

---

### Handout #3

---

Some remarks about grading Problem Set 1: Everybody has turned the problem set in on time - thanks, let this be a consistent pattern. Everyone got 100 percent on this problem set in accordance with our policy to base the grade on the efforts undertaken to solve the problems, and not solely on the results.

Favorite answers in the qualitative section on monopoly were the Stanford Bookstore (5), local cable companies (4), and PG&E (3). For those who were very brief on this part, I would like to see more analysis. Many had some difficulties on question 3c.) and would be well advised to review the corresponding sections on consumers' and producers' surplus in their textbooks. Overall comment: very good job!

By the way, you can pick up your Problem Sets in the EES&OR 341 box in the student mailroom.

---

### Solutions to Problem Set 2: Oligopoly and Game Theory

---

#### *Oligopoly I*

1a.) Set the industry MR to its MC with the cartel operating as "one" big firm.

$$Q = 240 - 10P \Rightarrow P = 24 - (Q/10) \Rightarrow \text{Rev} = (24 - (Q/10)) \cdot Q \Rightarrow \text{MR} = 24 - (Q/5)$$

$$\text{MC} = q = \Sigma q/10 = Q/10$$

$$\text{MR} = \text{MC} : Q/10 = 24 - (Q/5) \Rightarrow Q = \underline{80} \Rightarrow P = \underline{16}$$

b.) The output for each firm is just the industry output divided by the number of firms in the cartel:  $Q/10 = \underline{8}$ .

c.)  $\Pi_{\text{cartel}} = (\text{Price} - \text{Average Cost}) Q = (16-12) 80 = \underline{320}$

#### *Game Theory I*

2a.)  $\Pi_1 = (20 - P_1 + P_2) P_1$

$$\Pi_2 = (20 + P_1 - P_2) P_2$$

$$d \Pi_1 / d P_1 = 20 - 2P_1 + P_2 = 0$$

$$d \Pi_2 / d P_2 = 20 + P_1 - 2P_2 = 0$$

$$P_1 = \frac{1}{2} (20 + P_2)$$

$$P_2 = \frac{1}{2} (20 + P_1)$$

$$P_1 = P_2 = \underline{20}$$

$$Q_1 = Q_2 = \underline{20}$$

$$\Pi_1 = \Pi_2 = \underline{400}$$

b.) Substitute firm 2's (price) reaction function into firm 1's profit function.

$$\max_{P_1} \left(20 - P_1 + \frac{20 + P_1}{2}\right) P_1 = \max_{P_1} \left(30 - \frac{P_1}{2}\right) P_1$$

$$d \Pi_1 / d P_1 = 30 - P_1 = 0 \quad P_1 = \underline{30} \quad P_2 = \frac{1}{2} (20 + 30) = \underline{25}$$

$$Q_1 = 20 - 30 + 25 = \underline{15} \quad Q_2 = 20 + 30 - 25 = \underline{25}$$

$$\Pi_1 = 30 \cdot 15 = \underline{450} \quad \Pi_2 = 25 \cdot 25 = \underline{625}$$

c.) iii > ii > i, therefore, you should prefer to have your competitor set the price first. The Stackelberg strategy, however, is better for both over the Cournot equilibrium.

### **Game Theory I (qualitative)**

3.) Tit-for-tat in airline pricing: airlines often offer special promotional fares of one sort or another; many observers of the airline industry claim that these promotions can be used to signal competitors to refrain from cutting prices on key routes.

Northwestern introduced a special on all flights to West Coast cities in an effort to fill empty seats. Continental Airlines interpreted this as an attempt to gain market share at its expense and responded by cutting all its Minneapolis fares to Northwest's night-fare level. However, the Continental fare cuts were set to expire one or two days after they were introduced. Northwest interpreted this as a signal from Continental that it was not serious about competing in this market, but simply wanted Northwest to retract its night-fare cuts. But Northwest decided to send a message of its own to Continental: it instituted a set of cheap fares to the West Coast for its flights departing from Houston, Continental's home base. Northwest thereby signaled that it felt its cuts were justified, while Continental's response was inappropriate.

All these cuts had very short expiration dates; this feature seems to indicate that they were meant more as messages to the competition than as bids for larger market share. Playing tit-for-

tat in duopoly airline markets then seems to be a retaliation scheme aiming to keep overall prices high.

---

### Problem Set 3: Uncertainty

---

#### ***Uncertainty I***

1.) Which of the following utility functions have the expected utility property?

i.)  $u(c_1, c_2, \pi_1, \pi_2) = a (\pi_1 c_1 + \pi_2 c_2)$

ii.)  $u(c_1, c_2, \pi_1, \pi_2) = \pi_1 c_1 + \pi_2 c_2^2$

iii.)  $u(c_1, c_2, \pi_1, \pi_2) = \pi_1 \ln c_1 + \pi_2 \ln c_2$

where  $c_i$  is the future state and  $\pi_i$  its probability of occurrence, for  $i=1,2$ .

#### ***Uncertainty II***

2.) Show that the risk aversion coefficient for an exponential utility function is constant.

#### ***Uncertainty III***

3.) Consider the case of a quadratic expected utility function. Show that at some level of wealth utility is decreasing. More importantly, show that the risk aversion coefficient is increasing at any level of wealth.

#### ***Uncertainty IV***

4a.) Explain the idea of fire insurance in a small farm community assuming that each farmer is risk-averse to the same degree. (Use the notations of probability of a fire, utility in different future states. Graph the utility function for the average farmer and show why the expected utility of wealth is less than the utility of expected wealth.)

Explain further