EES 255   MIDTERM EXAM

1.   Sign the Honor Code, observing all honor code provisions. Do not sit directly next to any other student.
2.   Be sure to put your name on the cover of each blue book.
3.   The normal class time will be available for the exam.
4.   Each question is worth the same number of points.
5.   Open book, open notes.
6.   Your answers should be concise and to the point. All equations must be simplified as much as possible.
7.   In all cases you must demonstrate how you reached your results.
8.   We like diagrams which show that you really know how the results were obtained.
9.   Concise and neat solutions make graders feel happy. Happy graders are more likely to give the benefit of the doubt.

Whenever you need an interest rate for any problem, assume an annual interest rate, r, equal to 5% per year.

1.   Gasco is a natural gas retailer operating in a small country. Gasco owns all the natural gas pipelines and is the only retailer of gas. It faces little competition in the retail markets. However, the threat of regulation places some limits on its ability to increase gas prices. Currently Gasco is selling gas in the retail markets at a price of $4.00 per mcf (thousand cubic feet). The price-elasticity of demand for natural gas is -2 (measured for retail sales).

Gasco obtains gas by purchasing large quantities from Exxon. In particular, Gasco has a long-term contract under which it will receive natural gas from Exxon over the next fifteen years. It has a right to take 150 million mcf of gas during that time. It has agreed to pay Exxon $2.00 per mcf of gas for each of the 150 million mcf it has committed to buy, with payments equally spread across the 15 year contract life; thus Gasco has committed to pay $20 million per year over the next 15 years. This payment schedule is independent of the amount of gas Gasco takes each year. The payment is required under the contract even if Gasco does not take all 150 million mcf (this is a take-or-pay contract).

Gasco also faces costs of transporting and distributing the gas. All of these costs (other than the cost of acquiring the gas) total $1.00 per mcf.

Gasco has planned its purchases so that it would finish taking the gas in the 15th year. It could acquire gas from other sources at a price of $4.00 per mcf at any time.

(a)   What is Gasco's marginal cost for natural gas sold in the retail market during the first year of the contract? Be sure to include marginal costs of transporting, distributing, and acquiring the gas.

(b)   What is Gasco's opportunity cost of natural gas during the first year of the contract?

(c)   Is Gasco pricing at the monopolistic profit maximizing price? If not, approximate how much above or below the monopoly price Gasco is pricing its gas. In developing your approximation, you should not take into account the impact of changing price on the year that Gasco would finish taking its available gas.

(d)   Is Gasco pricing at the competitive equilibrium price? If not, approximate how much above or below the competitive equilibrium price Gasco is pricing its gas. In developing your approximation, you should not take into account the impact of changing price on the year that Gasco would finish taking its available gas.

2.   A renewable resource is characterized by a cost function:

$$C(E_t, S_{t-1}) = a E_t^2 - b S_{t-1}$$

where E is the extraction rate and S is the stock. a and b are positive constants. The adjustment dynamics are described by the following equation:

$$S_t = S_{t-1} + h - K S_{t-1} - E_t$$

where h and K are positive constants. K is strictly smaller than 1.0.

Once extracted, the resource can sell at a market price of P; P does not change over time. The firm owning the resource has a very long time horizon. Ignore any effects associated with approaching a final time period.

(a)   Find the first-order necessary conditions which characterize the optimal extraction rate and that characterize the evolution of the opportunity cost.

(b)   Draw a phase diagram for this renewable resource in a space of stock vs. opportunity cost. Draw arrows to show the direction of movement of the stock and the opportunity cost in the four regions of this diagram.

(c)   Using the phase diagram, show that along the convergent trajectory, the opportunity cost remains constant over time. Tell what is the value of the opportunity cost along the convergent trajectory.

(d)   Develop a closed-form equation for the optimal extraction rate. This expression can (but need not necessarily) include the stock and parameters of the problem. Does the extraction rate change over time? If so, describe the changes over time for the optimal extraction rate.

(e)   What is the steady state extraction rate and stock of the resource?