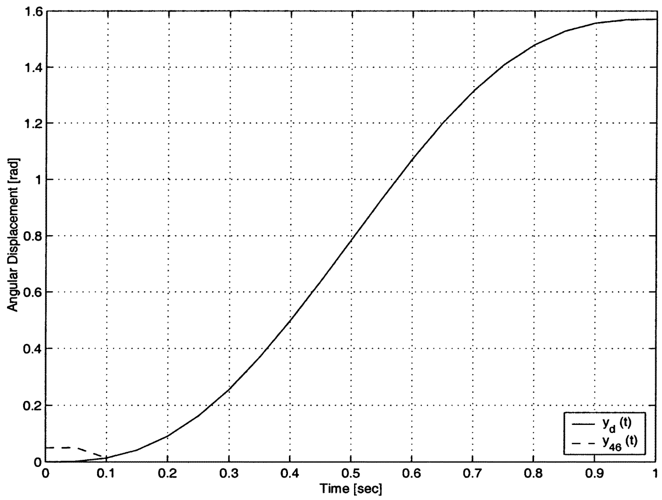

Fig. 2. Resultant output trajectory by (27).

dress the initial shift problem. The convergence and robustness properties of the scheme with respect to initial shifts have been presented by the developed analysis technique. Under certain conditions, the system output is ensured to converge to a neighborhood of the predefined trajectory and the error bound is proportional to the bound on initial shifts. The system undertaken has been shown to possess asymptotic tracking capability and the converged output trajectory can be assessed by the initial condition. The initial rectifying action has been shown effective to improve the tracking performance further, by which the complete tracking with specified transient is guaranteed.

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Joint Optimization of Communication Rates and Linear Systems

Lin Xiao, Mikael Johansson, Haitham Hindi, Stephen Boyd, and Andrea Goldsmith

***Abstract*—We consider a linear control system in which several signals are transmitted over communication channels with bit rate limitations. With the coding and medium access schemes of the communication system fixed, the achievable bit rates are determined by the allocation of communications resources such as transmit powers and bandwidths, to different communication channels. We model the effect of bit rate limited communication channels by uniform quantization and the quantization errors are modeled by additive white noises whose variances depend on the achievable bit rates. We optimize the stationary performance of the linear system by jointly allocating resources in the communication system and tuning parameters of the controller.**

***Index Terms*—Communication systems, control over networks, convex optimization, quantization noise, resource allocation.**

I. Introduction

We consider a linear system in which several signals are transmitted over wireless communication channels, as illustrated in Fig. 1. All signals are vector-valued: w is a vector of exogenous signals (such as disturbances or noises acting on the system); z is a vector of performance signals (including error signals and actuator signals); and y and

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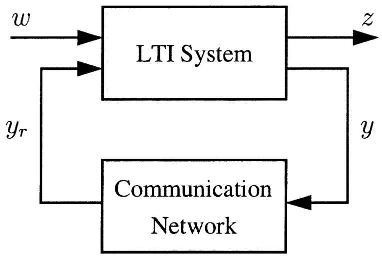

Fig. 1. Linear time-invariant (LTI) system closed over communication network.

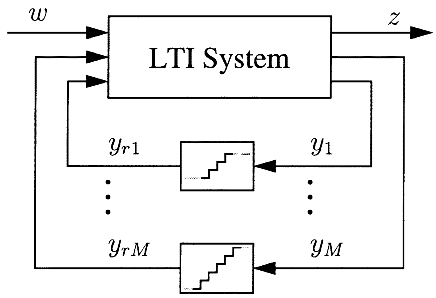

Fig. 2. Uniform quantization model of communication links.

y_r are the signals transmitted and received over the communication network, respectively. This general arrangement can represent a variety of systems, for example a controller or estimator in which actuator and sensor signals are sent over wireless channels. In this note, we address the problem of optimizing the stationary performance of the linear system by jointly allocating resources in the communication network and tuning parameters of the linear system.

