Lecture 2
Systems

- meaning & notation
- common examples & block diagram representations
- electronic realizations
- linearity
- interconnected systems
- differential equations

Systems

- a system transforms *input signals* into *output signals*
- a system is a *function* mapping input signals into output signals

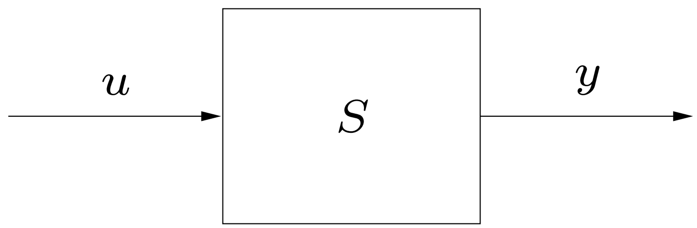
we concentrate on systems with one input and one output signal, $i.e.$, *single-input, single-output* (SISO) systems

notation:

- $y = Su$ or $y = S(u)$ means the system S acts on input signal u to produce output signal y
- $y = Su$ does *not* (in general) mean multiplication!

Block diagrams

systems often denoted by *block diagram*:



- lines with arrows denote signals (*not* wires)
- boxes denote systems; arrows show inputs & outputs
- special symbols for some systems

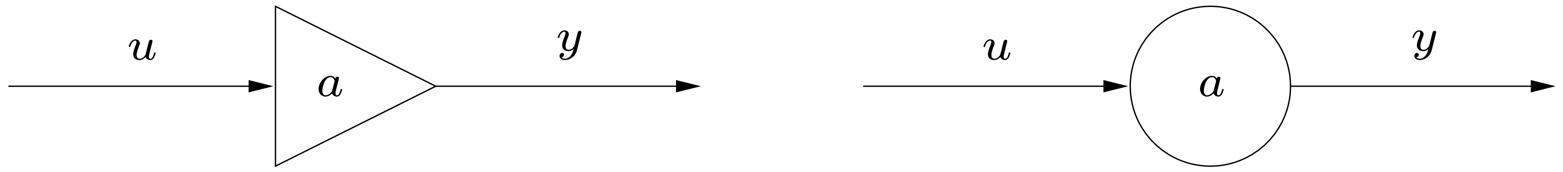
Examples

(with input signal u and output signal y)

scaling system: $y(t) = au(t)$

- called an *amplifier* if $|a| > 1$
- called an *attenuator* if $|a| < 1$
- called *inverting* if $a < 0$
- a is called the *gain* or *scale factor*

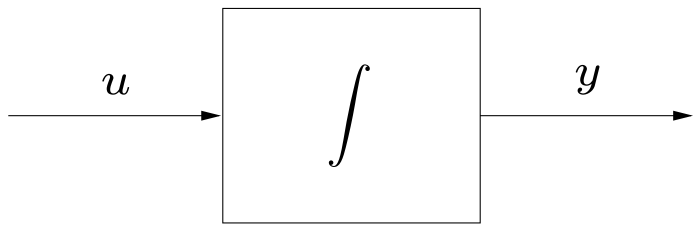
sometimes denoted by triangle or circle in block diagram:



differentiator: $y(t) = u'(t)$

integrator: $y(t) = \int_a^t u(\tau)\, d\tau$ (a is often 0 or $-\infty$)

common notation for integrator:



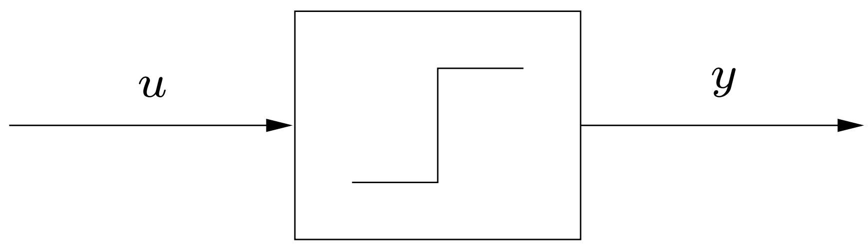
running average system: $y(t) = \frac{1}{t} \int_0^t u(\tau)\, d\tau$

time shift system: $y(t) = u(t - T)$

- called a *delay system* if $T > 0$
- called a *predictor system* if $T < 0$

sign detector or **1-bit limiter** system:

$$y(t) = \mathsf{sgn}(u(t)) = \begin{cases} 1, & u(t) \geq 0 \\ -1, & u(t) < 0 \end{cases}$$



convolution system:

