Handout #2

No class announcements.

Solutions to Problem Set 1: Warm-up and Monopoly

Warm-up

1.) $P = 100 - 0.01\,Q$ and $C = 50\,Q + 30{,}000$

where Q is the weekly production and P is price in cents/ unit.

a.) Find Q, P, and Π

$\text{Rev} = P \cdot Q = (100 - 0{,}01\,Q)\,Q = 100\,Q - 0.01\,Q^2$

$MR = 100 - 0.02\,Q$

$MC = 50$

$MR = MC \Rightarrow 100 - 0.02\,Q = 50 \Rightarrow Q^* = \underline{2{,}500}$

$P = 100 - 0.01\,(2500) = \underline{75\ (\text{cents/ unit})}$

$\Pi = \text{Rev} - \text{Cost} = (75)\,(2500) - 50\,(2500) - 30000 =$

$32500\ (\text{cents/ week}) = \underline{325\ (\$/\ \text{week})}$

b.) Government tax of 10 cent/ unit paid by firm:

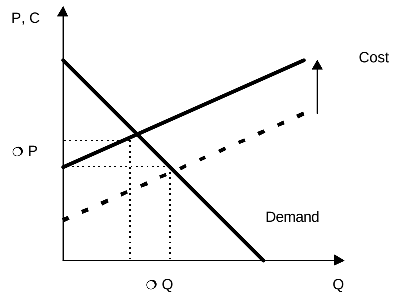
$C = 60\,Q + 30000 \Rightarrow MC = 60$

$MR = MC \Rightarrow 100 - 0.02\,Q = 60 \Rightarrow Q^* = \underline{2000}$

$P = 100 - 0.01\,(2000) = \underline{80\ (\text{cents/ unit})}$

$\Pi = (80)\,(2000) - 60\,(2000) - 30000 = 10000\ (\text{cents/ week}) = \underline{100\ (\$/\ \text{week})}$

Figure 1: firm-paid tax



Government tax of 10 cent/ unit paid by consumers:

$P + 10 = 100 - 0.01\ Q \Rightarrow P = 90 - 0.01\ Q$

$Rev = (90 - 0.01\ Q)\ Q = 90\ Q - 0.01\ Q^2 \Rightarrow MR = 90 - 0.02\ Q$

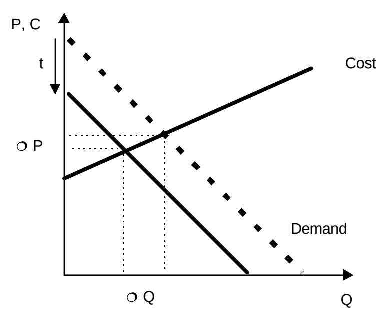
$MR = MC \Rightarrow 90 - 0.02\ Q = 50 \Rightarrow Q^* = \underline{2000}$

$P = 90 - 0.01\ (2000) = \underline{70\ \text{(cents/ unit)}}$

$\Pi = (70)\ (2000) - 50\ (2000) - 30000 =$

10000 (cents/ week) = <u>100 ($/ week)</u>

Figure 2: consumer-paid tax



Monopoly I

2.) From C = 35 Q + 200, we get MC = 35.

a.) The formula for the optimal pricing policy of the monopolist is

$$p(y) = \frac{MC(y)}{1 - \frac{1}{|e(y)|}}$$

For $|\varepsilon(y)| = 1.5$, we get

p(y) = 35 / 1 - (1/1.5) = <u>105</u>

Therefore, the markup multiple that the monoplist charges over his MC is given by

$$\frac{1}{1 - \frac{1}{|e(y)|}}$$

b.) In the markup formula, when $|\varepsilon(y)| \rightarrow \infty$, the firm faces an infinitely elastic demand curve. This means that for a competitve firm price simply equals marginal cost.

$$\frac{1}{1 - \frac{1}{|e(y)|}} \rightarrow 1 \qquad \text{for } |\varepsilon(y)| \rightarrow \infty$$

Monopoly II

3.) $Q_1 = 200 - 2\,P$

$Q_2 = 100 - 4\,P$

a.) Usually, you would combine the two demand curves and then set MR_{1+2} equal MC. However, in this example it turns out that the producer is better off to just sell to group 1.

Therefore,

MR = MC $\Rightarrow$ 100 - Q = 5 $\Rightarrow$ Q = <u>95</u>

P = 100 - (95/ 2) = <u>52.50</u> ($/ unit)

Π = 100 (95) - (95·2/ 2) - 5 (95) = <u>4512.5</u>

b.) For the case of price discrimination, we have two seperate demand functions as stated above.