Many issues arise in the design of networked controllers, including bit rate limitations, communication delays, packet loss, transmission errors and asynchronicity (see, e.g., [1]–[8]). In this note, we consider problems related to only the first issue, i.e., bit rate limitations. Much of the work on control with bit rate limitations has concentrated on joint design of control and coding to find the minimum bit rate required to stabilize a linear system. For example, [1] and [2] established various closed-loop stability conditions involving the feedback data rate and eigenvalues of the open-loop system and [8] and [3] studied control with communication constraints within the classical linear quadratic Gaussian (LQG) framework.

Our focus in this note is different. We assume that the source coding, channel coding and medium access scheme of the communication system are fixed and we concentrate on finding the allocation of communications resources such as transmit powers and bandwidths, that yields the optimal performance of the linear system. For a fixed sampling frequency of the linear system, the limit on communication rate translates into a constraint on the number of bits that can be transmitted over each communication channel during one sampling period. We assume that the individual signals y_i are coded using memoryless uniform quantizers, see Fig. 2. We impose lower bounds on the number of quantization bits, which correspond to lower bounds on the channel bit rates. These lower bounds ensure that the data rates are high enough for stabilization (i.e., much higher than lower bounds given in [1], [2]) and allow us to use the white-noise model for quantization errors introduced by Widrow (see, e.g., [15] and [16]). Memoryless uniform quantization is certainly not the optimal source coding scheme (see, e.g., [3], [8], and [12]), but it is conventional, easily implemented and leads to a simple model for how the system performance depends on the bit rates which, in turn, are determined by the allocation of communications resources to different channels.

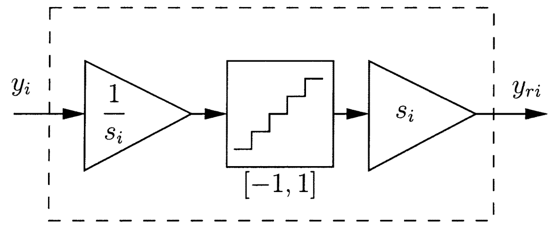

Fig. 3. Scaling of the uniform quantizer.

There has been significant research on control with quantized feedback information (see, e.g., [9]–[14]) and joint optimization of quantizer and estimator/controller has been considered in, e.g., [9]–[11]. However, joint optimization of communications resource allocation and linear system design, interacting through bit rate limitations and quantization, has not been addressed before in the literature. Even in the simplified setting under our assumptions, the joint optimization problem is quite nontrival and its solution requires concepts and techniques from communication, control and optimization. We show that for fixed linear system, the problem of optimally allocating communication resources is often convex and, hence, readily solved. We discuss efficient solution methods and suggest a heuristic for obtaining suboptimal integer solutions. The problem of jointly designing the linear system and allocating the communication resources is in general not convex and we present a iterative heuristic that exploits problem structure and appears to work very well in practice.

II. Linear System and Quantizer Model

A. Linear System Model

To simplify the presentation we assume a synchronous, single-rate discrete-time system. The LTI system (see Fig. 1) can be described as

$$z = G_{11}(\varphi)w + G_{12}(\varphi)y_r \quad y = G_{21}(\varphi)w + G_{22}(\varphi)y_r \tag{1}$$

where G_{ij} are LTI operators (i.e., convolution systems described by transfer or impulse matrices). Here, φ is the vector of design parameters, such as estimator or controller gains, that can be tuned to optimize performance. To give lighter notation, we suppress the dependence of G_{ij} on φ except when necessary. We assume that the signals sent (i.e., y) and received (i.e., y_r) over the communication links are related by memoryless scalar quantization.

