43rd IEEE Conference on Decision and Control
December 14-17, 2004
Atlantis, Paradise Island, Bahamas

ThA11.3

Moving Horizon Filter for Monotonic Trends

Sikandar Samar
Stanford University

Dimitry Gorinevsky
Honeywell Labs

Stephen Boyd
Stanford University

***Abstract*— This paper presents a novel approach for constrained state estimation from noisy measurements. The optimal trending algorithms described in this paper assume that the trended system variables have the property of *monotonicity*. This assumption describes systems with accumulating mechanical damage. The performance variables of such a system can only get worse with time, and their behavior is best described by monotonic regression. Unlike a standard Kalman filter problem, where the process disturbances are assumed to be gaussian, this paper considers a random walk model driven by a one-sided exponentially distributed noise. The main contribution of this paper is in studying recursive implementation of the monotonic regression algorithms. We consider a moving horizon approach where the problem size is fixed even as more measurements become available with time. This enables us to perform efficient online optimization, making embeded implementation of the estimation computationally feasible.**

I. Introduction

This work studies prognostic estimation algorithms for system health management applications. The initial motivation for the development of these algorithms was aircraft maintenance automation. The problem statement and algorithms can also be applied in many other applications where system performance needs to be monitored and trended. Such applications include automotive telematics, semiconductor fab equipment, industrial and chemical processes, and nuclear power plants. A specific focus of this paper is on embedded algorithms for mission-critical systems. These onboard algorithms are particularly useful for real time trending in aerospace and biomedical applications.

We develop trending algorithms that provide reliable online estimation of gradually deteriorating fault conditions in the presence of noise. Many efficiency loss parameters are associated with physical deterioration of hardware due to mechanical wear or erosion. These parameters are a priori known to grow (or decay) with time. They never decay (or grow) unless a maintenance action is performed. We use the *monotonic regression* framework presented in [5] to model such deterioration in fault parameters. Monotonic regression is an advanced statistical technique that has been used for some time; see [12], [13]. The prior publications in that area provide ad hoc algorithms. In this work we model the monotonic behavior and present algorithms within an optimal filtering formulation that can be implemented online. The model uses a one-sided exponential distribution for the random process driving the system state evolution. This is unlike the well-known Kalman filter approach where a symmetric (gaussian) distribution is used. See [5] for more information about the monotonic regression model. The work in [5] was focused on batch mode off-line processing. In this paper we extend that approach towards embedded online implementation.

One of the reasons why Kalman filtering is popular in practice is its recursive nature. Most modern navigation systems routinely use Kalman filters with simple models for trending motion of mobile platforms. However a batch optimization solution of the monotonic regression problem results in a growing size of the problem as more measurements become available. In many aerospace, automotive, and other applications there is a need for embedded implementation of the monotonic regression and the continuous growth of the problem size is unacceptable due to memory limitations. As a step towards recursive trending, we use moving horizon approach to the optimization-based solution. Such approaches were initially developed and applied to control problems, which can be considered dual to the estimation problems [6], [11]. More recently moving horizon optimization has been used in the estimation setting, *e.g.*, see [1], [4]. The moving horizon formulation allows us to keep the problem size fixed even as more sensor data becomes available. However considering only a subset of the total available measurements complicates the convergence analysis of the estimation method. This is a focus of current research. The prior work in this area, *e.g.*, see [9] however, considers problems without constraints that are linearizable. The receding horizon estimation formulation in this paper is different because it is based on constrained optimization.

The paper is organized as follows; Section II introduces our notation and the residual data model that we use in the later sections. Section III briefly discusses the monotonic regression framework used in this paper. Moving horizon formulation is presented in Section IV, where the deteriorating trend is estimated by solving a fixed dimension constrained quadratic programming (QP) problem at each step. Section V discusses some computational issues, and provides an example of the moving horizon trending algorithm. Concluding remarks are given in Section VI.

Sikandar Samar is with the Information Systems Laboratory, Department of Electrical Engineering, Stanford University, CA, 94305; Email: sikandar@stanford.edu.
Dimitry Gorinevsky is with Honeywell Labs., Fremont, CA, 94539; Email: dimitry.gorinevsky@honeywell.com.
Stephen Boyd is with the Department of Electrical Engineering, Stanford University, CA; Email: boyd@stanford.edu.