$MR = 100 - Q_1 \Rightarrow MR = MC \Rightarrow 100 - Q_1 = 5 \Rightarrow Q_1 = \underline{95}$

$MR = 25 - 0.25\ Q_2 \Rightarrow MR = MC \Rightarrow 25 - 0.25\ Q_2 = 5 \Rightarrow Q_2 = \underline{40}$

$\Pi = (95 \cdot 52.50) + (40 \cdot 15) - (95+40) \cdot 5 = \underline{4912.50}$

c.) In the case of no-price disrimination, the total of consumer surplus (CS) and producer surplus (PS) is the following:

$CS_{1+2} = 0.5\ (95)\ (100-52.5) = 2256.25$

$PS_{1+2} = (52.5-5)\ (95) = 4512.5 \Rightarrow$ equal to profit because there are no fixed costs

$CS_{1+2} + PS_{1+2} = 6768.75$

For the competition case, where price equals marginal cost:

$CS_{comp} = 0.5\ (100-25)\ (150) + (150)\ (25-5) + 0.5\ (25-5)\ (270-150) = 9825$

$PS_{comp} = 0$

The "efficiency loss" of the no-price discrimination situation is therefore

$9825 - 6768.75 = \underline{3056.25}$

In the case of price discrimination, the total consumers' plus producers' surplus is:

Group 1: $CS = 0.5\ (95)\ (100-52.5) = 2256.25$ and $PS = (95)\ (52.5-5) = 4512.5$

$\Rightarrow CS_1 + PS_1 = 6768.75$

For the competitive case: $CS + PS = 0.5\ (190)\ (100-5) + 0 = 9025 + 0$

$\Rightarrow CS_{comp1} + PS_{comp1} = 9025$

Group 2: same procedure

$\Rightarrow CS_2 + PS_2 = 600$

$\Rightarrow CS_{comp2} + PS_{comp2} = 800$

The "efficiency loss" of the price discrimination situation is therefore $9825 - 7368.75 = \underline{2456.25}$

d.) The loss in total surplus, or "efficiency loss", is lower in the situation of price discrimination. This is because without price discrimination, the producer would not sell to the people in the second group at all. With price discrimination, however, a larger number of people can buy the product, thus reducing the efficiency loss. In this particular case, it is evident that when we increase the producer's profit in the case of price discrimination by selling to group 1 on the same terms as without price discrimination and, in addition, by selling to group 2, the consumers' plus the producers' surplus overall is increased.

Monopoly III

4.) De Beers and the diamond cartel: the De Beers diamond cartel was formed by Sir Ernest Oppenheimer, a South African mine operator, in 1930. It has since grown into one of the world's most successful cartels. De Beers handles over 80% of the world's yearly production of diamonds and has managed to maintain this near-monoploy for several decades. Over the years, De Beers has developed several mechanisms to maintain control of the diamond market.

First, it maintains considerable stocks of diamonds of all types. If a producer attempts to sell outside the cartel, De Beers can quickly flood the market with the same type of diamond, thereby pushing the defector from the cartel. Second, large producers' quotas are based on the proportion of total sales. When the market is weak, everyone's production quota is reduced proportionally, thereby automatically increasing scarcity and raising price. Third, De Beers is involved at both the mining and the wholesaling levels of diamond production. In the wholesale market diamonds are sold to cutters in boxes of assorted diamonds: buyers take a whole box or nothing - they cannot choose individual stones. If the market is weak for a certain size of diamond, de Beers can reduce the number of those diamonds offered in the boxes, thereby making them more scarce. Finally, De Beers can influence the direction of demand for diamonds by the $110 million a year it spends on advertising. Again, this advertising can be adjusted to encourage demand for the types and sizes of diamonds that are in relatively scarce supply.

Recently, the situation in the former Soviet Union threatened the De Beers monopoly when the governement of Russia was expected to sell large quantities of diamonds to the international market in order to raise money for its federal budget deficit. Prices for diamonds were considered to be under pressure for quite some time but finally the Russian state decided that it was more lucrative to sell diamonds to the De Beers cartel and thus avoid a world market crash in price due to a sudden and large increase in supply.

Problem Set 2: Oligopoly and Game Theory (*due Monday, April 28*)

Oligopoly

1.) Assume that 10 identical firms in a purely oligopolistic industry form a centralized cartel. The total market demand function facing the cartel is $Q_d = 240 - 10\,P$ and each firm's short-run marginal cost is given by $q for q > 4.

a.) Find the optimal level of output and price for this cartel.

b.) How much should each firm produce if the cartel wants to minimize production costs.

c.) Find the cartel's profits if the short-run average costs of each firm at its best level of output is $12.

Game Theory

2.) Two firms compete by choosing price. Firm 1 produces product 1, firm 2 produces product 2. Their demand functions are

$$Q_1 = 20 - P_1 + P_2$$

$$Q_2 = 20 + P_1 - P_2 .$$

Marginal costs are zero.

a.) Suppose the two firms set their prices at the same time. Find the resulting Nash equilibrium. What price will each firm charge, how much will it sell, and what will its profit be?

b.) Suppose firm 1 sets its price first, and then firm 2 sets its price. What price will each firm charge, how much will it sell, and what will its profits be?

c.) Suppose you are one of these firms, and there are three ways you could play the game: i.) both firms set their price at the same time, ii.) you set your price first, iii.) your competitor sets his price first. If you could choose among these three alternatives, which would you prefer? Explain.

Game Theory

3.) Write up a real world example of a tit-for-tat strategy. (You might either rely on your own professional experience or business press/ Economic journal articles.)