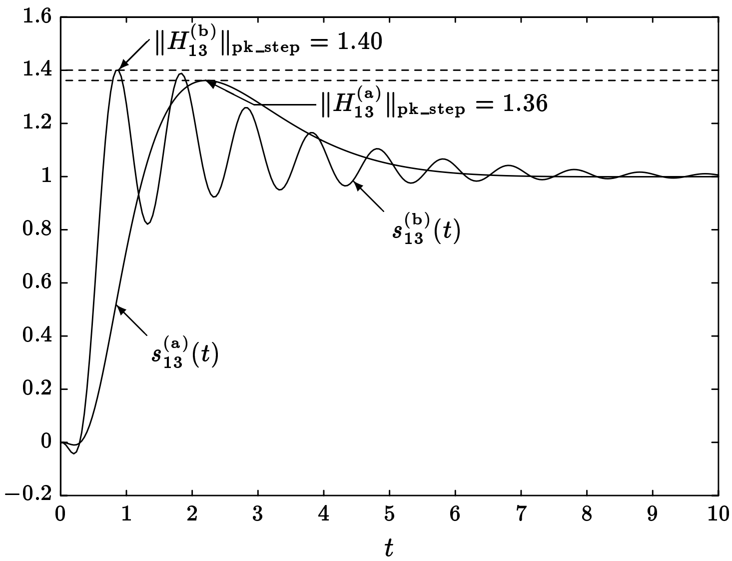

Figure 5.8 The step responses of the transfer functions in (5.17) and (5.18). Note that $\|H_{13}^{(a)}\|_{pk\_step} = 1.36$, and $\|H_{13}^{(b)}\|_{pk\_step} = 1.40$.

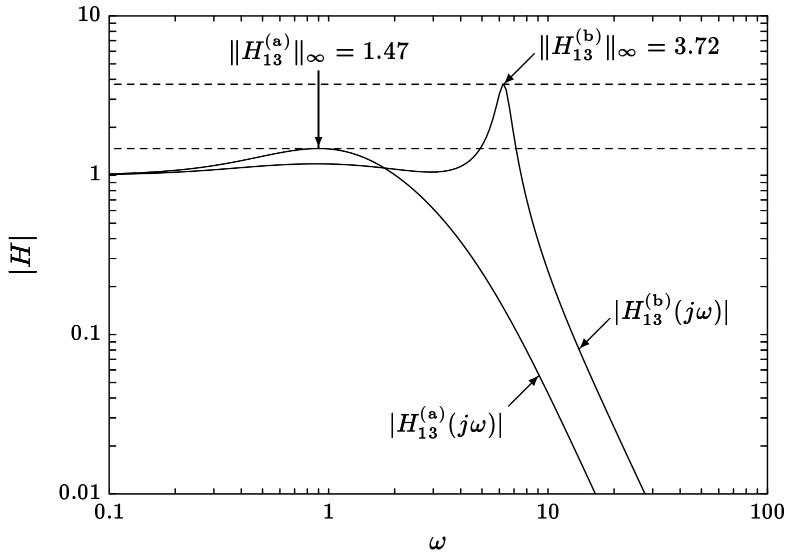

Figure 5.9 The magnitudes of the transfer functions in (5.17) and (5.18). Note that $\|H_{13}^{(a)}\|_\infty = 1.47$, and $\|H_{13}^{(b)}\|_\infty = 3.72$.