II. Preliminaries

The mathematical problem statement considered in this paper deals with trending gradually deteriorating fault parameters from noisy sensor data. Let $x(t)$ be such a monotonic fault parameter at the usage cycle t. As an example think of $x(t)$ as a variable describing wear in an engine which can only get worse with time due to accumulating mechanical damage. Let $y(t)$ denote the estimate of $x(t)$ at the cycle t. This estimate depends on internal sensor measurements or on data from ambient conditions. The estimate $y(t)$ may differ from the performance deterioration variable $x(t)$ due to modeling inaccuracies or sensor noise and is given as

$$y(t) = x(t) + \gamma(t), \tag{1}$$

where $\gamma(t)$ is the 'noise' variable. The data model (1) is used as a basis for the estimation algorithm design in this paper. For the remainder of this paper we assume that in (1) the observation $y(t)$, the underlying trend $x(t)$, and the noise $\gamma(t)$ are all scalars. $\gamma(t)$ is an uncorrelated (white) noise sequence, where variable γ is zero mean gaussian distributed with the covariance Γ

$$\gamma \sim N(0,\Gamma): \quad \left\{ P(\gamma \leq q) = \int_{-\infty}^{q} p(\gamma) d\gamma, \; p(x) = N_\Gamma(x) \equiv \frac{1}{\sqrt{2\pi\Gamma}} e^{-x^2/(2\Gamma)} \right\}. \tag{2}$$

The trending algorithms can be easily extended to include the case of multiple faults, $x \in \Re^n$, $n > 1$. We only consider the single fault case since it provides more clarity in presentation.

III. Monotonic Regression

Consider the data sequences $x(t)$, $y(t)$ in (1) on the interval $t = 1, \ldots, N$. We represent the sequences as

$$Y_N = \{y(1), \ldots, y(n)\}, \tag{3}$$

$$X_N = \{x(1), \ldots, x(n)\}. \tag{4}$$

For now we limit our attention to the fixed size fault trending problem, which is to estimate the underlying fault parameter sequence X_N based on the observed data sequence Y_N. This problem is discussed in greater depth in [5]. We only present a brief review of the results of [5] in this section. In Section IV we will use the moving horizon approach to develop recursive algorithms for the case when the size of the estimation problem grows beyond N as more measurements become available with time.

In addition to the statistical model of the observation noise (2) we also need a statistical model for the fault parameter sequence (4). A standard probabilistic method for modeling the unknown underlying sequence is to use a Random Walk model that has the form

$$x(t+1) = x(t) + \xi(t), \tag{5}$$

where $\xi(t)$ is a normally distributed uncorrelated white noise sequence with covariance Ξ

$$\xi \sim N(0, \Xi). \tag{6}$$

The initial state of the sequence is also assumed to be normally distributed with the mean x_0 and covariance Q_0

$$x(t=1) \sim N(x_0, Q_0). \tag{7}$$

The filtering problem is to estimate the *orbit* $x(t)$ of the Markov chain (1)–(7) from the noisy sensor data $y(t)$. The state of the chain at time t is fully defined by the distribution of the random variable $x(t)$. Since variables are gaussian, an optimal estimate of the trend for this classical random walk model is given by the Kalman Filter. The derivation of Kalman Filter is well-known and can be found in [7].

In the monotonic regression framework, we consider a non-standard model of the random walk driven by a random sequence with an exponential distribution for each term. The nonlinear filter implementing the monotonic regression is derived as an optimal estimator for this model. Let $\xi(t)$ in (5) be an *exponentially* distributed uncorrelated noise sequence. The exponential distribution is given as

$$\xi \sim E(\lambda): \quad \left\{ P(\xi \leq q) = \int_{-\infty}^{q} p(x) dx, \; p(\xi) = E_\lambda(x) \equiv \frac{1}{\lambda} e^{-x/\lambda} \right\}. \tag{8}$$

The probability distribution of the initial conditions is still given by (7). Now consider the problem of estimating the orbit $x(t)$ of the same Markov chain with the exponential noise distribution (8) replacing the gaussian distribution given in (6). The derivation is similar to the Kalman Filter derivation and is given in [5]. It results in an optimization problem that needs to be solved for finding the MAP estimate of the orbit $x(t)$. We just state the optimization problem whose solution yields the required estimate and refer the reader to [5] for further details of the derivation.

