Compensation of Multimode Fiber Dispersion Using Adaptive Optics via Convex Optimization

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***Abstract*—In this paper, we propose a provably optimal technique for minimizing intersymbol interference (ISI) in multimode fiber (MMF) systems using adaptive optics via convex optimization. We use a spatial light modulator (SLM) to shape the spatial profile of light launched into an MMF. We derive an expression for the system impulse response in terms of the SLM reflectance and the field patterns of the MMF principal modes (PMs). Finding optimal SLM settings to minimize ISI, subject to physical constraints, is posed as an optimization problem. We observe that our problem can be cast as a second-order cone program, which is a convex optimization problem. Its global solution can, therefore, be found with minimal computational complexity, and can be implemented using fast, low-complexity adaptive algorithms. We include simulation results, which show that this technique opens up an eye pattern originally closed due to ISI. We also see that, contrary to what one might expect, the optimal SLM settings do not completely suppress higher order PMs.**

***Index Terms*—Adaptive optics, optical fiber dispersion, spatial light modulators (SLMs).**

I. Introduction

MULTIMODE FIBER (MMF) is the dominant type of fiber used for data communications in current local-area networks. In achieving higher signalling rates, the dominant limiting factor is the intersymbol interference (ISI) caused by modal dispersion [1]. Light propagates in an MMF in modes, with each mode propagating at its group velocity. The set of modes excited depends on launch conditions at the input of the fiber. Thus, a pulse of light that excites many modes in the fiber arrives as several pulses at the output of the fiber—a phenomenon known as modal dispersion. This effect is analogous to multipath in wireless.

In describing modal dispersion, the relevant modes are typically not the modes of an ideal fiber—ideal modes (IM). Imperfections (bends, variations in refractive index profile, etc.) cause coupling between IMs. This means that a pulse launched into an IM will come out as a sequence of pulses. However, it has been shown [2] that there exists a complete set of orthonormal modes, called principal modes (PMs), such that a pulse launched into a PM at the input of the fiber emerges as a single pulse at the output, even in the presence of mode coupling. PMs in MMFs with mode coupling are analogous to principal states of polarization (PSPs) in single-mode fibers (SMF) with polarization-mode dispersion.

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In the past, electrical equalization [3], [4] has been used to mitigate ISI caused by modal dispersion. However, this can lead to noise enhancement, and, therefore, degradation of achievable bit-error ratio (BER) [5]. Instead, the use of adaptive optics was proposed in [6]. This involves shaping the spatial profile of the electric field at the input end of the fiber using a spatial light modulator (SLM) to excite only desired PMs. An SLM is a 2-D array of mirrors, capable of modifying the local phase and/or amplitude of an incident electric field. This technique leads to no noise enhancement. Experiments [7] have shown that this approach is capable of realizing high bit rates. In [6], a basic theoretical framework was developed. This framework, however, did not model all physics of the system pertinent to the problem. It also led to a very hard optimization problem—one with many local optima and no method to identify the global optimum. The algorithm proposed for adaptation of the SLM had no guarantee of convergence to the global optimum, or even a good local optimum.

In this paper, we develop a comprehensive theoretical framework for an MMF system that uses an SLM for ISI mitigation. Our objective is to find an SLM setting that minimizes the ISI. The system uses intensity detection, and the SLM modifies the electric field. Since intensity is a nonlinear function of electric field, this is a nonlinear filtering problem. Therefore, conventional techniques for optimization of linear filters are not applicable. Moreover, the resultant optimization problem is not in any standard form that leads to an efficient global solution. However, we observe that the optimal SLM settings may be obtained by solving an equivalent convex optimization problem. This convex problem is a second-order cone program (SOCP), and has $O(N^3)$ complexity. This means that the solution may be computed almost as easily as inverting an $N \times N$ matrix. The solution thus obtained is globally optimal. Moreover, the new framework enables us to develop efficient adaptive algorithms.

The remainder of this paper is organized as follows. In Section II, we introduce our transmission scheme, and develop a theoretical framework. We start by analyzing PMs and the SLM. From these, we derive the impulse response of the system. Finally, we derive an expression for the eye opening as a function of SLM settings. At the end of Section II, we pose minimization of ISI as an optimization problem. In Section III, we solve this optimization problem. We first prove the equivalence between the original optimization problem and the convex problem. We then solve this problem, and suggest adaptive techniques. In Section IV, we give simulation results, using parameters of commercially available components. We show that the algorithm cleans up the impulse response, and opens up the resulting eye pattern. We investigate the effect of SLM resolution, and adaptation after offset launch. Finally, we

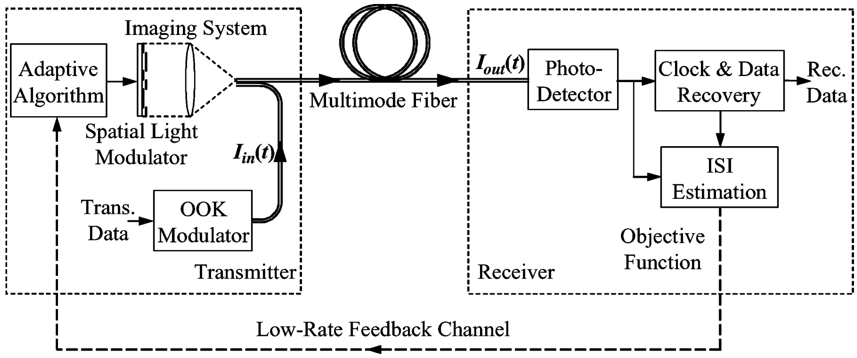

Fig. 1. Adaptive transmission system: output of the single mode fiber is imaged onto the SLM, SLM output is imaged onto MMF, and MMF output is detected and fed to an adaptive algorithm that controls the SLM.

see that the optimal impulse response, contrary to what might be expected, does not involve complete suppression of higher order PMs, even though the system is capable of it.