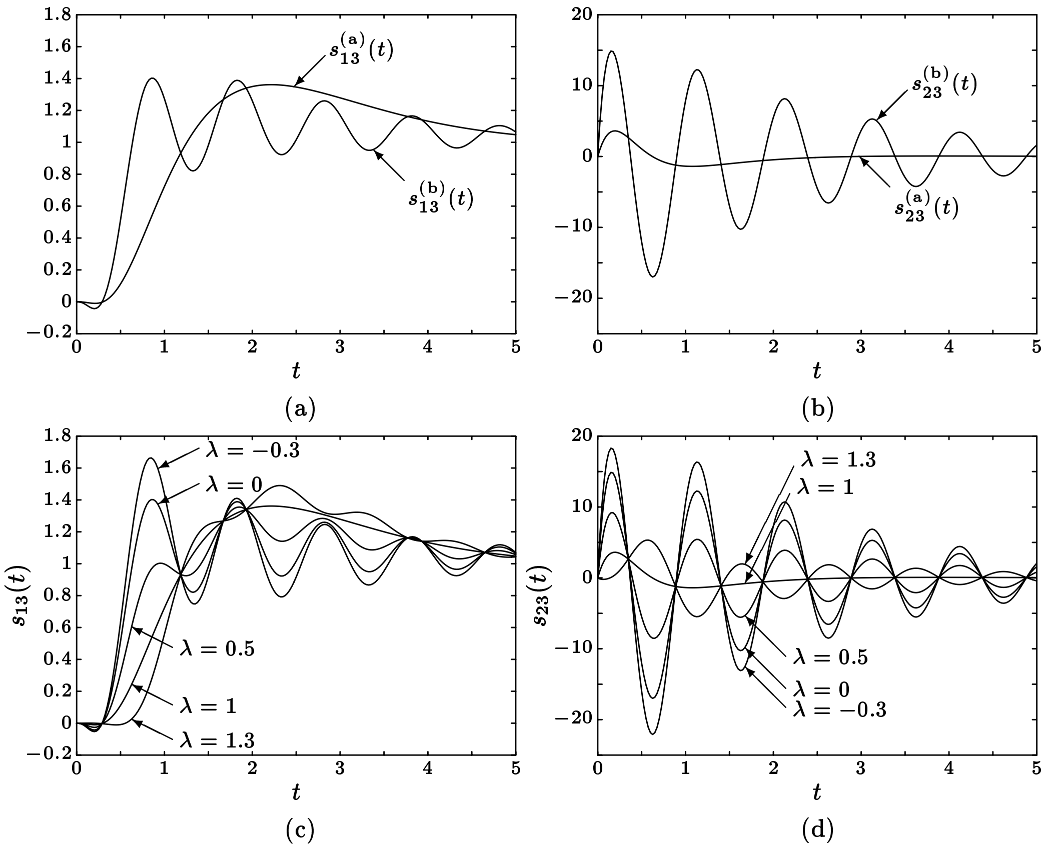

Figure 7.2 (a) shows the closed-loop step responses from r to y_p for the standard example with the two controllers $K^{(a)}$ and $K^{(b)}$. (b) shows the step responses from r to u. In (c) and (d) the step responses corresponding to five different values of λ are shown. Each of these step responses is achieved by some controller.

7.2.1 A Motivating Example

Consider our standard example SASS 1-DOF control system described in section 2.4, with the controller

$$K(s) = \frac{36 + 33s}{10 - s}.$$

This controller yields the closed-loop I/O transfer function

$$T(s) = \frac{33s + 36}{s^3 + 10s^2 + 33s + 36} = \frac{33s + 36}{(s+3)^2(s+4)},$$

which is a stable lowpass filter. Thus, we will have $y_p \approx r$ provided the reference signal r does not change too rapidly; the controller K yields good tracking of slowly where $H_{\tilde{z}\tilde{w}}$ is some entry or submatrix of H (*c.f.* robust stability, which involves the gain bound $\|H_{\tilde{z}\tilde{w}}\|_\infty < \infty$).

Throughout this section, we will consider the robustness specification that is formed from the perturbed plant set $\mathcal{P}$ and the RMS gain bound specification

$$\|H_{\tilde{z}\tilde{w}}\|_\infty \leq 1. \tag{10.75}$$

We will refer to this robust performance specification as $\mathcal{D}_{\rm rob\_perf}$. We will also assume that the perturbed plant set $\mathcal{P}$ is described by a perturbation feedback form for which the maximum RMS gain of the feedback perturbations is one, *i.e.*, $M = 1$ in (10.54).

The inner approximation of $\mathcal{D}_{\rm rob\_perf}$ is

$$\left\| \begin{bmatrix} H_{\tilde{z}\tilde{w}} & H_{\tilde{z}p} \\ H_{q\tilde{w}} & H_{qp} \end{bmatrix} \right\|_\infty < 1. \tag{10.76}$$

Like the inner approximation (10.57–10.60) of the robust stability specification $\mathcal{D}_{\rm rob\_stab}$, we can interpret (10.76) as limiting the size of $H_{\tilde{z}p}$, $H_{q\tilde{w}}$, and H_{qp}.

Let us show that (10.76) implies that the specification (10.75) holds robustly, *i.e.*,

$$\|H_{\tilde{z}\tilde{w}} + H_{\tilde{z}p}\Delta\left(I - H_{qp}\Delta\right)^{-1} H_{q\tilde{w}}\|_\infty \leq 1 \text{ for all } \Delta \in \boldsymbol{\Delta}. \tag{10.77}$$

Assume that (10.76) holds, so that for any signals $\tilde{w}$ and p we have

$$\left\| \begin{bmatrix} \tilde{z} \\ q \end{bmatrix} \right\|_{\rm rms} < \left\| \begin{bmatrix} \tilde{w} \\ p \end{bmatrix} \right\|_{\rm rms}, \tag{10.78}$$

where

$$\begin{bmatrix} \tilde{z} \\ q \end{bmatrix} = \begin{bmatrix} H_{\tilde{z}\tilde{w}} & H_{\tilde{z}p} \\ H_{q\tilde{w}} & H_{qp} \end{bmatrix} \begin{bmatrix} \tilde{w} \\ p \end{bmatrix}.$$