$$J = \frac{1}{2Q_0}[x(1) - x_0]^2 + \frac{1}{2\Gamma}\sum_{t=1}^{N}[x(t) - y(t)]^2 + \frac{1}{\lambda}\sum_{t=2}^{N}[x(t) - x(t-1)] \rightarrow \min, \tag{9}$$

subject to

$$x(1) \leq x(2) \leq \ldots \leq x(N). \tag{10}$$

If the initial condition covariance $Q_0 \to \infty$, $i.e.$, no a priori information about x_0 is available then the first term in the loss index disappears and solving the problem would yield a Maximal Likelihood estimate of the orbit $x(t)$. On the other hand if in (9) we assume that the initial state information is exactly available, and the initial condition covariance $Q_0 \to$

0 then it results in the following problem

$$J = \frac{1}{2\Gamma}\sum_{t=1}^{N}[x(t)-y(t)]^2 + \frac{1}{\lambda}\sum_{t=2}^{N}[x(t)-x(t-1)] \rightarrow \min, \quad (11)$$

subject to

$$x(1) = x_0; \quad x(1) \leq x(2) \leq \ \ldots \ \leq x(N). \quad (12)$$

The estimation of the *orbit* $x(t)$ in the monotonic regression framework is a QP problem with convex (linear) constraints, and very efficient computational techniques are available for solving such problems. A QP solver using sparse arithmetic has been developed during the course of this work. It is ideally suited to embedded implementation and is used for the simulation results of Section V. The solution to (9), (10) depends on the tuning parameter, $\beta := \lambda/\Gamma$ and the choice of the initial condition covariance. The last term in the loss function (9) provides a linear penalty $\beta^{-1}[x(N) - x(1)]$ for the overall increase of the fault estimate x through the observation time. The weight at this penalty is the ratio of the observation noise covariance Γ to the fault driving noise covariance λ. This parameter β can be tuned empirically to achieve the desired performance of the filter.

So far we have assumed a univariate problem formulation. In the multivariate form of (1)

$$y(t) = S(t)x(t) + \gamma(t),$$

where $x \in \Re^n$, $y \in \Re^m$, $S \in \Re^{m\times n}$ we can use a parameter $\lambda = \lambda_j$ to describe the exponential distribution of each monotonic component x_j of vector x. This will lead to the following loss function

$$\begin{aligned} J &= \frac{1}{2}[x(1)-x_0]^T Q_0^{-1}[x(1)-x_0] \\ &+ \frac{1}{2}\sum_{t=1}^{N}[S(t)x(t)-y(t)]^T\Gamma^{-1}[S(t)x(t)-y(t)] \\ &+ \sum_{t=2}^{N}\sum_{j=1}^{n}\frac{[x_j(t)-x_j(t-1)]}{\lambda_j} \rightarrow \min, \quad (13)\end{aligned}$$

where Q_0 is the initial condition covariance matrix and Γ is the covariance matrix for measurement noise. We consider the univariate case in this paper but the developed results can be easily extended to include the multivariate problem statement.

The main difficulty with the problem (9), (10) is in the presence of the constraints. When the statistical model of the underlying trend sequence (5) is driven by a gaussian noise distribution (6) there are no constraints in (9) and we just get a standard least-squares problem. If we assume a gaussian distribution for all the random processes, then the unconstrained least-squares problem takes the form

$$\begin{aligned} J &= \frac{1}{2Q_0}[x(1)-x_0]^2 + \frac{1}{2\Gamma}\sum_{t=1}^{N}[x(t)-y(t)]^2 + \\ &\frac{1}{2\Xi}\sum_{t=2}^{N}[x(t)-x(t-1)]^2 \rightarrow \min, \quad (14)\end{aligned}$$

where Γ and Ξ represent the covariances of the measurement and process noise respectively. This batch least-squares approach is used in Section V to compare the accuracy of the results of our moving horizon monotonic regression filter.

IV. Moving Horizon Estimation

The monotonic regression problem discussed in the previous section assumes batch processing of the data sequences and consequently limits the estimation problem size. Whereas this approach may be suitable in some cases many other safety-critical applications require embedded filters for online optimization. The problem size grows with time as an embedded filter processes more data resulting in an increase in computational complexity. The moving horizon formulation is specifically used to overcome this problem.