B. Quantization Model

Unit Uniform Quantizer and Scaling: A unit-range uniform b_i-bit quantizer partitions the range $[-1, 1]$ into 2^{b_i} intervals of uniform width 2^{1-b_i}. To each quantization interval a codeword of b_i bits is assigned. Given a received codeword, the input value y_i is approximated by (or reconstructed as) y_{ri}, the midpoint of the corresponding interval. As long as the quantizer does not overflow, i.e., $|y_i| < 1$, the quantization error lies in the interval $\pm 2^{-b_i}$. To avoid overflow, each signal $y_i(t)$ is scaled by a factor $s_i^{-1} > 0$ prior to encoding and rescaled by s_i after decoding (see Fig. 3). To minimize quantization error while ensuring no overflow (or that overflow is rare) the scaling factor s_i should be chosen as the maximum possible value of $|y_i(t)|$, or as a value that with very high probability is larger than $|y_i(t)|$. We will use the so-called 3σ-rule, $s_i = 3\,\mathbf{rms}(y_i)$, where $\mathbf{rms}(y_i) = (\lim_{t\to\infty} \mathbf{E}\, y_i(t)^2)^{1/2}$ denotes the rms (root-mean-square) value of y_i. For example, if y_i has a Gaussian distribution, then overflow occurs only about 0.3% of the time.

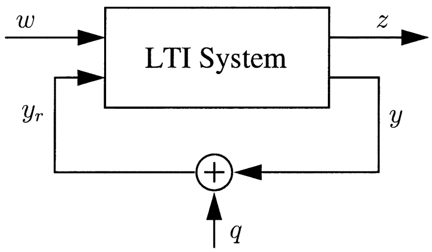

Fig. 4. Additive-white-noise model for quantization errors.

White-Noise Quantization Error Model: We adopt the stochastic quantization noise model introduced by Widrow (see, e.g., [15]). Assuming that overflow is rare, we model the quantization errors $q_i(t) = y_{ri}(t) - y_i(t)$ as independent random variables, uniformly distributed on the interval $s_i[-2^{-b_i}, 2^{-b_i}]$. In other words, we model the effect of quantizing $y_i(t)$ as an additive white-noise source $q_i(t)$ with zero mean and variance $\mathbf{E}\, q_i(t)^2 = (1/3)s_i^2 2^{-2b_i}$. we will impose a lower bound on each b_i, which corresponds to a lower bound on the bit rate for individual communication channels. This lower bound should be high enough for stabilizing the closed-loop system (cf. [1]–[3]) and make the white noise model a reasonable assumption in a feedback control context (cf. [15], [16]).

C. Performance of the Closed-Loop System

Using the white noise quantization error model, we obtain the system in Fig. 4. The LTI system is driven by exogenous inputs w and q. We express z and y in terms of w and q as

$$z = G_{zw}w + G_{zq}q \quad y = G_{yw}w + G_{yq}q$$

where $G_{zw},\ G_{zq},\ G_{yw}$, and G_{yq} are the closed-loop transfer matrices from w and q to z and y, respectively. They can be expressed as linear fractional transformations of the matrices G_{ij} in (1). The variance of z induced by the quantization is given by

$$V_q = \mathbf{E}\,\|G_{zq}q\|^2 = \sum_{i=1}^{M} \|G_{zqi}\|^2 \left(\frac{1}{3}s_i^2 2^{-2b_i}\right) \tag{2}$$

where G_{zqi} is the ith column of the transfer matrix G_{zq} and $\|\cdot\|$ denotes the $\mathbf{L}^2$ norm (see [17, Sec. 5.2.3]). This expression shows how V_q depends on the allocation of quantizer bits $b_1, \ldots, b_M$, as well as the scalings $s_1, \ldots, s_M$ and the LTI system. We can use V_q as a measure of the effect of quantization on the overall system performance. If w is also modeled as a stationary stochastic process (independent of q), the overall variance of z is given by

$$V = \mathbf{E}\,\|z\|^2 = V_q + \mathbf{E}\,\|G_{zw}w\|^2. \tag{3}$$

III. Communications Model and Assumptions

A. A Generic Model for Bit Rate Constraints

The capacities of communication channels depend on the media access scheme and the selection of certain critical parameters, such as transmission powers and bandwidths or time-slot fractions allocated to individual channels (or groups of channels). We refer to these critical communications parameters collectively as *communications variables* and denote the vector of communications variables by θ. The communications variables are themselves limited by various resource constraints, such as limits on the total power or total bandwidth available. We will assume that the medium access methods and coding and modulation schemes are fixed, but that we can optimize over the underlying communications variables θ.