The inequality (10.78) can be rewritten

$$\|\tilde{z}\|_{\rm rms}^2 + \|q\|_{\rm rms}^2 < \|\tilde{w}\|_{\rm rms}^2 + \|p\|_{\rm rms}^2. \tag{10.79}$$

Now assume that $p = \Delta q$, where $\Delta \in \boldsymbol{\Delta}$, so that these signals correspond to closed-loop behavior of the perturbed system, *i.e.*,

$$\tilde{z} = \left(H_{\tilde{z}\tilde{w}} + H_{\tilde{z}p}\Delta\left(I - H_{qp}\Delta\right)^{-1} H_{q\tilde{w}}\right)\tilde{w}. \tag{10.80}$$

Since $\|\Delta\|_\infty \leq 1$, we have

$$\|p\|_{\rm rms} \leq \|q\|_{\rm rms}. \tag{10.81}$$

From (10.79–10.81) we conclude that

$$\begin{aligned} \|\tilde{z}\|_{\rm rms} &= \left\| \left(H_{\tilde{z}\tilde{w}} + H_{\tilde{z}p}\Delta\left(I - H_{qp}\Delta\right)^{-1} H_{q\tilde{w}}\right)\tilde{w} \right\|_{\rm rms} \\ &\leq \|\tilde{w}\|_{\rm rms}. \end{aligned}$$

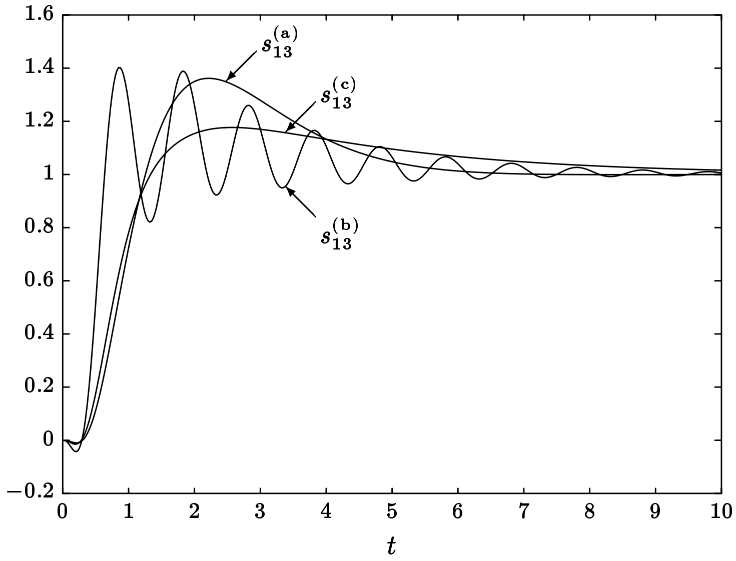

Figure 11.2 The step responses from the reference input, r, to plant output, y_p, for the closed-loop transfer matrices $H^{(a)}$, $H^{(b)}$, and $H^{(c)}$.

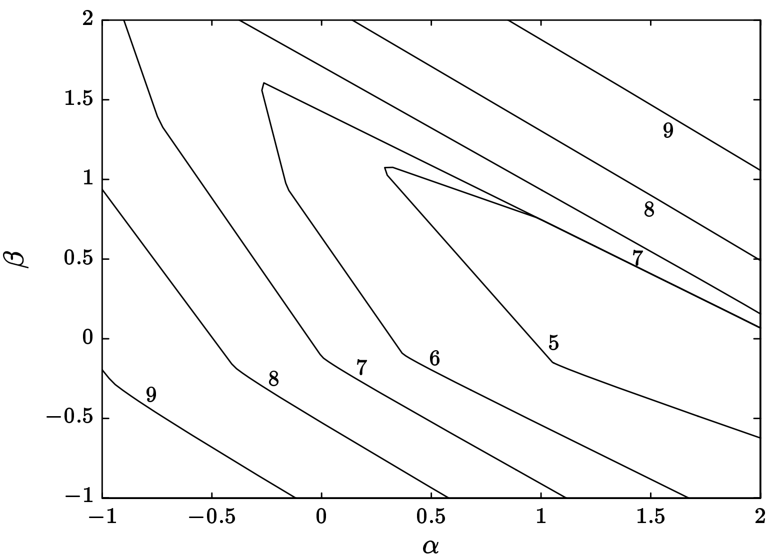

Figure 11.3 Level curves of the step response settling time, from the reference r to y_p, given by (11.1).