The basic idea in the moving horizon approach is to maintain a constant length of the estimation window by discarding the oldest sample as a new measurement becomes available. However working with only a subset of the total available information raises some important performance and stability issues which need to be addressed. Moving horizon control, commonly called model predictive control, has been studied extensively during the last decade; see [10], [2], [3]. More recently the moving horizon strategy, also referred to as sliding window, finite or limited memory or receding horizon, has been applied to estimation problems; see [8], [9]. Most of the work in constrained moving horizon estimation uses a least-squares framework, and the results reduce to a standard Kalman filter in the absence of constraints.

We present a moving horizon estimation algorithm for monotonic regression which computes a new estimate at each step by solving a finite horizon constrained QP problem. The data set at each step is defined by the current time to current time minus a fixed horizon. Recursive computations enable estimation of the fault parameters using the most recent time interval. This allows us to keep the problem size bounded as more measurements become available.

A. Trending approach

The observed data sequence Y_N in (3) yields a constrained QP problem of dimension N given by (9), (10). Define $\{\mathcal{I}\}_1^N := \{t = 1, \ldots, N\}$ as the information interval for this optimization problem. It is assumed that the estimate based on $\{\mathcal{I}\}_1^N$ is computed efficiently enough for us to fix the horizon size to N. In an embedded implementation of the monotonic regression filter new data for processing constantly becomes available, *e.g.*, as a result of one additional measurement the information interval $\{\mathcal{I}\}_1^N$ expands to $\{\mathcal{I}\}_1^{N+1} := \{t = 1, \ldots, N+1\}$. For an arbitrary number of additional measurements we define the estimation problem over the interval $\{\mathcal{I}\}_1^M$, with $M > N$, as

$$J = \frac{1}{2Q_0}[x(1) - x_0]^2 + \frac{1}{2\Gamma}\sum_{t=1}^{M}[x(t) - y(t)]^2 + \frac{1}{\lambda}\sum_{t=2}^{M}[x(t) - x(t-1)] \rightarrow \min, \quad (15)$$

$$x(1) \leq x(2) \leq \ldots \leq x(M) \quad (16)$$

We refer to (15), (16) as the full information problem since it takes into account all the available data at that instant. This however results in a QP problem of dimension M. As more and more data becomes available for processing, the information interval expands further and the dimension of the constrained QP problem continues to grow.

B. Moving Horizon Scheme

The standard approach to proving convergence of moving horizon estimation (MHE) for a purely quadratic loss function is the use of *arrival cost* strategy. This is analogous to the *cost to go* concept used in control problems. Arrival cost is used to summarize all the information prior to the current horizon. The convergence of MHE algorithms based on the arrival cost concept depends on the accuracy with which the old data is approximated by the arrival cost. In the specific case of monotonic regression, we have a loss function with a linear penalty term and constraints. The convergence results of [8], [9] cannot be applied directly as a result. The arrival cost cannot be computed in a convenient way for our problem. In this paper we propose to use a quadratic cost as an approximation that leads to practically convenient algorithms. The matching of the MHE and the full information estimate in monotonic regression framework is governed by the quadratic penalty term for the initial condition of each horizon.

Consider a Moving Horizon Estimation problem over the interval $\{\mathcal{I}\}_{M-N+1}^M$. From now onwards we use the end point of the interval in subscripts to refer to the horizon that it indicates. The MHE problem is defined over the interval $\{\mathcal{I}\}_{M-N+1}^M$ as

$$J = \frac{1}{2Q_{0(M)}}[x_{(M-N+1)} - x_{0(M)}]^2 + \frac{1}{2\Gamma}\sum_{t=M-N+1}^{M}[x(t) - y(t)]^2 + \frac{1}{\lambda}\sum_{t=M-N+2}^{M}[x(t) - x(t-1)] \rightarrow \min \quad (17)$$

$$x(M-N+1) \leq \ldots \leq x(M), \quad (18)$$

where $x_{0(M)}$ and $Q_{0(M)}$ define the initial state mean and covariance for the moving horizon estimation problem over the interval $\{\mathcal{I}\}_{M-N+1}^M$. The above MHE is a constrained QP problem of dimension N. As more measurements become available (M increases), we continue to slide the window accordingly. At every instant in the MHE, computations starting from the new measurement are repeated over a shifted horizon. This limits the MHE problem to a fixed size while the full information problem given by (15), (16) grows without bound. Note that for $t \leq N$, MHE and the full information problem are the same.

In a moving horizon regression filter we solve the fixed dimension constrained QP problem of (17), (18) at each step. One important aspect of computing the estimate in each horizon is to define the initial state estimate $x_{0(M)}$ for each next window.