We let $b \in \mathbf{R}^M$ denote the vector of bits allocated to each quantized signal. The associated communication rate r_i (in bits per second) is proportional to b_i and their relationship can be expressed as $b_i = \alpha r_i$. The constant α has the form $\alpha = c_s/f_s$, where f_s is the sample frequency of the linear system and c_s is the channel coding efficiency (in source bits per transmission bit) for a fixed coding scheme. This relationship will allow us to express capacity constraints in terms of bit allocations rather than communication rates.

We will use the following general model to relate the vector of bit allocations b and the vector of communications variables θ:

$$\begin{aligned} f_i(b,\theta) &\le 0, & i &= 1,\ldots,m_f \\ h_i^T\theta &\le d_i, & i &= 1,\ldots,m_h \\ \theta_i &\ge 0, & i &= 1,\ldots,m_\theta \\ \underline{b}_i \le b_i &\le \overline{b}_i, & i &= 1,\ldots,M. \end{aligned} \tag{4}$$

We make the following assumptions about this generic model.

- The first set of inequalities describe capacity constraints on the communication channels. The functions f_i are convex in (b,θ), monotone increasing in b and decreasing in θ. We will show below that many classical capacity formula satisfy these assumptions.
- The second set of constraints describes resource limitations, such as a total available power or bandwidth for a group of channels.
- The third constraint specifies that the communications variables are nonnegative.
- The last group of inequalities specify lower and upper bounds for each bit allocation. We assume that $\underline{b}_i$ and $\overline{b}_i$ are nonnegative integers. The lower bounds are imposed to ensure that the white noise model for quantization errors is a reasonable assumption (see Section III-B). The upper bounds can arise from hardware limitations.

This generic model will allow us to formulate the communication resource allocation problem, i.e., choosing θ to optimize overall system performance, as a convex optimization problem.

There is also one more important constraint on b not included in the model above: the b_i's should all be integers. We ignore this constraint for now and will return to it in Section IV.

B. Examples of Channel Capacity Constraints

In this section, we describe some classical channel models and show how they fit the generic model (4). Detailed descriptions of more channel models can be found in, e.g., [18] and [19]. Channels with gain variations (fading) as well as rate constraints based on bit-error rates can be formulated in a similar manner (see, e.g., [20]).

Frequency Division Multiple Access (FDMA) Gaussian Channels: In the Gaussian broadcast channel with FDMA, a transmitter sends information to n receivers over disjoint frequency bands with bandwidths $W_i \ge 0$ and assigns a transmit power $P_i \ge 0$ to each band. The communications variables are P_i and W_i for each individual channel. The receivers are subject to independent additive white Gaussian noises with power spectral densities N_i. The classical Shannon capacity result (see, e.g. [18]) relates the achievable bit allocations b_i and the communications variables by $b_i \le \alpha W_i \log_2(1 + (P_i/N_iW_i))$, which is equivalent to

$$\begin{aligned} f_i(b_i, W_i, P_i) &= b_i - \alpha W_i \log_2\left(1 + \frac{P_i}{N_iW_i}\right) \le 0, \\ i &= 1,\ldots,n. \end{aligned} \tag{5}$$

It is easily verified that f_i is jointly convex in the variables (b_i, W_i, P_i), monotone increasing in b_i and monotone decreasing in W_i and P_i.

So, (5) is in the generic form of the first set of constraints in (4). The communications variables are constrained by total resource limits

$$P_1 + \cdots + P_n \leq P_{\mathrm{tot}} \quad W_1 + \cdots + W_n \leq W_{\mathrm{tot}}$$

which have the generic form for total resource limits (the second set of constraints) in (4).