II. Adaptive Transmission System

First, we describe a physical system which enables ISI mitigation using an SLM. Then, we set up a mathematical framework for analysis. We look at two key components, PMs and the SLM, and derive an equation for the continuous-time impulse response of the system. Then, we link system performance—the eye opening—to the impulse response, and pose minimization of ISI as an optimization problem.

A. Adaptive Transmission System

The system configuration is shown in Fig. 1, and it was originally proposed in [6]. It consists of a transmitter, the MMF itself, and a receiver in which received light pulses are detected and decoded. At the transmitter, we have an SMF, from which light carrying the data signal is imaged onto an SLM. This is followed by an imaging system that focuses the light from the SLM into the input end of the MMF. The SLM is a 2-D array of pixels and is used to manipulate the phase and/or amplitude of the electric field incident on the MMF, in the spatial domain. The ISI at the receiver depends on the excitation pattern of modes in the MMF. Therefore, we try to set the SLM pixels to optimally shape the light field incident on the MMF and selectively excite modes in the MMF. At the receiver, light from the MMF is detected by a photodetector, and decoded by a recovery circuit. The detected signal is also used for ISI estimation, which drives an adaptive algorithm that controls the SLM.

B. Principal Modes

PMs [2] provide a framework for modeling pulse propagation in MMFs. If a pulse of light is launched in one of the IMs of a fiber (i.e., Bessel, Hermite–Gaussian, or Laguerre–Gaussian modes), in the presence of mode coupling, we get a sequence of pulses at the output. Fundamentally, this is because IMs are not eigenvectors of the group delay operator. It is shown in [2] that the eigenvectors of the group delay operator are in fact PMs. This means that a pulse of light launched into an input PM will come out as a single pulse in the corresponding output PM even in the presence of mode coupling. PMs also form an orthogonal basis over all propagating modes.

We now derive an explicit orthogonality relationship for PMs.[1] Let $\vec{\mathbf{E}}_{\mathrm{PM},i}(x,y)$ and $\vec{\mathbf{H}}_{\mathrm{PM},i}(x,y)$ be the electric and magnetic fields corresponding to the ith input PM, where the fiber supports $2M$ modes in both polarizations. Let $\hat{z}$ be the unit vector along the fiber axis.

Theorem 2.1:

$$\int \left[\vec{\mathbf{E}}_{\mathrm{PM},i}(x,y) \times \vec{\mathbf{H}}^{*}_{\mathrm{PM},j}(x,y)\right] \cdot \hat{z}dxdy = \delta_{ij}. \quad (1)$$

Proof: We know that the IMs of a fiber form an orthonormal basis for propagating modes; so PMs can be expanded in the basis of IMs. Let $\vec{\mathbf{E}}_{\mathrm{IM},i}(x,y)$ and $\vec{\mathbf{H}}_{\mathrm{IM},i}(x,y)$ be the electric and magnetic fields corresponding to the ith IM at the input face of the fiber. Also

$$\int \left[\vec{\mathbf{E}}_{\mathrm{IM},i}(x,y) \times \vec{\mathbf{H}}^{*}_{\mathrm{IM},j}(x,y)\right] \cdot \hat{z}dxdy = \delta_{ij} \quad (2)$$

[8]. Therefore

$$\begin{aligned}\vec{\mathbf{E}}_{\mathrm{PM},i}(x,y) &= \sum a_{ij}\vec{\mathbf{E}}_{\mathrm{IM},j}(x,y)\\ \vec{\mathbf{H}}_{\mathrm{PM},i}(x,y) &= \sum a_{ij}\vec{\mathbf{H}}_{\mathrm{IM},j}(x,y)\end{aligned} \quad (3)$$

for an appropriate choice of a_{ij}. Let $\mathbf{a_i} = [a_{i1} \ldots a_{i2M}]^T$. Since PMs are orthonormal, $\mathbf{a_i}^H\mathbf{a_j} = \delta_{ij}$. Therefore

$$\begin{aligned}&\int \left[\vec{\mathbf{E}}_{\mathrm{PM},i}(x,y) \times \vec{\mathbf{H}}^{*}_{\mathrm{PM},j}(x,y)\right] \cdot \hat{z}dxdy\\ &= \int \left\{ \left[\sum_k a_{ik}\vec{\mathbf{E}}_{\mathrm{IM},k}(x,y)\right] \right.\\ &\qquad \left. \times \left[\sum_l a_{jl}\vec{\mathbf{H}}_{\mathrm{IM},l}(x,y)\right]^{*} \right\} \hat{z}dxdy\\ &= \sum_k \sum_l a_{ik}a^{*}_{jl} \int \left[\vec{\mathbf{E}}_{\mathrm{IM},k}(x,y) \times \vec{\mathbf{H}}^{*}_{\mathrm{IM},l}(x,y)\right] \cdot \hat{z}dxdy\\ &= \sum_k a_{ik}a^{*}_{jk} = \mathbf{a_j}^H\mathbf{a_i} = \delta_{ij}.\end{aligned} \quad (4)$$