Say we compute an estimate for the first horizon $\{\mathcal{I}\}_1^N$. This equals the full information estimate as $t = N$. Then as an additional measurement becomes available, we slide the window to compute the estimate over the interval $\{\mathcal{I}\}_2^{N+1}$. In doing so, we assume that the starting point for the estimate in the interval $\{\mathcal{I}\}_2^{N+1}$ is known from the previous estimate over the interval $\{\mathcal{I}\}_1^N$. Let $\hat{x}(2|N)$ denote the estimate for the second point of the sequence given measurements over the interval $\{\mathcal{I}\}_1^N$. Then we define the initial condition estimate in the next horizon $\{\mathcal{I}\}_2^{N+1}$ by setting $x_{0(N+1)} = \hat{x}(2|N)$. In a similar manner we set the initial condition for the next horizon $\{\mathcal{I}\}_3^{N+2}$, denoted $x_{0(N+2)}$, to equal $\hat{x}(3|N+1)$. By explicitly defining the initial conditions for computing the estimate in each horizon, we can prove exactness of the moving horizon and the full information monotonic regression filter for some special cases. Mathematically we can express the specification of initial condition for each next horizon as

$$x_{0(t)} = \hat{x}(t - N + 1|t - 1), \quad (19)$$

where the notation $\hat{x}(k|j-1)$ denotes the estimate for a point k given measurements over the interval terminating at $j-1$. This way of specifying the initial condition works well for the monotonic regression framework. It is easy to keep track of the estimate from the most recent window to specify the initial point of the next horizon. This does not raise any complicating memory issues if the horizon size is chosen appropriately.

C. Hard Constraint

Assume now that the initial condition covariance $Q_0 \rightarrow 0$. Then the initial state is defined by the hard constraint (19) and the MHE problem (17), (18) reduces to

$$J = \frac{1}{2\Gamma}\sum_{t=(M-N+1)}^{M}[x(t) - y(t)]^2 + \frac{1}{\lambda}\sum_{t=(M-N+2)}^{M}[x(t) - x(t-1)] \rightarrow \min, \quad (20)$$

$$x(M-N+1) = x_{0(M)},$$
$$x(M-N+1) \leq \ \dots\ \leq x(M). \tag{21}$$

Using the hard constraint (21) is different from the commonly used techniques in literature [8], [9]. These results assume gaussian distribution and work with a loss function with quadratic penalty terms.

We now state an important assumption which guarantees convergence of the MHE monotonic regression algorithm when the initial state is defined by (19).

Assumption 1: Assume that the moving average of the sequence of measurements is increasing as we move from one window to the next to compute the receding horizon estimate. Let $\overline{Y}$ denote the moving average of a sequence of measurements, *i.e.*,

$$\overline{Y}(t) = \frac{1}{N}\sum_{t-N}^{t} Y(t), \tag{22}$$

then an increase in moving average for each next horizon implies

$$\overline{Y}(t-N) \leq \overline{Y}(t-N+1) \tag{23}$$

This assumption is reasonable if the underlying trend is monotonic nondecreasing and the estimation horizon is considered large enough to provide sufficient statistical averaging for reducing the noise influence on the fault parameter estimates. We can now state the main proposition for this MHE scheme.

Proposition 1: Given a monotonic regression framework, the moving horizon estimate given by (20), (21) exactly equals the full information estimate of (15), (16) if the assumption in (23) is satisfied.

D. Other Covariance Choices

Proposition 1 is valid only for an appropriately chosen horizon length for which the assumption in (23) holds true. This assumption is somewhat limiting and not realistic in most practical cases. The quadratic penalty term in (17) will in general determine the error between the full information estimate and the MHE. The scheme discussed in (20), (21) is just one limit case of the penalty term. The other limit case for the penalty term will be when the initial condition covariance is very large, *i.e.*, $Q_{0(M)} \to \infty$. This implies that the initial condition for each moving window is left free as the penalty term in the loss index disappears. Choosing an appropriate initial state to guarantee convergence of the MHE algorithm is a subject of current research. The choice of measurement noise covariance to define the initial state is suitable in some cases. The measurement noise covariance is an easily available problem parameter that describes the order of magnitude for the arrival cost distribution covariance.