In the Gaussian multiple access channel with FDMA, n transmitters send information to a common reveiver, each using power P_i over bandwidth W_i. It has the same set of constraints as for the broadcast channel, except that $N_i = N,\ i = 1, \ldots, n$ (since they have a common receiver).

IV. Resource Allocation for Fixed Linear System

In this section, we assume that the linear system is fixed and consider the problem of choosing the communications variables to optimize the system performance. We take as the objective (to be minimized) the variance of the performance signal z, given by (3). Since this variance consists of a fixed term (related to w) and the variance induced by the quantization, we can just as well minimize the variance of z induced by the quantization error, i.e., the quantity V_q defined in (2). This leads to the optimization problem

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{M} a_i 2^{-2b_i} \\ \text{subject to} \quad & f_i(b, \theta) \leq 0, \qquad i = 1, \ldots, m_f \\ & h_i^T \theta \leq d_i, \qquad i = 1, \ldots, m_h \\ & \theta_i \geq 0, \qquad i = 1, \ldots, m_\theta \\ & \underline{b}_i \leq b_i \leq \overline{b}_i, \qquad i = 1, \ldots, M \end{aligned} \tag{6}$$

where $a_i = (1/3)\|G_{zqi}\|^2 s_i^2$ and the optimization variables are θ and b. We note that while the formula (2) was derived assuming that b_i are integers, the objective function makes sense for $b_i \in \mathbf{R}_+$. Since the objective function and each constraint in problem (6) are convex, this is a convex optimization problem. It can be solved globally and efficiently using a variety of methods, e.g., interior-point methods (e.g., [21]). In many cases, the problem (6) has an separable structure, which can be efficiently exploited by dual decomposition (e.g., [21] and [22]).

We now return to the requirement that the bit allocations must be integers. Since general-purpose integer programming techniques have high computational complexity, it is of interest to develop efficient heuristic methods that give good suboptimal integer solutions. We propose to use a simple *variable threshold rounding*: for a given threshold $0 \leq t \leq 1$, we round b_i down if its fractional part is no larger than t and round it up otherwise. Given the rounded bit allocations, we find the associated communications variables θ by solving a convex feasibility problem with the constraints in (6). We then find the smallest t that admits a feasible solution. In [23], we discuessed some theoretical properties of this scheme and demonstrated its effectiveness on a networked least-squares estimator.

V. Joint Design of Communication and Linear Systems

We have seen that when the linear system is fixed, the problem of optimally allocating communication resources is often convex (ignoring integrality constraints) and can be efficiently solved. In order to achieve optimal system performance, however, one should optimize the linear system parameters *and* the communications variables *jointly*. Unfortunately, this joint design problem is in general not convex.

In some cases, however, the joint design problem is convex in subsets of the variables. For example, the globally optimal communications variables can be computed very efficiently (ignoring the integrality constraints) when the linear system is fixed. Similarly, when the communications variables are fixed, it is often possible to compute the globally optimal linear system variables. Finally, when the linear system and the communications variables are fixed, it is straightforward to find the quantizer scalings, e.g., by the 3σ-rule. This naturally leads to an approach where we sequentially fix one set of variables and optimize over the others.

A. Alternating Optimization for Joint Design

The fact that the joint problem is convex in certain subsets of the variables while others are fixed can be exploited by the following iterative optimization procedure.

given initial linear system variables $\varphi^{(0)}$, communications variables $\theta^{(0)}$, scalings $s^{(0)}$
repeat
1. Fix $\varphi^{(k)}$, $s^{(k)}$ and optimize over θ. Let $\theta^{(k+1)}$ be the optimal value.
2. Fix $\theta^{(k+1)}$, $s^{(k)}$ and optimize over φ. Let $\varphi^{(k+1)}$ be the optimal value.
3. Fix $\varphi^{(k+1)}$, $\theta^{(k+1)}$. Let $s^{(k+1)}$ be appropriate scaling factors.
until convergence

Many variations on this basic heuristic method are possible. We can, for example, add trust region constraints to each of the optimization steps to limit the variable changes in each step. Another variation is to convexify (by, for example, linearizing) the jointly nonconvex problem and solve in each step using linearized versions for the constraints and objective terms in the remaining variables; see, e.g., [24] and the references therein.