■

[1]In our analysis, we introduce the electric and magnetic field patterns of the IMs and of the PMs. These fields are functions of the optical frequency ω. Throughout this paper, we suppress the dependence on ω, assuming that the optical signals of interest occupy a narrow bandwidth, over which the modal field patterns can be considered independent of frequency. In particular, when we consider data-modulated signals, these are assumed to occupy a bandwidth smaller than the coherence bandwidth of the PMs. This assumption is valid when using quasi-monochromatic (laser) sources modulated at rates (e.g., 10 Gb/s) less than the coherence bandwidth of the PMs (e.g., tens to hundreds of gigahertz [7]). It is not expected to be valid when using light-emitting diode sources, which have bandwidths much greater than the bandwidths of the PMs.

C. Spatial Light Modulator

The SLM is a passive reflective device comprising a 2-D array of mirrors (the analysis remains identical for a transmission SLM). It can, therefore, be represented by a 2-D complex reflectance function $V(x,y)$. Since it is a passive device, $|V(x,y)| \leq 1$. Also, since it comprises a discrete array of mirrors

$$V(x,y) = \sum_{i=1}^{N} v_i s_i(x,y) \tag{5}$$

where N is the number of SLM blocks, $v_i \in \mathbb{C}$ is the reflectance of the ith SLM block, and $s_i(x,y)$ is an indicator function over the ith SLM block, defined as

$$s_i(x,y) = \begin{cases} 1, & (x,y) \text{ in the interior of the } i\text{th block} \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

The imaging from the SLM to the fiber face is assumed to be done using a linear optical device, e.g., lens. Therefore, propagation of light from the SLM to the fiber face may be represented by a linear operator L. Let the electric field incident on the SLM be $\vec{\mathbf{E}}_{\mathrm{SLMin}}(x,y)$, and let the output electric field of the SLM at the SLM face be $\vec{\mathbf{E}}_{\mathrm{SLMout}}(x,y)$. Then

$$\begin{aligned} \vec{\mathbf{E}}_{\mathrm{SLMout}}(x,y) &= V(x,y)\vec{\mathbf{E}}_{\mathrm{SLMin}}(x,y) \\ &= \sum_{i=1}^{N} v_i s_i(x,y)\vec{\mathbf{E}}_{\mathrm{SLMin}}(x,y) \qquad (7) \\ \vec{\mathbf{E}}_{\mathrm{fiberIn}}(x,y) &= L\left[\vec{\mathbf{E}}_{\mathrm{SLMout}}(x,y)\right] \\ &= L\left[\sum_{i=1}^{N} v_i s_i(x,y)\vec{\mathbf{E}}_{\mathrm{SLMin}}(x,y)\right] \\ &= \sum_{i=1}^{N} v_i \vec{\mathbf{E}}_i(x,y) \qquad (8) \end{aligned}$$

where $\vec{\mathbf{E}}_i(x,y) = L[s_i(x,y)\vec{\mathbf{E}}_{\mathrm{SLMin}}(x,y)]$, and the electric field at the input face of the fiber is $\vec{\mathbf{E}}_{\mathrm{fiberIn}}(x,y)$. We thus see that the electric field at the input face of the fiber is a linear function of the SLM reflectances.

D. Continuous-Time Impulse Response

We now derive an expression for the continuous-time impulse response of the system. Any electric field profile at the input face of the fiber may be decomposed in the basis of input PMs using the orthogonality relationship derived in Theorem 2.1

$$\vec{\mathbf{E}}_{\mathrm{fiberIn}}(x,y) = \sum_i c_i \vec{\mathbf{E}}_{\mathrm{PM},i}(x,y) + \text{radiation modes} \tag{9}$$

where

$$c_i = \int \left[\vec{\mathbf{E}}_{\mathrm{fiberIn}}(x,y) \times \vec{\mathbf{H}}^*_{\mathrm{PM},i}(x,y)\right] \cdot \hat{z}dxdy. \tag{10}$$

