TensorFlow
DEV SUMMIT 2020

Convex optimization layers

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Convex optimization

$$\begin{array}{ll} \text{minimize} & f(x;\theta) \\ \text{subject to} & g(x;\theta) \leq 0 \\ & A(\theta)x = b(\theta) \end{array}$$

- $x \in \mathbf{R}^n$ is the variable
- $\theta \in \mathbf{R}^m$ is the parameter
- f and g are convex (curve upwards)
- find x that minimizes f while satisfying the constraints

Why convex optimization?

- Convex optimization problems can be solved quickly, reliably, and *exactly*

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```
import cvxpy as cp

x = cp.Variable(n)
objective = cp.Minimize(cp.sum_squares(A @ x - b) + cp.pnorm(x, p=1))
constraints = [0 <= x, x <= 1]
prob = cp.Problem(objective, constraints)
result = prob.solve()
```

Why convex optimization?

- Convex optimization problems can be solved quickly, reliably, and *exactly*
- Software libraries like **CVXPY** make convex optimization easy
- ***Tons* of applications**

Controlling self-driving cars

Landing rockets

Designing airplanes

Until now ...

- Difficult to use convex optimization problems in TensorFlow pipelines
- Parameters θ were chosen and tuned by hand

CVXPY Layers

CVXpy

$$
\begin{aligned}
x^\star(\theta) = \quad & \text{argmin} && f(x;\theta) \\
& \text{subject to} && g(x;\theta) \leq 0 \\
& && A(\theta)x = b(\theta)
\end{aligned}
$$

TensorFlow 2.0

```python
import cvxpy as cp
import tensorflow as tf
from cvxpylayers.tensorflow import CvxpyLayer

n, m = 2, 3
x = cp.Variable(n)
A, b = cp.Parameter((m, n)), cp.Parameter(m)
constraints = [x >= 0]
objective = cp.Minimize(0.5 * cp.pnorm(A @ x - b, p=1))
problem = cp.Problem(objective, constraints)

cvxpylayer = CvxpyLayer(problem, parameters=[A, b], variables=[x])

A_tf = tf.Variable(tf.random.normal((m, n)))
b_tf = tf.Variable(tf.random.normal((m,)))
with tf.GradientTape() as tape:
    solution, = cvxpylayer(A_tf, b_tf)
    summed_solution = tf.math.reduce_sum(solution)
gradA, gradb = tape.gradient(summed_solution, [A_tf, b_tf])
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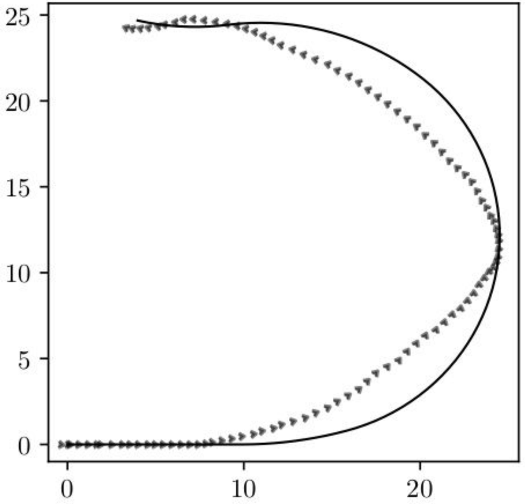
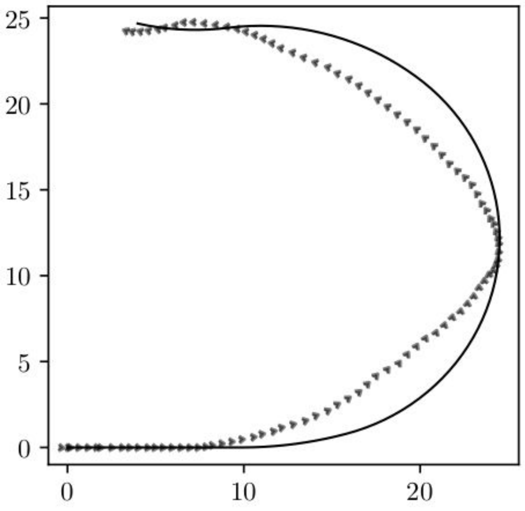
Learning to control a car

Learning to control a car



iteration 0

Learning to control a car



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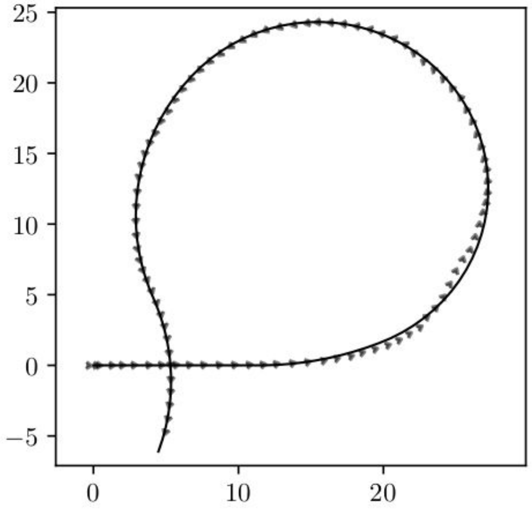

iteration 100

For more ...

github.com/cvxgrp/cvxpylayers

NeurIPS paper

Learning control policies (L4DC)

with examples in

- controlling a car
- managing a supply chain
- allocating financial portfolios

Thank you!

TensorFlow

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