EES 255   FINAL EXAM

1. Sign the Honor Code, observing all honor code provisions. Do not sit directly next to any other student.
2. Be sure to put your name on the cover of each blue book.
3. The exam will last for three hours, from 8:30 - 11:30 a.m.
4. Each question is worth the points indicated below.
5. Open book, open notes.
6. Your answers should be concise and to the point. All equations must be simplified as much as possible.
7. In all cases you must demonstrate how you reached your results.
8. We like diagrams which show that you really know how the results were obtained.
9. Concise and neat solutions make graders feel happy. Happy graders are more likely to give the benefit of the doubt.

Question Weights:

Q1	15 Points
Q2	25 Points
Q3	30 Points
Q4	30 Points
Total	100 Points

Whenever you need an interest rate for any problem, assume an annual interest rate, r, equal to 10% per year.

1.   (15 Points)   Optimal Depletion Paths (No Stock Effects)

The optimal extraction paths for three different depletable resource optimization problems are given below. In each case cost is a function of extraction rate but there are no stock effects:
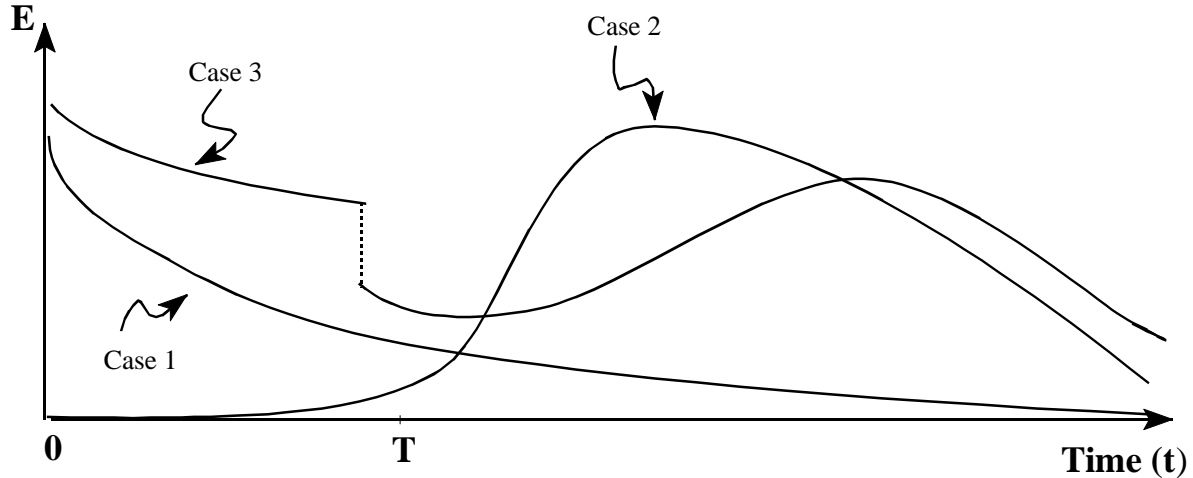
a)   (4 Points)   If the technology is not changing in any of the three cases, what can be said about expectations of price changes and the actual price changes over time for each of the three cases?

b)   (4 Points)   If the price remains constant over time in each of the three case, what can be said about expectations of technology changes (cost function changes) and actual technology changes over time for each of the three cases?

c)   (3 Points)   For Case 1 only, sketch the optimal extraction path if the interest rate were to suddenly and unexpectedly increase at time T.

d)   (4 Points)   For Case 1 only, sketch the optimal extraction path if at $t = 0$ the government were to impose a marginal tax of \$5 per unit of the resource extracted. Sketch the optimal extraction path and compare it to that shown here for Case 1. Would the new opportunity cost at $t = 0$ be higher or lower if the tax were imposed?

2.   (25 Points)   Competitive Market Clearing

Assume that there are 100 identical perfectly competitive firms each producing a depletable resource. Each has an large initial but not unlimited stock of the resource. Each has an extraction cost function:

$$C(E_t) = \$5\ E_t + 0.1\ E_t^2$$

In the economy the demand for the output is 2500 units per year, independent of price. A backstop technology can be used to produce a perfect substitute for the output of the depletable resource industry. This substitute can be produced in unlimited quantities for $30 per unit. Thus the demand for the output of the depletable resource industry is 2500 units per year for all prices below $30, is zero for all prices above $30, and can be anything between 0 and 2500 at a price of $30 per unit.

Assume that a competitive equilibrium has been obtained. In that competitive equilibrium, it is observed that the price rises to $30 per unit at t = 20.

a)   (15 Points)   Derive a closed form expression for competitive equilibrium price as a function of time.   What is the numerical value of the market clearing price at t = 0?

b)   (5 Points)   Derive a closed form expression for the competitive equilibrium quantities of the depletable resource extraction over time. What is the numerical value of industry's extraction of the depletable resource at t = 22?

c)   (5 Points)   Sketch the following:
(i)   the market clearing price as a function of time;
(ii)   the market clearing quantity as a function of time.