If a time-domain pulse is sent into the fiber with an electric field profile $\vec{\mathbf{E}}_{\mathrm{fiberIn}}(x,y)$, at the output, we expect a sequence of pulses, with time delays τ_i corresponding to group delays of the PMs, and amplitudes $|c_i|$, along with an overall attenuation due to loss in the fiber. Therefore, for an input $\vec{\mathbf{E}}_{\mathrm{fiberIn}}(x,y)\delta(t)$, we expect the output intensity to be

$$\begin{aligned} h(t) &= e^{-\alpha l}\sum_{i=1}^{2M}|c_i|^2\delta(t-\tau_i) \\ &= e^{-\alpha l}\sum_{i=1}^{2M}\left|\int\left[\vec{\mathbf{E}}_{\mathrm{fiberIn}}(x,y)\times\vec{\mathbf{H}}^*_{\mathrm{PM},i}(x,y)\right]\cdot\hat{z}dxdy\right|^2 \\ &\quad \times \delta(t-\tau_i) \qquad (11) \end{aligned}$$

where α is the fiber loss coefficient (approximated as mode-independent[2]), l is the fiber length, $2M$ is the number of propagating PMs in all polarizations, and τ_i is the group delay of the ith PM. We see that $h(t)$ is the impulse response of the fiber.[3] Using (8)[4]

$$h(t) = e^{-\alpha l}\sum_{i=1}^{2M}\left|\int\sum_{j=1}^{N}\left[v_j\vec{\mathbf{E}}_j(x,y)\times\vec{\mathbf{H}}^*_{\mathrm{PM},i}(x,y)\right]\cdot\hat{z}dxdy\right|^2 \times\delta(t-\tau_i). \tag{12}$$

Let $u_{ij} = \int[\vec{\mathbf{E}}^*_j(x,y) \times \vec{\mathbf{H}}_{\mathrm{PM},i}(x,y)] \cdot \hat{z}\, dx\, dy$. Then

$$\begin{aligned} h(t) &= \sum_{i=1}^{2M}\left|\sum_j v_j u^*_{ij}\right|^2 \delta(t-\tau_i) \\ &= \sum_{i=1}^{2M}\left|\mathbf{u}_i^H\mathbf{v}\right|^2\delta(t-\tau_i) \qquad (13) \end{aligned}$$

where $\mathbf{v} = [v_1 \ \dots \ v_N]^T$ and $\mathbf{u}_i = [u_{i1} \ \dots \ u_{iN}]^T$. Therefore

$$h(t) = \mathbf{v}^H\left[\sum_{i=1}^{2M}\mathbf{u}_i\mathbf{u}_i^H\delta(t-\tau_i)\right]\mathbf{v} \tag{14}$$

with the constraint $|v_i| \leq 1, i = 1, \dots, N$.

E. Minimization of ISI as an Optimization Problem

1) Minimization of ISI Is Equivalent to Maximization of Eye Opening: If 1 bit, described by an input intensity waveform $p(t)$, is transmitted, and the receiver impulse response is $r(t)$, the receiver output waveform is given by $g(t) = p(t) * h(t) * r(t)$. This is the pulse response of the continuous-time system. When a sequence of 0 and 1 bits is transmitted, and the receiver output is sampled once per bit, the effect of ISI is characterized by the impulse response of the discrete-time system $g(nT;t_0) = g(t)|_{t_0+nT}$, where n is an integer, T is the bit duration, and t_0 is an initial offset [9]. In a well-designed receiver with low group-

[2]Our approximation of loss as mode-independent is expected to be accurate for low-order modes in silica fibers, which is the main application intended for this work. This approximation may not be valid for higher order modes in silica fibers or in plastic fibers.

[3]It is to be noted that the impulse response $h(t)$ is not to be used without convolution with a narrowband time-domain pulse. In this paper, that pulse is the convolution of the transmit pulse and the receiver impulse response, and it will ensure that any analysis is consistent with the narrowband nature of PMs.

[4]Equation (12) is a correction to (1) in [7].

delay distortion and time-domain overshoot, $g(t)$, the filtered intensity waveform, will be such that $g(t) \geq 0$, to a very good approximation. An objective function that quantifies ISI is

$$F = g(0;t_0) - \sum_{n\neq 0} g(nT;t_0). \tag{15}$$

At high signal-to-noise ratio (SNR), the effect of ISI on the BER depends on $g(nT;t_0)$ only through F [9]. F is directly proportional to the eye opening, with $F < 0$ when the eye is closed, and $F > 0$ when the eye is open. Thus, minimization of ISI is equivalent to maximization of F.

2) Maximization of Eye Opening as an Optimization Problem: Let $q(t) = p(t) * r(t)$. Then

$$\begin{aligned} g(t) &= h(t) * q(t) \\ &= \mathbf{v}^H \left[\sum_{m=1}^{2M} \mathbf{u}_m \mathbf{u}_m^H q(t-\tau_m) \right] \mathbf{v}. \end{aligned} \tag{16}$$

Therefore, from (15)

$$\begin{aligned} F &= \mathbf{v}^H \left[\sum_{m=1}^{2M} \mathbf{u}_m \mathbf{u}_m^H q(t_0-\tau_m) \right] \mathbf{v} \\ &\quad - \sum_{n\neq 0} \mathbf{v}^H \left[\sum_{m=1}^{2M} \mathbf{u}_m \mathbf{u}_m^H q(t_0+nT-\tau_m) \right] \mathbf{v} \\ &= \mathbf{v}^H \mathbf{P} \mathbf{v} \end{aligned} \tag{17}$$

where

$$\mathbf{P} = \sum_{m=1}^{2M} \mathbf{u}_m \mathbf{u}_m^H \left[q(t_0-\tau_m) - \sum_{n\neq 0} q(t_0+nT-\tau_m) \right]. \tag{18}$$

