tail loss and probability of insolvency for the different mid-bid spreads scenarios. (Insolvency probability estimates are based on 200,000 trials)

a. VaR

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	7.050	7.300	7.624	8.606
Variable Spreads, $\rho_1 = \rho_2 = 0$	7.050	7.330	7.620	8.715
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	7.050	7.389	7.730	9.138
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	7.050	7.423	7.826	9.326

b. ETL

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	8.376	8.696	9.041	10.155
Variable Spreads, $\rho_1 = \rho_2 = 0$	8.376	8.710	9.071	10.262
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	8.376	8.809	9.277	10.774
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	8.376	8.875	9.420	11.095

c. Insolvency Probability (in %)

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	0.240	0.292	0.388	0.808
Variable Spreads, $\rho_1 = \rho_2 = 0$	0.240	0.288	0.396	0.836
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	0.240	0.324	0.464	1.084
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	0.240	0.340	0.504	1.208

Table 2: 99%-Value at Risk, expected tail loss and probability of insolvency with fat tails.

a. VaR

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	8.622	8.752	8.871	9.392
Variable Spreads, $\rho_1 = \rho_2 = 0$	8.622	8.755	8.872	9.414
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	8.622	8.774	8.908	9.577
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	8.622	8.786	8.932	9.689

b. ETL

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	8.864	9.014	9.183	9.935
Variable Spreads, $\rho_1 = \rho_2 = 0$	8.864	9.013	9.183	9.961
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	8.864	9.041	9.247	10.173
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	8.864	9.060	9.290	10.301

c. Insolvency Probability (in %)

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	0.136	0.272	0.512	2.668
Variable Spreads, $\rho_1 = \rho_2 = 0$	0.136	0.268	0.516	2.840
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	0.136	0.308	0.620	4.052
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	0.136	0.332	0.692	4.916

Table 3: 99%-Value at Risk, expected tail loss and probability of insolvency in the high-volatility case.

a. VaR

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	11.076	11.402	11.734	12.694
Variable Spreads, $\rho_1 = \rho_2 = 0$	11.076	11.402	11.711	12.669
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	11.076	11.470	11.837	12.955
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	11.076	11.519	11.932	13.184

b. ETL

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	13.903	14.198	14.488	15.350
Variable Spreads, $\rho_1 = \rho_2 = 0$	13.903	14.206	14.506	15.421
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	13.903	14.287	14.666	15.811
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	13.903	14.341	14.774	16.054

c. Insolvency Probability (in %)

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	2.136	2.356	2.616	4.232
Variable Spreads, $\rho_1 = \rho_2 = 0$	2.136	2.376	2.636	4.360
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	2.136	2.424	2.724	5.144
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	2.126	2.452	2.772	5.556

Table 4: 99%-Value at Risk, expected tail loss and probability of insolvency in the high-volatility case with fat tails.

a. VaR

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	8.622	8.752	8.871	9.392
Variable Spreads, $\rho_1 = \rho_2 = 0$	8.622	8.754	8.875	9.547
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	8.622	8.803	8.966	9.930
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	8.622	8.831	9.021	10.111

b. ETL

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	8.864	9.014	9.183	9.935
Variable Spreads, $\rho_1 = \rho_2 = 0$	8.864	9.014	9.193	10.100
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	8.864	9.076	9.339	10.540
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	8.864	9.121	9.440	10.770

c. Insolvency Probability (in %)

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	0.136	0.272	0.512	2.668
Variable Spreads, $\rho_1 = \rho_2 = 0$	0.136	0.272	0.544	3.624
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	0.136	0.348	0.844	6.660
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	0.136	0.424	1.124	8.740

Table 5: 99%-Value at Risk, expected tail loss and probability of insolvency with high spread volatility.