3.   (30 Points)   Atmospheric Carbon Dioxide

Carbon dioxide is emitted into the atmosphere as fossil fuels are burned. Here you will be asked to use <u>depletable</u> resource theory (with stock effects) to model the optimal emission rate of carbon dioxide, although initially the formulation might seem like a renewable resource problem.

Let M_t denote the rate per year that carbon dioxide is emitted into the atmosphere. Assume that carbon dioxide is removed from the atmophere at a constant rate K per year, independent of the stock of carbon dioxide. Let E_t represent the emissions rate <u>net of</u> the removal rate:

$$E_t = M_t - K$$

E_t will be referred to as the net emissions rate.

The stock of carbon dioxide in the atmosphere is denoted by S_t. It varies over time according to the equation:

$$S_t = S_{t-1} + E_t$$

Assume that competitive markets would lead to a net emissions rate of E_0 and that increases or decreases of the net emissions rate from E_0 would lead to cost increases to the users of fossil fuels. These cost increases are quadratic in the variation from E_0. Thus the cost to fossil fuel users of varying the emissions rate is:

$$C^E(E_t) = A [E_t - E_0]^2$$

There is also a externality associated with increases in the concentration of carbon dioxide above its natural level of S_0. This additional social cost is quadratic in stock increases above S_0:

$$C^S(S_t) = B [S_t - S_0]^2$$

The coefficients A and B are positive. Total cost to all society is the sum of $C^E(E_t)$ and of $C^S(S_t)$. Our goal is to find the net emissions over time which minimizes the discounted present value of the total cost to all society. Assume an infinite time horizon.

a)   (8 Points)   State the constrained minimization problem. Then derive the first order necessary conditions for the optimization.

b)   (4 Points)   Draw a phase diagram in a space of S and current value oppotunity cost (ϕ), illustrating the locus of constant S, the locus of constant ϕ, the directions of change of S and ϕ, and the convergent trajectory.

c)   (4 Points)   What is the optimal steady state value of net emission rate, E_t? of S_t? of ϕ_t?

d)   (4 Points)   Demonstrate for $S_t < S_0$, that ϕ_t must always grow, but at a rate of growth smaller than r.

e)   (2 Points)   Show that the optimal value of net emissions must decline over time.

f)   (8 Points)   Assume now that the interest rate is increased. Show that the steady state stock of carbon dioxide in the atmosphere is an increasing function of r. Show that initially E_t must be increased as a result of the interest rate increase.

4.   (30 Points)   California Whaling Industry

There exists a very rare whale sometimes sighted off the shores of the goled gate, the "cardinal whale." This particular whale is highly valued by local fisherpeople (who fish in blue and gold ships). Costs of harvesting include a private cost borne by the fisherpeople, $C^F(E_t)$ plus a social cost borne by non-fisherpeople, $C^S(S_{t-1})$:

$$C^F(E_t) \quad = \quad 3\ E_t \ + \ 0.02\ E_t^2$$
$$C^S(S_{t-1}) \quad = \quad 2\ [\ 1000\ -\ S_{t-1}]$$

Note that the units have been scaled down to facilitate easier CALculations. Note also that the social cost will occur even if no cardinal whales are being harvested.

Let the growth rate of cardinal whales be $G(S_{t-1})$:

$$G(S_{t-1}) \quad = \quad 0.1\ S_{t-1}\ [1-S_{t-1}/1000]$$

The price of harvested whales is $20 for all t.

a)   (5 Points)   Calculate the first order conditions for optimal harvesting of the cardinal whale, including the social cost.

b)   (5 Points)   Assuming no harvesting of the stock, calculate <u>all</u> steady state equilibrium stocks of the cardinal whale.

c)   (10 Points)
    (i)   Calculate the maximum sustainable yield (MSY) of the cardinal whale, and the stock which corresponds to the MSY.
    (ii)   Calculate the annual profit, <u>excluding</u> the social cost, at this level of harvest.
    (iii)   Calculate the annual profit, <u>including</u> the social cost, at this level of harvest.
    (iv)   Calculate the NPV of profits (including the social cost) for this level of harvest assuming an infinite time horizon.

d)   (7 Points)   Now assume that the opportunity costs and social costs are completely ignored by the short-sighted and selfish blue and gold fisherpeople.

    (i)   Determine the short-run private-profit-maximizing harvest rate for cardinal whales for each year. Assume that the initial stock of whales, S_0 is 1,000. Calculate the NPV of profits <u>excluding</u> the social cost from harvesting the cardinal whale. (Be sure to state your equation showing the stock adjustment dynamics.)
    (ii)   Using the harvest rate calculated in part (i), calculate the NPV of profits <u>including</u> the social cost from the cardinal whale.

e)   (3 Points)   What would you conclude from your answers to c) and d) about the relationship between harvest rates chosen by blue and gold fisherpeople and the optimal harvest rates?