Rearranging terms in (18), we see that

$$\begin{aligned} \mathbf{P} &= \sum_{m=1}^{2} \mathbf{u}_m \mathbf{u}_m^H \left[q(t_0-\tau_m) - \sum_{n\neq 0} q(t_0+nT-\tau_m) \right] \\ &\quad - \sum_{m=3}^{2M} \mathbf{u}_m \mathbf{u}_m^H \left[\sum_{n\neq 0} q(t_0+nT-\tau_m) - q(t_0-\tau_m) \right] \\ &= w_1 \mathbf{u}_1 \mathbf{u}_1^H + w_2 \mathbf{u}_2 \mathbf{u}_2^H - \sum_{m=3}^{2M} w_m \mathbf{u}_m \mathbf{u}_m^H. \end{aligned} \tag{19}$$

In Section II-E3, we show that w_m, $m = 1, \ldots, 2M$ are positive.

Maximization of eye opening is therefore equivalent to maximization of $F = \mathbf{v}^H \mathbf{P} \mathbf{v}$.

3) Structure in the Optimization Problem—Insights From Physics: In this section, we explore the structure of the matrix $\mathbf{P}$, since this determines the nature of the optimization problem we need to solve. $\mathbf{P}$ is an $N \times N$ Hermitian matrix, and, therefore, has N real eigenvalues. We assume that modes 1 and 2 correspond to the two polarizations of the strongest spatial mode. We first show that $w_m > 0$, $m = 1, \ldots, 2M$.

In a typical MMF system, polarization-mode dispersion is weak. This means that modes 1 and 2 have small delay separations in comparison to the width of $q(t)$ and are therefore indistinguishable at the receiver. In a well-designed system, the sampling instant t_0 is chosen to be close to τ_1 and τ_2. Therefore, τ_1 and τ_2 lie in the first bit interval. Also, $q(t)$ is a unimodal pulse, since $q(t) = p(t) * r(t)$, and $p(t)$ and $r(t)$ are both unimodal; so $q(t_0-\tau_1)$ and $q(t_0-\tau_2)$ are large, while $q(t_0-\tau_m)$ is small for $m > 2$. Therefore, since $w_m = q(t_0-\tau_m) - \sum_{n\neq 0} q(t_0+nT-\tau_m)$ for $m = 1, 2$, w_1 and w_2 are positive. Otherwise, even if all power is directed into modes 1 and 2 (both of which are expected to lie within the first bit interval), the eye will not be open. Also

$$w_m = \sum_{n\neq 0} q(t_0+nT-\tau_m) - q(t_0-\tau_m), \qquad m = 3, \ldots, 2M. \tag{20}$$

When higher order modes ($m \geq 3$) have delays that put them outside the first bit interval, and $q(t)$ and t_0 satisfy properties stated previously, $w_m > 0$ for $m \geq 3$. Thus, $w_m > 0$, $m = 1, \ldots, 2M$. Therefore, $\mathbf{P}$ has *at most* 2 *positive* eigenvalues and *at most* $2M-2$ *negative* eigenvalues.

If $\mathbf{P}$ has no positive eigenvalues, $\mathbf{v}^H \mathbf{P} \mathbf{v}$ will be nonpositive for all $\mathbf{v}$, meaning that the eye will be closed for all possible SLM settings. However, experiments [7] show that it is possible to open the eye with suitable SLM settings. This suggests that $\mathbf{P}$ has at least one positive eigenvalue. We know that the SLM only does spatial filtering and cannot control polarization. If $\mathbf{P}$ has two positive eigenvalues, it means that modes 1 and 2, which are expected to be spatially similar but polarization degenerate, can be independently controlled using spatial filtering alone. However, this is not physically possible. Since $\mathbf{P}$ has a total of N eigenvalues, we conclude that, *in practice*, $\mathbf{P}$ *has* 1 *positive eigenvalue and* $N-1$ *nonpositive eigenvalues*.

4) Final SLM Optimization Problem: The optimization problem we wish to solve is, therefore

$$\begin{array}{ll} \text{maximize} & \mathbf{v}^H \mathbf{P} \mathbf{v} \\ \text{subject to} & |v_i|^2 \leq 1, \qquad i = 1, \ldots, N \end{array} \tag{21}$$

where $\mathbf{v}$ (the vector of SLM reflectances) is the optimization variable, and $\mathbf{P}$ (the matrix characterizing the fiber and the optical system) is problem data. Here, $\mathbf{v} \in \mathbb{C}^N$, $\mathbf{v} = [v_1 \ldots v_N]^T$, and $\mathbf{P} \in \mathbb{C}^{N\times N}$ are Hermitian ($\mathbf{P}^H = \mathbf{P}$). Since $\mathbf{P}$ has one positive and $N-1$ nonpositive eigenvalues, we may write it as $\mathbf{P} = \mathbf{p}_1 \mathbf{p}_1^H - \mathbf{P}_2 \mathbf{P}_2^H$, where $\mathbf{p}_1 \in \mathbb{C}^N$, $\mathbf{P}_2 \in \mathbb{C}^{N\times N-1}$, and $\mathbf{P}_2^H \mathbf{p}_1 = 0$. We may, therefore, rewrite (21) as

$$\begin{array}{ll} \text{maximize} & \left|\mathbf{p}_1^H \mathbf{v}\right|^2 - \left\|\mathbf{P}_2^H \mathbf{v}\right\|^2 \\ \text{subject to} & |v_i|^2 \leq 1, \qquad i = 1, \ldots, N. \end{array} \tag{22}$$

Henceforth, we refer to (22) as the SLM optimization (SLMO) problem.