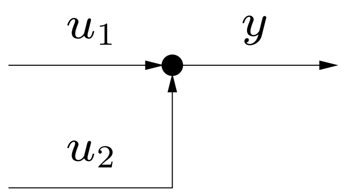
$$y(t) = \int u(t-\tau)h(\tau)\, d\tau,$$

where h is a given function (you'll be hearing much more about this!)

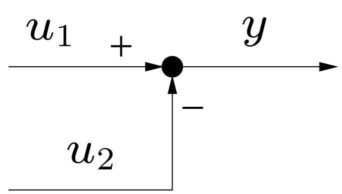
Examples with multiple inputs

(with inputs u_1, u_2, and output y)

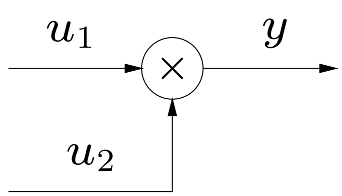
- **summing system:** $y(t) = u_1(t) + u_2(t)$



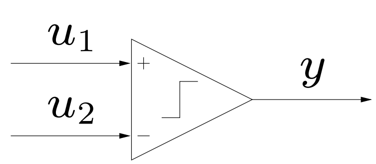
- **difference system:** $y(t) = u_1(t) - u_2(t)$



- **multiplier system:** $y(t) = u_1(t)u_2(t)$



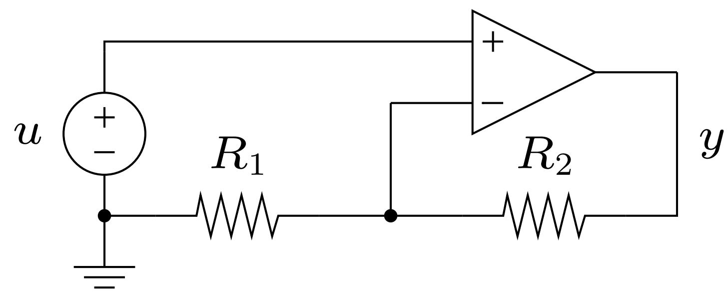
- **comparator system:** $y(t) = \begin{cases} 1, & u_1(t) \geq u_2(t) \\ -1, & u_1(t) < u_2(t) \end{cases}$



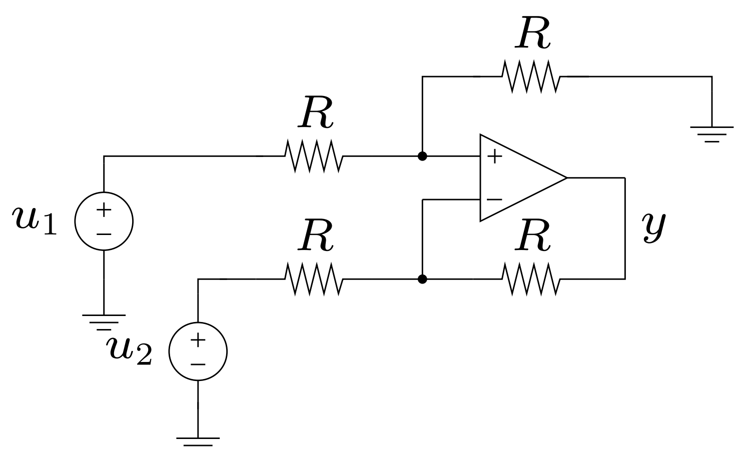
Electronic realizations

the systems described above can be *realized* as electronic circuits, $e.g.$, with op-amps