Since the joint problem is not convex, there is no guarantee that this heuristic converges to the global optimum. On the other hand it appears to work well in practice.

B. Control Over Communication Networks

We consider a system with distributed sensors and actuators. The sensors send their measurements to a central controller through a multiple access channel and the controller sends control signals to the actuators through a broadcast channel, as shown in Fig. 5.

The linear dynamical system has a state-space model

$$\begin{aligned} x(t+1) &= Ax(t) + B\left(u(t) + w(t) + p(t)\right) \\ y_r(t) &= Cx(t) + v(t) + q(t) \end{aligned}$$

where $u(t) \in \mathbf{R}^m$ and $y(t) \in \mathbf{R}^n$, $w(t)$ is the process noise, $v(t)$ is the sensor noise and $p(t)$ and $q(t)$ are quantization noises due to the bit rate limitations of the communication channels. Assume that $w(t)$ and $v(t)$ are independent zero-mean white noises with covariance matrices R_w and R_v, respectively. Using the independent white-noise model for the quantization noises, we can define the equivalent process noise and sensor noise

$$\widetilde{w}(t) = w(t) + p(t) \quad \widetilde{v}(t) = v(t) + q(t)$$

with covariance matrices $R_{\widetilde{w}} = R_w + R_p$ and $R_{\widetilde{v}} = R_v + R_q$, respectively, where

$$\begin{aligned} R_p &= \mathbf{diag}\left(\frac{s_{a1}^2}{3} 2^{-2b_{a1}}, \ldots, \frac{s_{am}^2}{3} 2^{-2b_{am}}\right) \\ R_q &= \mathbf{diag}\left(\frac{s_{s1}^2}{3} 2^{-2b_{s1}}, \ldots, \frac{s_{sn}^2}{3} 2^{-2b_{sn}}\right). \end{aligned} \tag{7}$$

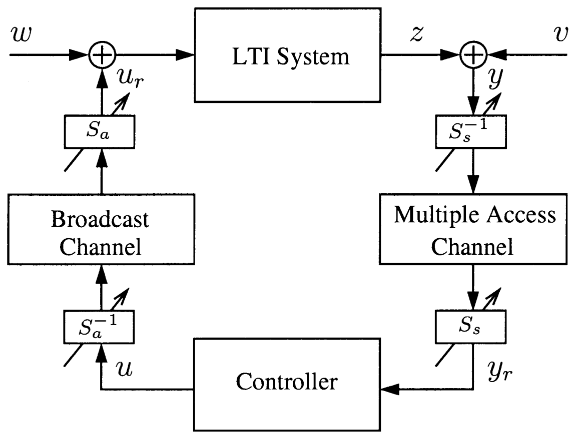

Fig. 5. Control over communication networks.

Here, b_{ai} and b_{sj} are numbers of bits allocated to the actuators and sensors and s_{ai} and s_{sj} are corresponding scaling factors for the quantizers, found by the 3σ-rule.