III. Optimizing the SLM

In this section, we solve the SLMO problem (22). This problem is not in any standard form that leads to an efficient global solution; for example, it is not convex. To solve it, we will transform it to an equivalent problem that is convex and is easily solved. We start by observing that replacing $\mathbf{v}$ by $e^{j\theta}\mathbf{v}$ leaves the objective function and constraints unchanged. Therefore, we may arbitrarily choose an overall phase θ for $\mathbf{v}$.

In particular, we can choose θ so that $\mathbf{p}_1^H \mathbf{v} \in \mathbb{R}$ and $\mathbf{p}_1^H \mathbf{v} \geq 0$. The SLMO problem can, therefore, be written as

$$\begin{aligned} \text{maximize} \quad & t^2 - \mathbf{x}^H \mathbf{x} \\ \text{subject to} \quad & \mathbf{p}_1^H \mathbf{v} = t \\ & \mathbf{P}_2^H \mathbf{v} = \mathbf{x} \\ & t \geq 0 \\ & |v_i|^2 \leq 1, \qquad i = 1, \ldots, N \end{aligned} \tag{23}$$

where $t \in \mathbb{R}$ and $\mathbf{x} \in \mathbb{C}^{N-1}$ are additional variables introduced.

The choice $\mathbf{v} = 0$ is feasible, and has zero objective value. It follows that when we solve the SLMO problem, the objective value must be nonnegative, i.e., we can assume that $t^2 - \mathbf{x}^H \mathbf{x} \geq 0$. We introduce another variable $y \in \mathbb{R}$ and rewrite our problem as

$$\begin{aligned} \text{maximize} \quad & y^2 \\ \text{subject to} \quad & t^2 - \mathbf{x}^H \mathbf{x} \geq y^2 \\ & \mathbf{p}_1^H \mathbf{v} = t \\ & \mathbf{P}_2^H \mathbf{v} = \mathbf{x} \\ & t \geq 0 \\ & |v_i|^2 \leq 1, \qquad i = 1, \ldots, N. \end{aligned} \tag{24}$$

This is equivalent to SLMO, because the first inequality constraint must be tight at the optimal point. Since y^2 is monotonically increasing in y for $y \geq 0$, we can take the square root of the objective function and rewrite the problem as

$$\begin{aligned} \text{maximize} \quad & y \\ \text{subject to} \quad & \sqrt{\mathbf{x}^H \mathbf{x} + y^2} \leq t \\ & \mathbf{p}_1^H \mathbf{v} = t \\ & \mathbf{P}_2^H \mathbf{v} = \mathbf{x} \\ & |v_i|^2 \leq 1, \qquad i = 1, \ldots, N. \end{aligned} \tag{25}$$

Here, $\mathbf{v} \in \mathbb{C}^N$, $\mathbf{x} \in \mathbb{C}^{N-1}$, $t \in \mathbb{R}$, and $y \in \mathbb{R}$ are the optimization variables, and $\mathbf{p}_1$ and $\mathbf{P}_2$ are problem data. This problem is a second-order cone program (SOCP), which can be globally solved very efficiently [10]. The computational cost of solving this problem is N^3—the same order as solving a set of N linear equations. We will refer to the SOCP (25) as the convex form SLMO. Many publicly distributed software packages can be used to solve this convex form SLMO, e.g., `CVX` [11] or `SeDuMi`.

We note that in real fiber systems, $\mathbf{P}$ is not known explicitly, since it captures the details of the mode coupling and the optical system and depends on the exact refractive profile, every bend and twist in the fiber, and so on. However, if for every SLM setting $\mathbf{v}$, we can observe $\mathbf{v}^H \mathbf{P} \mathbf{v}$, we can still find the optimal solution. $\mathbf{v}^H \mathbf{P} \mathbf{v}$ is the eye opening and can often be measured directly. When the eye is closed, it may be estimated from the response to a training sequence.

One simple algorithm to solve the problem using only measurements of eye opening is sequential coordinate ascent (SCA). We choose one component of $\mathbf{v}$, v_i (corresponding to the ith block on the SLM), and maximize the objective function (the eye opening) with respect to (w.r.t.) v_i. This can be done empirically. We then choose v_{i+1} and repeat this procedure. The value of the objective function increases to the global maximum, although this may require several passes over the SLM. This is a very simple algorithm, both in computational complexity and hardware implementation. Since it is adaptive, it is also capable of tracking drift in the system. All these factors make SCA very promising as a practical solution.

We note that convexity of the problem (25) can be used to show that any locally optimal point for the original SLMO problem must in fact be global. In particular, we can be sure that when the SCA method converges to a locally optimal solution, it is in fact global.