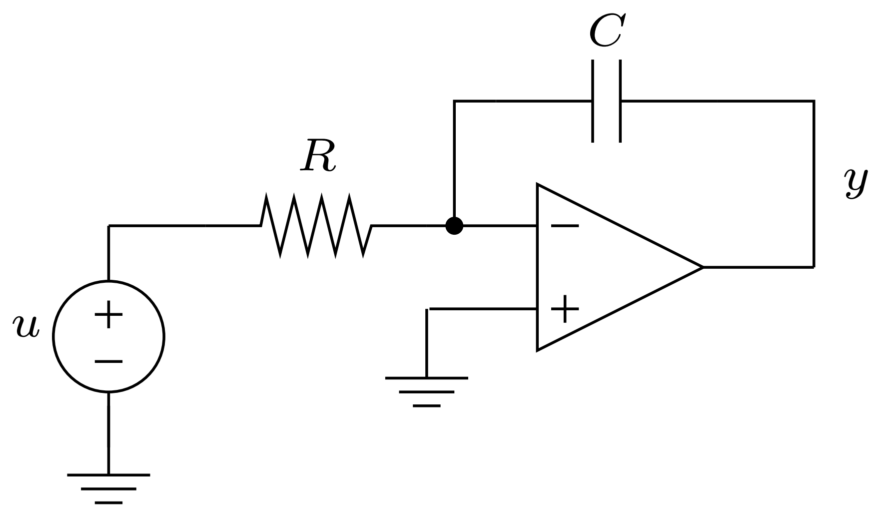
scaling: $y(t) = (1 + R_2/R_1)u(t)$



difference: $y(t) = u_1(t) - u_2(t)$



integrator: $y(t) = -1/(RC) \int^t u(\tau)\, d\tau$



- these are *circuit schematics*, not *block diagrams*
- signals are represented by *voltages* (which is common but *not* universal)

Linearity

a system F is **linear** if the following two properties hold:

1. **homogeneity:** if u is any signal and a is any scalar,

$$F(au) = aF(u)$$

2. **superposition:** if u and $\tilde{u}$ are any two signals,

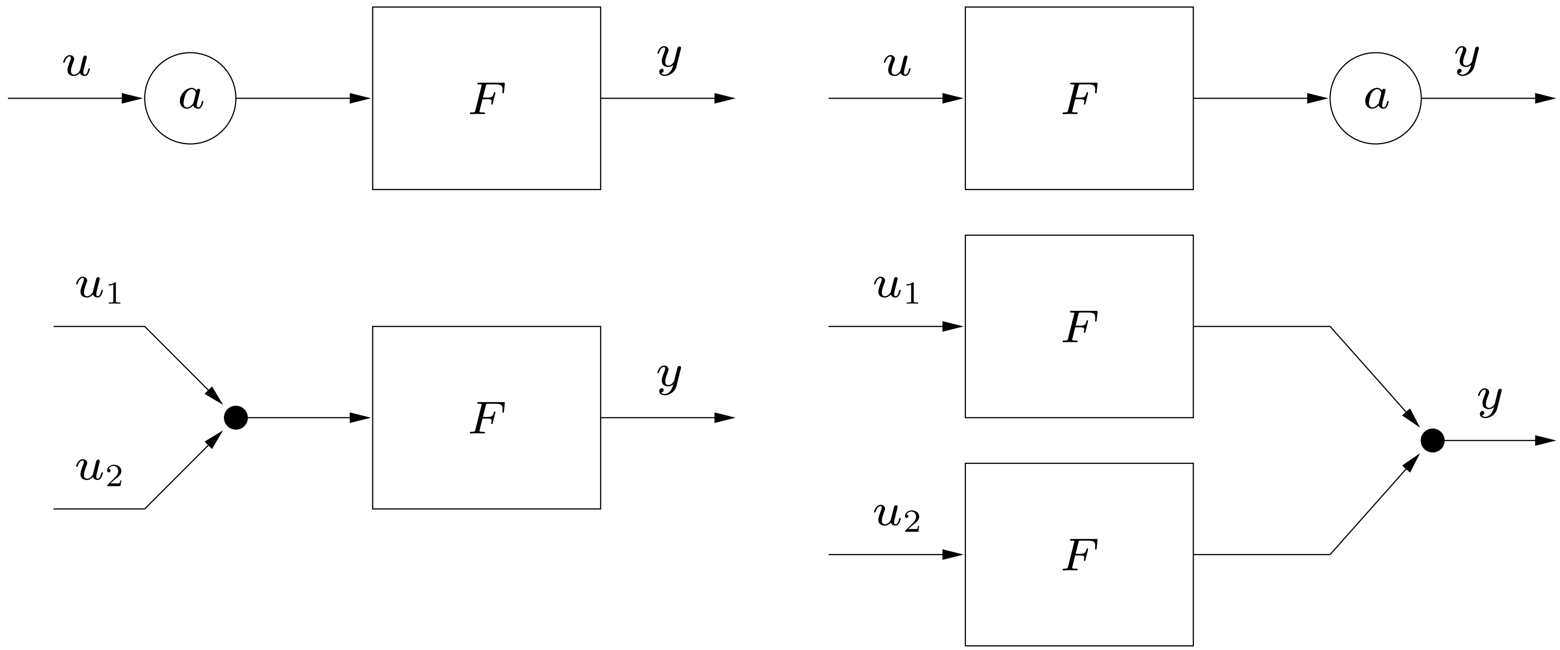
$$F(u + \tilde{u}) = Fu + F\tilde{u}$$

(watch out — just a few symbols here express a *very complex* meaning)

in words, linearity means:

- scaling before or after the system is the same
- summing before or after the system is the same

linearity means the following pairs of block diagrams are equivalent, *i.e.*, have the same output for any input(s)



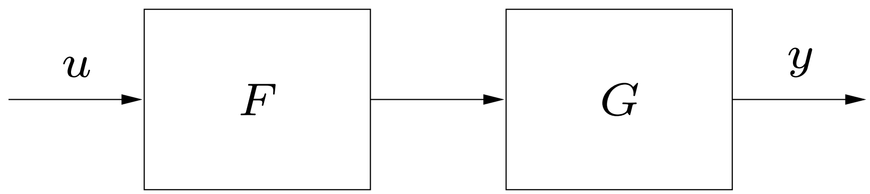
examples of linear systems: scaling system, differentiator, integrator, running average, time shift, convolution, summer, difference systems

examples of nonlinear systems: sign detector, multiplier, comparator

Interconnections of systems

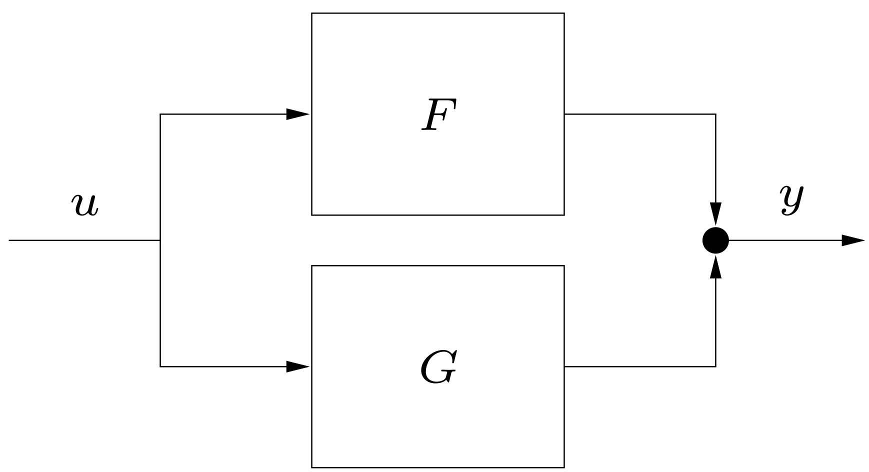
we can interconnect systems to form new systems, $e.g.$,