Our goal is to design a controller that minimizes the root-mean square (rms) value of $z = Cx$, subject to some upper bound constraints on the rms values of the control signals:

$$\begin{aligned} &\text{minimize } \mathbf{rms}(z) \\ &\text{subject to } \mathbf{rms}(u_i) \leq \beta_i, \qquad i = 1, \ldots, m. \end{aligned} \tag{8}$$

The constraints are added to avoid actuator saturation. It can be shown that the optimal controller for this problem has the standard estimated state feedback form

$$\begin{aligned} \hat{x}(t+1|t) &= A\hat{x}(t|t-1) + Bu(t) + L\left(y(t) - C\hat{x}(t|t-1)\right) \\ u(t) &= -K\hat{x}(t|t-1) \end{aligned}$$

where K is the state feedback control gain and L is the estimator gain, found by solving an appropriately weighted LQG problem. Finding the appropriate weights, for which the LQG controller solves the problem (8), can be done via solving the dual problem; see, e.g., [17] and [25].

Iterative Procedures for Controller Design: First, we allocate an equal number of bits to each actuator and sensor. This means that we assign power and bandwidth (in the case of FDMA) uniformly across all channels. We can design a controller for this fixed uniform resource allocation via an iterative design on the scaling factors and the controller. The iterative procedure is very similar to the one in Section V-A, but without the resource allocation step. For the joint optimization problem, we use the alternating optimization procedure in Section V-A. Here, the controller parameters φ are the state feedback gain K and estimator gain L and the communications variables θ are the powers and bandwidths allocated to the multiple access and broadcast channels. Step 1) of the iterative procedure solves the resource allocation problem (6); step 2) solves the controller design problem (8); step 3) computes the rms values of u_i and y_i and find the scaling factors using the 3σ-rule.

C. Numerical Example: Control of a Mass-Spring System

Now, we consider the system shown in Fig. 6. The position sensors on each mass send measurements $y_i = x_i + v_i$, where v_i is the sensor noise, to the controller through a Gaussian multiple access channel using FDMA. The controller receives $y_{ri} = x_i + v_i + q_i$, where q_i is the quantization error. The controller sends control signals u_j to actuators on each mass through a Gaussian broadcast channel using FDMA. The actual force acting on each mass is $u_{rj} = u_j + w_j + p_j$, where w_j is the exogenous disturbance and p_j is the quantization error.

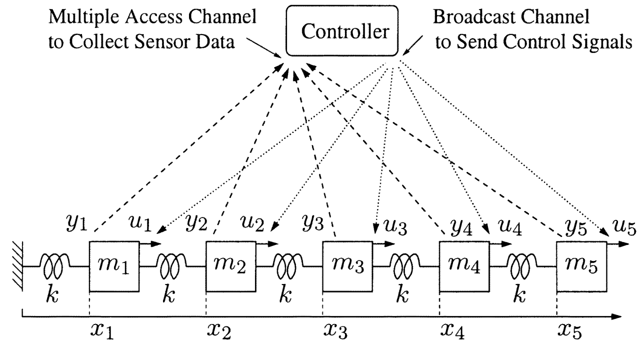

Fig. 6. Control of a mass-spring system.

The mechanical system parameters are $m_1 = 10$, $m_2 = 5$, $m_3 = 20$, $m_4 = 2$, $m_5 = 15$, and $k = 1$. The discrete-time system dynamics is obtained using a sampling frequency which is five times faster than the fastest mode of the continuous-time dynamics. The independent zero-mean white noises w and v have covariance matrices $R_w = R_v = 10^{-6}I$. The actuators impose rms constraints on the control signals: $\mathbf{rms}(u_i) \leq 1$, $i = 1, \ldots, 5$.

The multiple access channel and the broadcast channel have separate total power limits $P_{\text{mac,tot}} = P_{\text{bc,tot}} = 7.5$, but they share a total bandwidth limit $W_{\text{tot}} = 10$. All receivers have the same noise power density $N = 0.1$. The proportional coefficient in the capacity formula is $\alpha = 2$. We impose an upper bound $\bar{b} = 12$ and a lower bound $\underline{b} = 5$ for all quantizers.[1]

First, we allocate power and bandwidth evenly to all sensors and actuators, which results in a uniform allocation of eight bits for each channel. For this fixed resource allocation, the iterative controller and scaling design yields $\mathbf{rms}(u_i) = 1$ for all i's and $\mathbf{rms}(z) = 0.549$. Then we used the alternating procedure in Section V-A. to do joint optimization of bit allocation and controller design. After four iterations, it resulted in $\mathbf{rms}(u_i) = 1$ for all i's and $\mathbf{rms}(z) = 0.116$. The variable threshold rounding procedure [23] yields the threshold $t^\star = 0.615$ and $\mathbf{rms}(z) = 0.126$, which is quite close to the relaxed noninteger solution. We see a significant 77% reduction in rms value compared with the uniform bit allocation.