IV. Results

A. Fiber and System Parameters

For simulation, we use parameters from the experimental setup used in [7]. We use a 50-μm-core graded-index multimode fiber, 1 km in length. We use the infinite-core approximation, and therefore approximate the IMs as Hermite–Gaussian modes. Mode sizes and propagation constants (β_i) are computed analytically. The group delays of the IMs are scaled up by a factor of 10, so as to make them comparable to modal delays observed in experiment. This is because the differential group delay in a fiber with perfectly quadratic refractive index profile is less than that observed in most real fibers by about an order of magnitude. We operate at a wavelength of 1550 nm. We use a bit rate of 10 Gb/s. Note that the actual delay spreads do not affect our results, because our analysis is independent of the time scale of modal delays. At a wavelength of 1550 nm, the fiber supports 55 modes in each polarization.

Light from a standard 10.4-μm-diameter core SMF is imaged onto the SLM through a 10.4-mm focal length lens. The guided mode of the SMF is approximated as Gaussian. Light reflected by the SLM is imaged onto the MMF through a 10.4-mm focal-length lens. Both fibers are in the focal plane of their respective lenses.

Although the SLM in [7] provided phase control only, here, the SLM is assumed to control both amplitude and phase with a 128×128 array of pixels, covering a region containing 95% of the incident power. Each pixel is 18×18 μm^2. These pixels are grouped into larger square blocks during operation. A typical block size is 16×16 pixels, to have a 2-D array of 8×8 blocks on the SLM, though block size is part of the parameter space explored in this simulation. The total area of the SLM is always kept constant. Grouping a large number of pixels into blocks means that even if, in practice, the SLM has only phase control, both amplitude and phase control may be achieved at the block level by introducing high spatial frequency (pixel-level) phase gratings to diffract light away from the fiber core. Simulations show that a block size of 4×4 pixels is sufficient to mimic amplitude-and-phase control with a phase-only SLM for the imaging system, wavelength, and fiber parameters used in this simulation.

Mode coupling is simulated by coupling the IMs with randomly generated complex unitary matrices. PMs are thus unitary combinations of IMs. Unitarity ensures conservation of energy. High mode-coupling regime is simulated using unitary

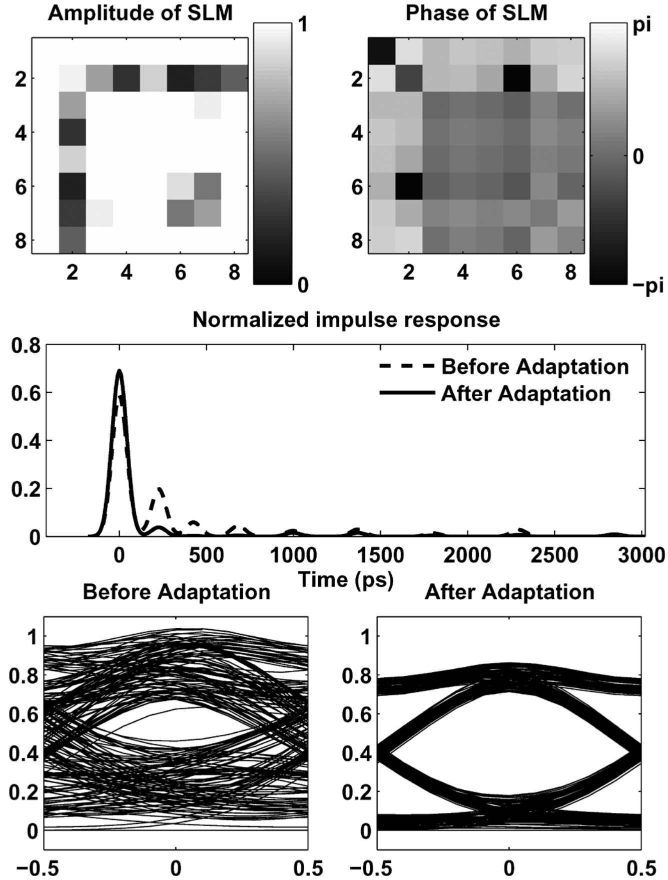

Fig. 2. SLM settings after adaptation, and impulse response and eye diagram before and after adaptation—1-km, 50-μm-core graded-index MMF, with random mode coupling, center launch.