The moving horizon estimate is much easier to compute than the full information estimate of (15), (16). As additional measurements become available, the size of the full information problem continues to increase, making it computationally intractable. On the other hand the moving horizon estimation algorithm can perform efficient online estimation without being limited by excessive memory requirements.

V. Simulation and Computational Issues

Many computational methods are available for solving the convex optimization problem that we encounter in moving horizon fault estimation. When dealing with specific applications, a routine that best exploits the problem structure is usually more efficient than a general purpose solver. We used primal-dual interior-point methods to develop a solver that is based on sparse arithmetic. This solver is used in estimation and numerical analysis of our receding horizon monotonic regression filter. For a constrained QP problem over a fixed length horizon of 50 data points, the developed solver outperforms the QP solver in the MATLAB Optimization Toolbox by two orders of magnitude. In an embedded implementation where repeated estimates are required for each moving horizon this fast processing is extremely important. The performance of the developed solver is comparable to MOSEK which is the commercially available benchmark in speed. The solver is written in only 25 lines of MATLAB code which makes it much easier to implement in an embedded framework. The memory requirements for onboard processing depend upon the length of the moving horizon. The memory allocated by the developed solver for computing the fault estimates is equal to the space required for storing an array of six times the chosen horizon length. The efficient performance and limited memory requirements make the developed solver ideally suited for online implementation.

Fig. 1 shows the moving horizon estimate based on the initial condition constraint (19) for a randomly generated data sequence that satisfies the assumption in (23). The full information estimate and the moving horizon estimate overlap and appear as one curve. For the 200 data point sequence a moving window of size 20 with a tuning parameter value of $\beta = 4$ was used for this simulation. Solution of batch least-squares using (14) is also plotted to provide a comparison for the accuracy of our monotonic regression filter. As shown, the MHE gives a much better estimation of the underlying orbit than the least-squares solution. The random scatter in the data is ignored and the deterioration in performance is accurately captured by the moving horizon estimate.

The convergence of MHE was found to be as good as that of the corresponding full information problem. Whereas the full information problem may converge in slightly fewer iterations, it requires more complex computations at each iteration. The MHE algorithm is more practical because the problem size remains fixed at each step and the individ-

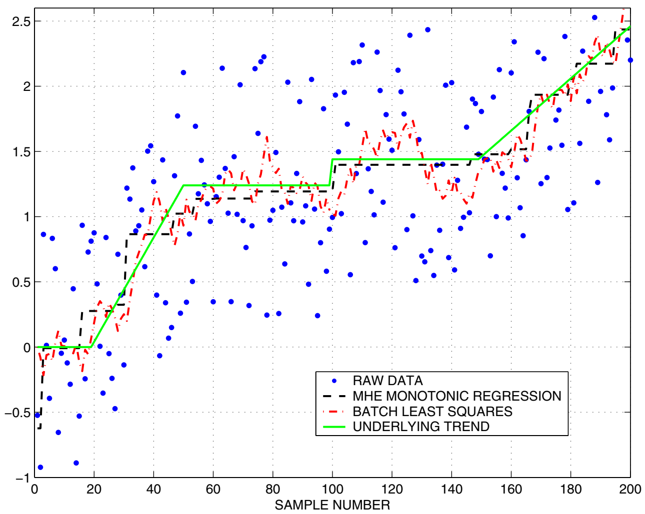

Fig. 1. Comparison of MHE and batch least square

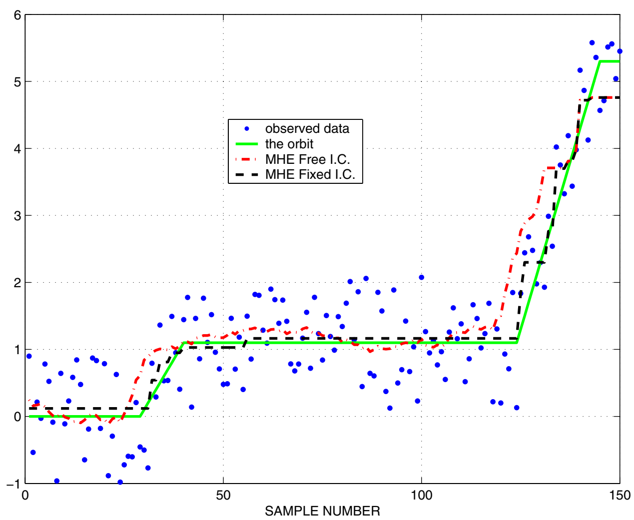

Fig. 2. Comparison of MHE with fixed and free initial condition

ual iterations are faster requiring much less computational power and memory usage.

