1. Open book, open notes.
2. NO COMMUNICATION OF ANY KIND WITH ANYONE BUT THE INSTRUCTOR AND THE COURSE ASSISTANT
3. Write your answers on $81 / 2$ by 11 sheets of paper and staple them together.
4. Sign the Honor Code on the sheet attached to this exam. Staple that sheet to the top of your answers.
5. Your answers should be concise and to the point. All equations must be simplified as much as possible.
6. In all cases you must demonstrate how you reached your results.
7. Graders like to see diagrams to show that you really do know how the results were obtained.
8. Concise and neat solutions make graders feel happy. Happy graders are more likely to give the benefit of the doubt.

## Question 1 (50 Points)

The Alpha company is planning to introduce a new product into a new market. It must decide which of two types of manufacturing plants to construct for this product. The company can use a non-automated process which would lead to annual costs of $8 q_{a}$, where $q_{a}$ is the company's total output. This process is publicly available and other firms could manufacture the product using this technology. Alternatively, it could develop a highly automated factory which could not be duplicated by competitors. The total annual costs would be $60+\mathrm{q}_{\mathrm{a}}$ if the highly automated process were used.

The firm believes that the market inverse demand function for the product is:

$$
\mathrm{P}=20-\mathrm{Q},
$$

where P is the market price and Q is the total industry sales.
a) (10 Points) Suppose that Alpha company were certain it could maintain a monopoly position for sales of this product and that it would never face any threats of entry by competitors.
(i) Which technology should the Alpha company adopt?
(ii) What is the profit maximizing quantity of production and price of the product?
(iii) What would be the annual profits that it could earn?
b) (20 Points) Suppose that Alpha company believed that Beta firm would enter the market immediately after Alpha company began selling its product. Beta firm would have access only to the non-automated process. Once Beta firm entered the market, sales and pricing would be set as a Nash-Cournot equilibrium, with each firm choosing quantity as its decision variable.
(i) Which technology should the Alpha company adopt?
(ii) What is the profit maximizing quantity of production by Alpha company and Beta firm. What is the equilibrium price of the product?
(iii) What would be the annual profits that Alpha Company could earn?
c) (20 Points) Suppose that Alpha company believed that Beta firm would enter the market immediately after Alpha company began selling its product. Beta firm would have access only to the non-automated process. Once Beta firm entered the market, sales and pricing would be set as a Bertrand equilibrium, with each firm choosing price as its decision variable.
(i) Which technology should the Alpha company adopt?
(ii) What is the profit maximizing quantity of production by Alpha company and Beta firm. What is the equilibrium price of the product?
(iii) What would be the annual profits that Alpha Company could earn?

Question $2 \quad(80$ Points)
Consider a simple economy in which there are two factors of production, land (N) and labor (L), in fixed quantities. The initial endowment in the economy consists of 1000 units of land and 3000 units of labor. Two goods can be produced using these two factors: Food (F) and Clothing (C). Food can be produced using a technology that can be approximated by a constant-returns-to-scale Cobb-Douglas production function:

$$
F=10 L_{F}^{.5} N_{F}^{.5}
$$

where $F$ is the quantity of food produced, $L_{F}$ is the quantity of labor used in food production and $N_{F}$ is the quantity of land used in food production.

Similarly, clothing can be produced using a technology that can be approximated by a constant-returns-to-scale Cobb-Douglas production function:

$$
C=20 L_{C}{ }^{8} N_{C}{ }^{.2}
$$

where $C$ is the quantity of clothing produced, $L_{C}$ is the quantity of labor used in clothing production and $N_{C}$ is the quantity of land used in clothing production.

Let $\mathrm{P}_{\mathrm{N}}$ and $\mathrm{P}_{\mathrm{L}}$ be the prices of land and labor, respectively.
a) (10 Points) Using an Edgeworth box, diagram the locus of quantities of land and labor to be allocated to the production of the two goods, food and clothing, which could be consistent with a competitive equilibrium. (No calculation required here.) In your diagram, be sure to label the axes, indicating the lengths of the box sides. Be sure to show the relevant isoquants and diagram the competitive equilibrium allocations.
b) (10 Points) For the competitive equilibrium allocations of land and labor, derive simple mathematical expressions relating the ratio $L_{F} / N_{F}$ and the ratio $L_{C} / N_{C}$ to the ratio of prices, $\mathrm{P}_{\mathrm{N}} / \mathrm{P}_{\mathrm{L}}$.
c) (10 Points) Develop closed form expressions for $L_{F}, N_{F}, L_{C}$, and $N_{C}$, each as a function of the ratio of prices, $\mathrm{P}_{\mathrm{N}} / \mathrm{P}_{\mathrm{L}}$.
d) (10 Points) For what range of the ratio $\mathrm{P}_{\mathrm{N}} / \mathrm{P}_{\mathrm{L}}$ will the competitive equilibrium production of food be equal to zero? For what range of the ratio $\mathrm{P}_{\mathrm{N}} / \mathrm{P}_{\mathrm{L}}$ will the competitive equilibrium production of clothing be equal to zero?
e) (10 Points) Use a spreadsheet or similar mathematical software to plot the production possibility set for this economy. On this plot, write the numerical values of food and clothing production for the following points:

- Zero production of food
- Zero production of clothing
- One half of all land allocated to food production, the other half allocated to clothing production
- One half of all labor allocated to food production, the other half allocated to clothing production
f) (10 Points) Develop closed form expressions for the competitive equilibrium prices of clothing and of food, as functions of $\mathrm{P}_{\mathrm{N}}$ and $\mathrm{P}_{\mathrm{L}}$. Show that the prices of clothing and food are homogeneous of degree one in $\mathrm{P}_{\mathrm{N}}$ and $\mathrm{P}_{\mathrm{L}}$.
f) (10 Points) Assume that this country exports food and clothing and that the world price of food is $\$ 20$ per unit and the world price of clothing is $\$ 6$ per unit. Numerically calculate:
- The price of labor
- The price of land
- The competitive equilibrium output of food
- The competitive equilibrium output of clothing
- $L_{F}$
- $\quad N_{F}$
- $L_{C}$
- $\quad N_{C}$

Question 3. (50 Points)
Assume there exists an economy for which the value of imports justs equals the value of exports, so that there is never any net exports or imports. In this economy there exists a government and a private sector. Investment (I) expenditure is a decreasing function of the interest rate, r.
Consumption (C) expenditure is an increasing function of Disposable Income, where Disposable Income is Gross National Product (denoted by Y) minus Total Taxes Paid. Consumption does not depend on the interest rate. Total Taxes Paid (denoted by T) is an aggregate of all taxes. These are assumed to be lump sum taxes so that T is not a function of Y . The government can issue money or issue bonds. Corporations also issue bonds. Government expenditure (G) is a variable controlled directly as a policy instrument, as is the supply of money, M.

For the purpose of this problem, you should ignore any non-linearities of any of the functions, calculating only first order differentials when you are asked to show changes. Assume that the economy is not at full employment.
a) (10 Points) Derive an expression showing the change in $\mathrm{Y}(\Delta \mathrm{Y})$ that would result from a small increase in government spending $(\Delta \mathrm{G})$, financed by an equal increase in taxes. The ratio of the change in Y to the change in G (or T) is called the "balanced budget multiplier." What is the numerical value of this expression? Does Disposable Income increase or decrease in response to an increase in taxes matched by an equal increase in government spending?
b) (20 Points) Derive an expression showing the change in $\mathrm{Y}(\Delta \mathrm{Y})$ that would result from a small increase in government spending $(\Delta \mathrm{G})$, financed by an equal increase in government bonds. Is the ratio of $\Delta \mathrm{Y} / \Delta \mathrm{G}$ smaller than, greater than, or equal to 1.0 ?
c) (20 Points) Derive an expression showing the change in $\mathrm{Y}(\Delta \mathrm{Y})$ that would result from a small increase in government spending $(\Delta \mathrm{G})$, financed by an increase in the supply of money. Is the ratio of $\Delta \mathrm{Y} / \Delta \mathrm{G}$ smaller than, greater than, or equal to 1.0 ?

Question 4. (50 Points)
Consider the market for natural gas in a small country. Assume that many firms produce natural gas, but that all natural gas, once produced, is purchased by only one firm, who has a monopoly in the wholesale market of natural gas. Assume that that monopolistic wholesaler also owns all the natural gas pipelines. Assume that there are many retailers for natural gas competing with each other, but each must buy natural gas from the single wholesaler. Once they buy that gas they must pay to have it transported through the pipelines. Assume that the monopolist maximizes profits. Assume that the supply of natural gas can be described by an upward sloping supply function (ignore any issues associated with the natural gas as a depletable resource).
a) (10 Points) Develop necessary conditions to show the profit maximizing wholesale price and profit maximizing price of natural gas pipeline transportation services.
b) (10 Points) Show that the equilibrium of part a) is not economically efficient, under the assumption that all other markets in the economy are operating competitively. Show that the consumer price is higher and the producer price is lower than would be economically efficient.
c) (10 Points) The government of the country now regulates the price that can be charged for natural gas pipeline transportation services, so as to force a reduction in the transportation charges. Show that this action will not influence the price that consumers pay for natural gas nor the price received by those who produce natural gas. Explain why this is so.
d) (10 Points) The government tries another approach and forces the monopolist to sell the pipeline to a entirely independent firm. That firm is allowed to operate as a monopolist and to set its own price for transportation services but is not allowed to enter into any negotiations about prices or quantities with the wholesale monopolist. Show that this action will increase the price that consumers pay for natural gas and decrease the price received by those who produce natural gas. Explain why this is so.
e) (10 Points) Beginning with the situation in part d), the pipeline monopolist and the wholesale monopolist are now allowed to make agreements with each other about their pricing rules, so this becomes a cooperative game. Show that in the solution to this game, both firms can make more profits than would be the case in part d). Show, in addition, that the consumer price of gas and the producer price will be identical to those in part a) and c).