cascade (or series): $y = G(Fu) = GFu$

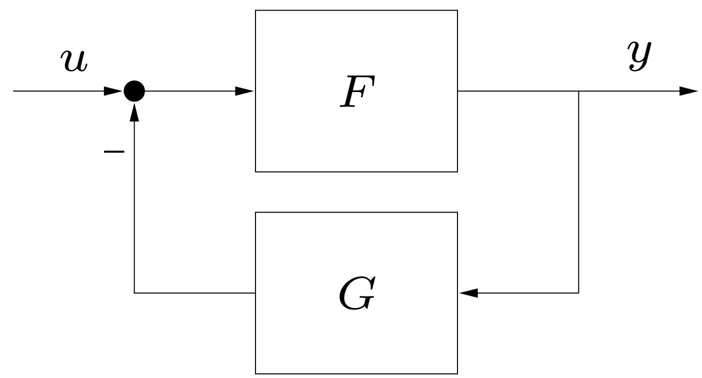


(note that block diagrams and algebra are *reversed*)

sum (or parallel): $y = Fu + Gu$



feedback: $y = F(u - Gy)$

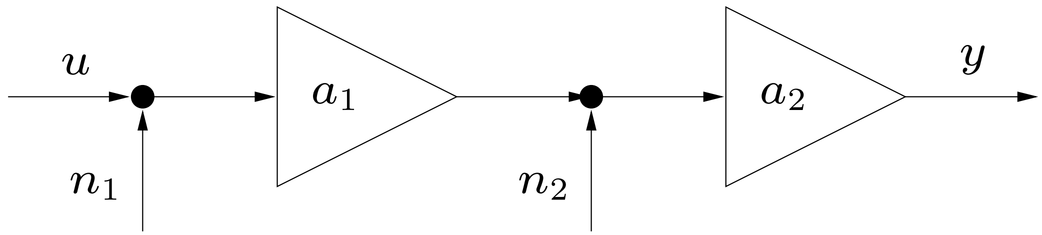


- the minus sign is just a tradition, and often isn't there
- we'll study this arrangement later

in general,

- block diagrams are just a symbolic way to describe a connection of systems
- we can just as well write out the equations relating the signals

Example: Two-stage amplifier

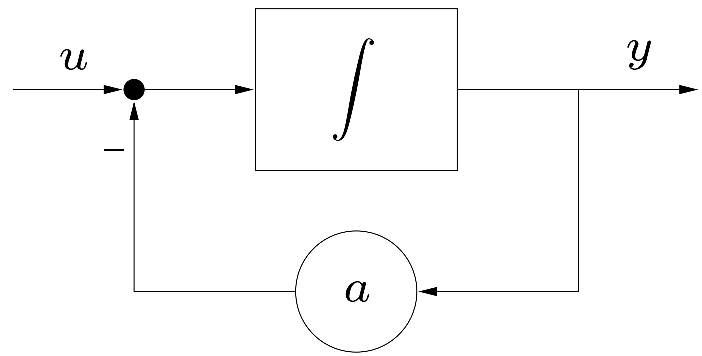


- input signal u, output signal y
- noise signals n_1, n_2
- first stage gain a_1, second stage gain a_2

$$y = a_2(a_1(u + n_1) + n_2) = (a_1a_2)u + (a_1a_2)n_1 + (a_2)n_2$$

- input to first amplifier is $u + n_1$
- output of first amplifier is $a_1(u + n_1)$
- input to second amplifier is $a_1(u + n_1) + n_2$
- output of second amplifier is $a_2(a_1(u + n_1) + n_2)$

Example: Integrator with feedback



input to integrator is $u - ay$, so

$$\int^t (u(\tau) - ay(\tau))\, d\tau = y(t)$$

(soon we'll be able to give an explicit expression for y in terms of u)

another (useful) method: the *input* to an integrator is the derivative of its output, so we have

$$u - ay = y'$$

(of course, same as above)

Systems described by differential equations

many systems are described by a *linear constant coefficient ordinary differential equation* (LCCODE):

$$a_n y^{(n)} + \cdots + a_2 y'' + a_1 y' + a_0 y = b_m u^{(m)} + \cdots + b_1 u'' + b_1 u' + b_0 u$$

with given *initial conditions*

$$y^{(n-1)}(0), \quad y^{(n-2)}, \quad \ldots \quad , y'(0), \quad y(0)$$

(which fixes y, given u)

