A MATLAB implementation for best approximation of two-term log-sum-exp function

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Abstract

In this note we first formulate the problem of finding the best piecewise linear convex approximation of the two-term log-sum-exp function. Then we present how to use `battlse.m` to find the solution.

1 Problem formulation

The bivariate convex function $\mathbf{lse2} : \mathbf{R}^2 \to \mathbf{R}$, defined by

$$\mathbf{lse2}(y) = \log(e^{y_1} + e^{y_2}), \tag{1}$$

is called the *two-term log-sum-exp* function.

For given integer $r \geq 2$, the problem of finding the best r-term piecewise linear (PWL) convex *lower* approximation of the two-term log-sum-exp function can be formulated as the following optimization problem:

$$\begin{array}{ll} \text{minimize} & \sup_{x \in \mathbf{R}^2} \left(\mathbf{lse2}(x) - \max_{i=1,\ldots,r} \{a_i^T x + b_i\} \right) \\ \text{subject to} & \mathbf{lse2}(x) \geq \max_{i=1,\ldots,r} \{a_i^T x + b_i\}, \quad \forall\, x \in \mathbf{R}^2, \end{array} \tag{2}$$

where the optimization variables are $(a_i, b_i) \in \mathbf{R}^2 \times \mathbf{R},\ i = 1, \ldots, r$. Suppose $\{(\underline{a}_1, \underline{b}_1), \ldots, (\underline{a}_r, \underline{b}_r)\}$ solves the above optimization. The best r-term PWL convex lower approximation, $\underline{f}_r : \mathbf{R}^2 \to \mathbf{R}$, of the two-term log-sum-exp function $\mathbf{lse2}$ is

$$\underline{f}_r(x) = \max \left\{ \underline{a}_1^T x + \underline{b}_1, \ldots, \underline{a}_r^T x + \underline{b}_r \right\}, \tag{3}$$

where r is called the *degree of approximation* of the best r-term lower approximation (3). The optimal value of (2), denote by $\underline{\epsilon}(r)$, is defined as the *approximation error* of the best r-term lower approximation (3).

We refer to [HKB06] for more details about the best PWL convex approximation of log-sum-exp functions.

2 Matlab function: `battlse.m`

Syntax: `[A,b,apx_err] = battlse(r)`

2.1 Purpose

`battlse.m` is a MATLAB function for solving the best PWL convex 'lower' approximation problem (2).

2.2 Best upper approximation

`battlse.m` is designed to find the best lower approximation (3). The best r-term PWL convex *upper* approximation $\overline{f}_r$ can be easily obtained by adding $\underline{\epsilon}(r)$ to each b_i in (3), *i.e.*,

$$\overline{f}_r(x) = \max\left\{\underline{a}_1^T x + \underline{b}_1 + \underline{\epsilon}(r), \ldots, \underline{a}_r^T x + \underline{b}_r + \underline{\epsilon}(r)\right\}. \qquad (4)$$

(See [HKB06] for the definition of best upper approximation and more details.)

2.3 Storage convention

The output data $A \in \mathbf{R}^{r\times 2}$ and $b \in \mathbf{R}^r$ store the optimal solution of (2) according to the following convention:

$$A = \begin{bmatrix} \underline{a}_1^T \\ \vdots \\ \underline{a}_r^T \end{bmatrix}, \quad b = \begin{bmatrix} \underline{b}_1 \\ \vdots \\ \underline{b}_r \end{bmatrix}. \qquad (5)$$

2.4 Input argument

- `r`: Degree of approximation, as described in §1. `r` must be an integer greater than one.

2.5 Output arguments

- `A`: Matrix of size r by 2, as described in §2.3.
- `b`: Vector of length r, as described in §2.3.
- `apx_err`: Approximation error $\underline{\epsilon}(r)$, as described in §1.

References

[HKB06] K.-L. Hsiung, S.-J. Kim, and S. Boyd. Tractable approximate robust geometric programming. Technical Report of Department of Electrical Engineering, Stanford University, 2006. Available at `http://www.stanford.edu/~boyd/rgp.html`.