Fig. 2 shows comparison of the two limit cases of the moving horizon scheme discussed in the previous section. The 150 point randomly generated data set was estimated using a 20 point sliding window with a tuning parameter value of $\beta = 3.7$. The MHE algorithm based on the fixed initial condition constraint of (19) matched the full information estimate and gave a good approximation of the underlying trend. The MHE based on the other limit case of free initial condition performed better than batch least squares (not shown) but was not as accurate as the estimate based on initial condition constraint. In this problem if the measurement noise covariance is used to define the initial state for each moving horizon then the resulting estimate is better than the one with free initial condition.

VI. Conclusion

Moving horizon estimation algorithms have been developed for monotonic trends. The moving horizon approach allows for recursive implementation of the estimation algorithm and is most useful for embedded filters. The developed technique shows superior performance in trending deteriorating parameters in comparison with batch least-squares. We show that the moving horizon estimate exactly matches the full information estimate in a special case of monotonic regression when an initial condition is specified for each next horizon by a hard constraint. The developed algorithms solve a constrained optimization problem of a fixed size at each step and are well suited to online implementation.

Current work includes extending the MHE algorithms to the case where the underlying statistical model of the signal sequence in (5) is assumed to be second-order. A second-order model is useful for describing the accumulation of secondary damage in a system caused by some primary fault condition. Another area of current focus is to define the initial condition state for each sliding interval in such a way that it can provide a global convergence guarantee for the moving horizon estimates.

References

[1] Alessandri, A., Baglietto, M., Battistelli, G. "Receding-horizon estimation for discrete-time linear systems," *IEEE Transactions on Automatic Control*, Vol. 48, No. 3, March 2003, pp. 473–478

[2] Bemporad, A., Borrelli, F., Morari, M. "Model predictive control based on linear programming—The explicit solution," *IEEE Transactions on Automatic Control*, Vol. 47, No. 12, December 2002, pp. 1974–1985

[3] Bemporad, A., Morari, M., Dua, V., Pistikopoulos, E.N. "The explicit linear quadratic regulator for constrained systems," *Automatica*, Vol. 38, No. 1, March 2003, pp. 3–20

[4] Ferrari-Trecate, G., Mignone, D., Morari, M. "Moving horizon estimation for hybrid systems," *IEEE Transactions on Automatic Control*, Vol. 47, No. 10, October 2002, pp. 1663–1676

[5] Gorinevsky, D. "Monotonic regression filters for trending gradual deterioration faults," *American Control Conference*, June 30 - July 2, 2004, Boston, MA

[6] Mayne, D.Q., Michalska, H. " Receding horizon control of nonlinear systems," *IEEE Transactions on Automatic Control*, Vol. 35, No. 7, October 1990, pp. 814–824

[7] Oppenheim, A.V., Schafer, R.W., and Buck, J.R. *Discrete-Time Signal Processing*, Prentice Hall, 1999

[8] Rao, C.V., Rawlings, J.B., Lee, J.H. "Constrained linear state estimation—a moving horizon approach," *Automatica*, Vol. 37, No. 10, February 2001, pp. 1619–1628

[9] Rao, C.V., Rawlings, J.B., Mayne, D.Q. "Constrained state estimation for nonlinear discrete-time systems: Stability and moving horizon approximations," *IEEE Transactions on Automatic Control*, Vol. 48, No. 2, February 2003, pp. 246–258

[10] Rawlings J.B. "Tutorial overview of model predictive control," *IEEE Control Systems Magazine*, Vol. 20, No. 3, 2000, pp. 38–52

[11] Rawlings J.B., Muske, K.R. " The stability of constrained receding horizon control ," *IEEE Transactions on Automatic Control*, Vol. 38, No. 10, October 1993, pp. 1512–1516

[12] Restrepo, A., Bovik, A. C. "Locally monotonic regression," *IEEE Trans. on Signal Processing*, Vol.41, Sept. 1993, pp. 2796–2810,

[13] Sidiropoulos, N.D., Bro, R. "Mathematical programming algorithms for regression-based nonlinear filtering in R^N," *IEEE Transactions on Signal Processing*, Vol. 47, No. 3, March 1999, pp. 771–782