- n is called the *order* of the system
- $b_0, \ldots, b_m, a_0, \ldots, a_n$ are the *coefficients* of the system
- when initial conditions are all zero, LCCODE systems are **linear**

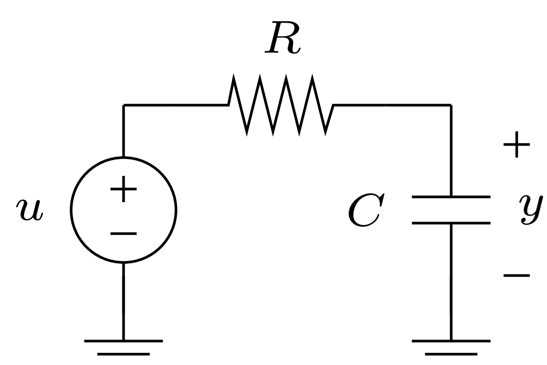
an LCCODE gives an *implicit* description of a system; soon we'll be able to *explicitly* express y in terms of u

Examples

simple examples

- scaling system ($a_0 = 1$, $b_0 = a$)
- integrator ($a_1 = 1$, $b_0 = 1$)
- differentiator ($a_0 = 1$, $b_1 = 1$)
- integrator with feedback (page 2–15)

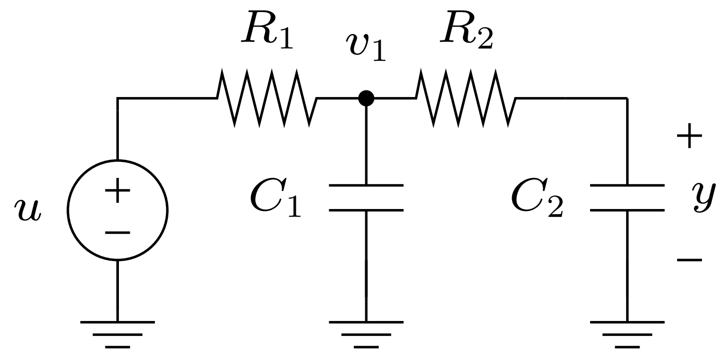
RC circuit



current flowing into capacitor is $Cy'(t) = \dfrac{u(t) - y(t)}{R}$

rewrite as first-order LCCODE: $RCy'(t) + y(t) = u(t)$

second-order RC circuit



- current into C_2 is $C_2 y' = \dfrac{v_1 - y}{R_2}$
- current into C_1 is $C_1 v_1' = \dfrac{u - v_1}{R_1} - \dfrac{v_1 - y}{R_2}$

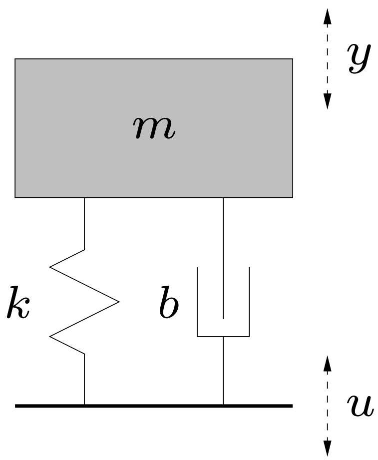
using $v_1 = y + R_2 C_2 y'$ in the 2nd equation yields:

$$C_1(y + R_2C_2y')' = \frac{u}{R_1} + \frac{y}{R_2} - \left(\frac{1}{R_1} + \frac{1}{R_2}\right)(y + R_2C_2y')$$

rewrite (eventually) as second-order LCCODE

$$(R_1C_1R_2C_2)y'' + (R_1C_1 + R_1C_2 + R_2C_2)y' + y = u$$

mechanical system (mass-spring-damper)



(can represent suspension system, building during earthquake, . . .)

- $u(t)$ is displacement of base; $y(t)$ is displacement of mass
- spring force is $k(u-y)$; damping force is $b(u-y)'$
- Newton's equation is $my'' = b(u-y)' + k(u-y)$

rewrite as second-order LCCODE

$$my'' + by' + ky = bu' + ku$$