Fig. 7 shows the rounded resource allocation. We see that more bandwidth and, hence, more bits are allocated to the broadcast channel than to the multiple access channel. This means that the closed-loop performance is more sensitive to the equivalent process noises than to the equivalent sensor noises.

VI. Conclusion

We have addressed the problem of jointly optimizing the parameters of a linear system and allocating resources in the communication system that is used for transitting sensor and actuator information. We considered a scenario where the coding and medium access scheme of the communication system are fixed, but the available communications resources, such as transmit powers and bandwidths, can be allocated to different channels in order to influence the achivable communiucation rates. To model the effect of limited communication rates on the performance of the linear system we assumed conventional uniform quantization and used a simple white-noise model for quantization errors.

[1]Here the open-loop system is critically stable and the lower bound for stabilization given in [1]–[3] is zero. More generally, if we discretize an unstable continuous-time open-loop system using a sampling rate which is at least twice the largest magnitude of the eigenvalues (a traditional rule-of-thumb in design of digital control systems; see, e.g., [16]), then the lower bound given in [1]–[3] is less than one bit and $b_i \geq 3$ or 5 is usually high enough for assuming the white noise model for quantization errors.

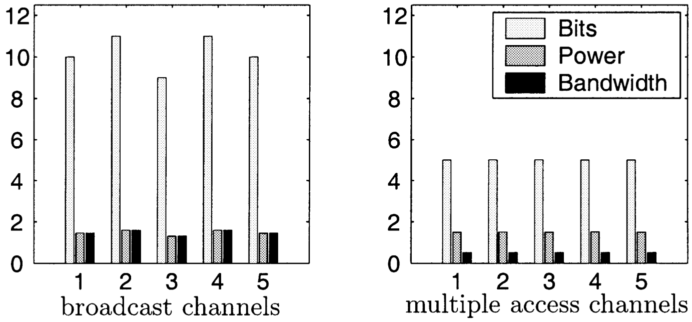

Fig. 7. Allocation of quantization bits and communications resources.

We showed that the problem of allocating communication resources to optimize the stationary performance of the linear system is often convex (ignoring the integrality constraint), hence readily solved. The problem of jointly allocating communication resources and designing the linear system is in general not convex, but is often convex in subsets of variables while the others are fixed. We suggested an iterative heuristic for the joint design problem that exploits this special strucutre, and demonstrated its effectiveness on the design of a multivariable networked LQG controller.

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Control of Integral Processes With Dead Time—Part 3: Deadbeat Disturbance Response

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***Abstract*—A deadbeat disturbance response of integral processes with dead-time is obtained by intentionally using two adjustable delay elements in the controller. These delays are tuning parameters of the controller. The shorter delay is optimally determined to minimize the robustness indicator (equivalently, to maximize the allowable uncertainty bound) while the longer delay (i.e., the deadbeat time itself) is determined with compromise of robustness. An example with comparison to conventional control schemes has been given to show the effectiveness of the proposed controller. Some interesting topics, such as input shaping techniques and dual-locus diagrams, are involved in this note.**

***Index Terms*—Dead-time compensator, deadbeat, disturbance observer, input shaping, integral process, robustness.**

I. INTRODUCTION

In recent years, the control of processes with an integrator and dead time has attracted much attention because of the inherent *critical stability* or *instability*. The problem originated from the fact that the well-known Smith predictor (SP) cannot be applied to these systems because of the

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