a. VaR

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	5.957	6.157	6.379	7.137
Variable Spreads, $\rho_1 = \rho_2 = 0$	5.957	6.157	6.376	7.168
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	5.957	6.189	6.460	7.433
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	5.957	6.218	6.505	7.613

b. ETL

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	6.370	6.565	6.774	7.486
Variable Spreads, $\rho_1 = \rho_2 = 0$	6.370	6.568	6.782	7.544
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	6.370	6.607	6.867	7.809
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	6.370	6.633	6.922	7.967

c. Insolvency Probability (in %)

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	0	0	0	0
Variable Spreads, $\rho_1 = \rho_2 = 0$	0	0	0	0
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	0	0	0	0
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	0	0	0	0.001

Table 6: 99%-Value at Risk, expected tail loss and probability of insolvency for the "cash-last" liquidation strategy. (Insolvency probabilities estimated from 200,000 trials.

a. Cash first

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	0	0.049	0.104	0.319
Variable Spreads, $\rho_1 = \rho_2 = 0$	0	0.049	0.105	0.323
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	0	0.055	0.118	0.375
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	0	0.059	0.127	0.410

b. Cash last

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	0	0.069	0.147	0.448
Variable Spreads, $\rho_1 = \rho_2 = 0$	0	0.069	0.147	0.453
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	0	0.077	0.164	0.515
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	0	0.081	0.174	0.554

Table 7: Comparison of average transactions costs of the two liquidation strategies over the 10-day simulation period.

a. VaR

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	6.901	7.172	7.439	8.380
Variable Spreads, $\rho_1 = \rho_2 = 0$	6.901	7.143	7.458	8.516
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	6.901	7.235	7.579	8.797
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	6.901	7.261	7.668	8.976

b. ETL

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	8.162	8.489	8.832	9.850
Variable Spreads, $\rho_1 = \rho_2 = 0$	8.162	8.502	8.856	9.960
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	8.162	8.598	9.051	10.424
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	8.162	8.663	9.184	10.714

c. Insolvency Probability (in %)

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	0.192	0.256	0.340	0.684
Variable Spreads, $\rho_1 = \rho_2 = 0$	0.192	0.264	0.340	0.700
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	0.192	0.288	0.412	0.868
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	0.192	0.296	0.444	0.984

Table 8: 99%-Value at Risk, expected tail loss and probability of insolvency with fat tails for the “cash-last” liquidation strategy.

a. VaR

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	8.307	8.458	8.587	8.987
Variable Spreads, $\rho_1 = \rho_2 = 0$	8.307	8.451	8.583	8.994
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	8.307	8.476	8.616	9.152
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	8.307	8.495	8.637	9.270

b. ETL

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	8.573	8.715	8.861	9.489
Variable Spreads, $\rho_1 = \rho_2 = 0$	8.573	8.715	8.860	9.507
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	8.573	8.737	8.907	9.697
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	8.573	8.752	8.936	9.813

c. Insolvency Probability (in %)

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	0.072	0.104	0.176	0.964
Variable Spreads, $\rho_1 = \rho_2 = 0$	0.072	0.108	0.180	0.988
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	0.072	0.128	0.208	1.368
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	0.072	0.132	0.224	1.648

Table 9: 99%-Value at Risk, expected tail loss and probability of insolvency in the high-volatility case for the "cash-last" liquidation strategy.

a. Cash first

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	0	0.131	0.274	0.802
Variable Spreads, $\rho_1 = \rho_2 = 0$	0	0.131	0.275	0.807
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	0	0.142	0.299	0.886
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	0	0.149	0.315	0.933

b. Cash last

Spread (liquid/illiquid asset)	No spread	0.1%/0.5%	0.2%/1%	0.5%/2.5%
Constant Spreads	0	0.154	0.323	0.938
Variable Spreads, $\rho_1 = \rho_2 = 0$	0	0.154	0.323	0.943
Variable Spreads, $\rho_1 = \rho_2 = -0.5$	0	0.166	0.349	1.022
Variable Spreads, $\rho_1 = \rho_2 = -0.8$	0	0.173	0.365	1.068

Table 10: Comparison of average transactions costs of the two liquidation strategies in the high-volatility case.