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MIDTERM EXAM

Open book, open notes. Your answers should be concise and to the point. In all cases where numerical answers are requested, you should demonstrate how you reached your results.

Question 1 (30 Points)

Two sports goods stores in a small town are competing with one another. Each week they advertise the prices for many items in the local newspapers. Thus, neither knows what prices their rival will offer until they set their own prices. As they set prices each week, they can use either a moderate price strategy or a low price strategy.

Assume that the following weekly profit payoff matrix exists for the two rivals; e.g. if A chooses a moderate price strategy and B chooses a low price strategy, then A will earn a weekly profit of $2,000 and B will earn a weekly profit of $6,000.

		Player B: Moderate	Player B: Low
Player A	Moderate	\$10,000; \$10,000	\$2,000; \$6,000
	Low	\$6,000; \$2,000	K_A; K_B

a.) What is (are) the pure strategy Nash-Equilibrium if $K_A = K_B = \$4,500$?

b.) What is (are) the pure strategy Nash-Equilibrium if $K_A = \$4,500$, and $K_B = \$1,500$?

c.) What is the mixed strategy equilibrium and the expected profits for each rival if $K_A = K_B = \$4,500$?

d.) Suppose you are A and B is not playing the Nash equilibrium strategy. Should you play your Nash equilibrium strategy? Give an example if your answer is "no".

e.) Briefly discuss whether you expect these firms to play the Nash equilibrium strategies you have identified above. In your answer you should take into account the repeated nature of the game, the need to commit to prices one week in advance, and communications options that might be available in a small town.

Question 2 (40 Points)

There are two firms in a market selling audio cassette tapes. The tapes are not perfect substitutes; they are differentiated products. Firm 1 sells a product called "Ultra Chrome" and Firm 2 calls its product "Pro Ferrox". Let x_1 and p_1 be the quantity and the price of Ultra Chrome, and x_2 and p_2 be those of Pro Ferrox.

The market demand functions for the two products are:

$$x_1 = (100 + p_2 - 2 p_1) / 6$$
$$x_2 = (100 + p_1 - 2 p_2) / 6$$

Equivalently, these demand functions can be written as inverse demand functions:

$$p_1 = 100 - 4x_1 - 2x_2$$
$$p_2 = 100 - 4x_2 - 2x_1$$

The cost function of Firm *i* is given by $C(x_i) = 5 x_i$ for $i = 1,2$ (the marginal cost is constant.)

a.) Suppose that the two firms each simultaneously choose a quantity, playing as a Cournot duopoly:
   - i.) Specify the profit maximization problem of Firm 1 and the profit maximization problem of Firm 2.
   - ii.) Derive the best reaction function of Firm 1.
   - iii.) Derive the best reaction function of Firm 2.
   - iv.) Find the Cournot equilibrium quantity, price, and profit of each firm.

b.) Suppose that the two firms simultaneously choose a price, operating as a Bertrand Duopoly.
   - i.) Specify the profit maximization problem of Firm 1 and the profit maximization problem of Firm 2.
   - ii.) Derive the best reaction function of Firm 1.
   - iii.) Derive the best reaction function of Firm 2.
   - iv.) Find the Bertrand equilibrium price, quantity, and profit of each firm.

c.) Suppose firm 1 is the Stackleberg leader setting a price:
   - i.) Specify the profit maximization problem of Firm 1 and the profit maximization problem of Firm 2.
   - ii.) Find the Stackleberg equilibrium price, quantity, and profit of each firm.

d.) Compare the prices between the three market structures and give an intuitive explanation for the price differences that you predict in parts a) through c).

Question 3 (30 Points)

Assume that there are N people and M goods in an exchange economy, where N is a large number. Each person has some initial endowment of the various goods. Let the vector w_i represent the initial endowments of the various commodities held by person i. Let the vector x_i represent the final allocation of goods chosen by person i. Let the (transpose of the) equilibrium price vector be represented by P. Each person has continuous, convex, monotonic preferences.

In addition, there is a government in this economy. This government charges a tax to each person based on the final allocation to that person. The sales tax is the same for all commodities.
The total amount of tax that person i pays is

$$T \;\; P \cdot x_i$$

where T is the tax rate. T is a positive number much smaller than 1.0. (The tax rate T is multiplied by the inner product of the price vector and the quantity vector.) The government spends all the revenue it collects on commodity k.

The government maintains a balanced budget: its total expenditure just equals its total revenue.

a) Define the budget constraint for each person.

b) Derive and mathematically state Walras' Law for this economy. In your statement, explicitly separate the government taxation and expenditure from the supplies and demands for goods by the group of people in the economy. We do not need a rigorous proof but rather a demonstration of how you obtained this mathematical statement of Walras' Law.

c) Give the mathematical statement for general equilibrium in this economy in terms of the price vector and the excess demand vector.

d) Explain how one could prove that a general equilibrium exists for this economy. Give a statement of the assumptions that underlie a proof and sketch out the basic ideas of a proof. You do not have to complete the formal proof as long as you provide the main ideas.