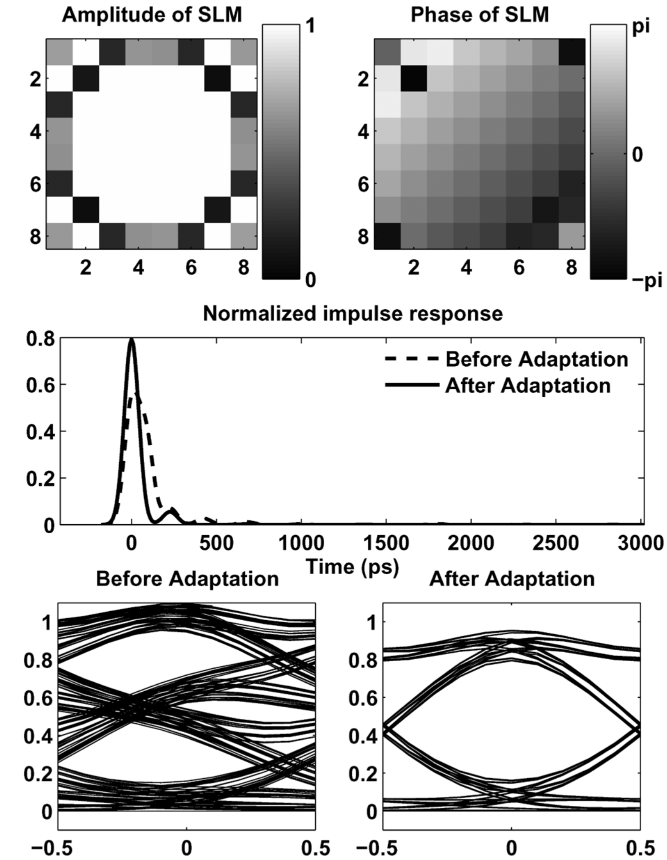

Fig. 3. SLM settings after adaptation, and impulse response and eye diagram before and after adaptation—1-km, 50-μm-core graded-index MMF, 5-μm offset launch.

matrices that have large off-diagonal terms. The fiber loss coefficient α may be assumed to be zero without loss of generality. Correspondence between simulated impulse responses and those observed in experiment suggests that this is a reasonable model.

Our figure of merit is the objective function $\mathbf{v}^H\mathbf{P}\mathbf{v}$. $\mathbf{P}$ is normalized so that, when the SLM blocks are all set to amplitude 1 and phase 0 $(v_i = 1)$, the total power in all modes excited in the MMF is unity. Note that a negative value of the objective function indicates a closed eye.

The optimal solution is computed by solving using CVX, a freely distributed convex optimization library for Matlab [11]. We compare our solution to those obtained by other algorithms currently available— two-phase and four-phase SCA. These involve selecting an SLM block, trying two or four different phase settings, respectively, picking the best one, and repeating this for another SLM block, cycling over the SLM many times till the algorithm converges. These were the algorithms used in the experiments reported in [7] and [12].

B. Simulation Results

Fig. 2 shows the impulse response of the system before and after adaptation of the SLM. The SLM is a 2-D array of 8× 8 blocks, each of 16 × 16 pixels. Before adaptation, all SLM blocks have unit amplitude and zero phase. The fiber has random mode coupling, simulated as described in Section IV-A. We see that, after adaptation, higher order PMs are no longer excited. This leads to reduction in ISI and a larger eye opening, as is evident from the eye diagrams before and after adaptation. Fig. 2 also shows the optimal SLM settings. Note that these settings are achieved irrespective of the initial state of the SLM.

Next, we look at a system with offset launch. The fiber has no mode coupling, the SLM uses 8 × 8 blocks, each of 16 × 16 pixels, and the beam launched into the SLM is offset from the center of the MMF along the diagonal by 5 μm. Fig. 3 shows the impulse response and eye diagrams before and after adaptation, as well as the optimal SLM settings. We see that the optimal SLM setting has a phase that appears to vary linearly along the diagonal, as expected, because the SLM tries to steer the beam back on center.

Table I compares the objective function before and after adaptation for this and the previous example, as well as for other values of offset at launch. We also give comparisons with two-phase and four-phase SCA.

Table II gives a comparison of performance across various blocks sizes, in a system with random mode coupling. A smaller sized block translates to higher resolution in spatial frequency. We see that an SLM with 8 × 8 blocks, each of 16 × 16 pixels, gives good performance, while minimizing complexity.

The optimal SLM settings in Fig. 2 and Fig. 3 clearly show that at optimum, the SLM reflectances do not have unit magnitude. This is contrary to what is stated in [6]. However, Fig. 4

TABLE I
PERFORMANCE COMPARISON OF PROPOSED OPTIMAL TECHNIQUE, TWO-PHASE SCA, AND FOUR-PHASE SCA, FOR FIBER WITH RANDOM MODE COUPLING AND CENTER LAUNCH, AND FOR FIBER WITH NO MODE COUPLING AND OFFSET LAUNCH, 8×8 BLOCK SLM

	Objective function			
	Before adaptation	Optimal solution	2-phase SCA	4-phase SCA
Random mode coupling				
Low coupling	0.1234	0.5163	0.3990	0.4430
High coupling	-0.2885	0.2964	-0.0234	0.2178
Offset launch				
2 μm	0.4710	0.6430	0.5093	0.5449
5 μm	-0.0950	0.6290	0.1224	0.4343
10 μm	-0.8795	0.5837	-0.0836	0.4808

TABLE II
OPTIMAL SOLUTION FOR VARIOUS SLM RESOLUTIONS: TOTAL ACTIVE AREA OF THE SLM IS KEPT CONSTANT AT 128×128 PIXELS

Number of SLM blocks	**Objective function**	
	Before adaptation	Optimal solution
4×4	0.1234	0.3767
8×8	0.1234	0.5163
16×16	0.1234	0.5452
32×32	0.1234	0.5575

shows that at higher spatial resolution, at optimal setting, most of the SLM blocks have unit magnitude. This means that the difference in performance of phase-only and amplitude-and-phase SLMs decreases with increasing spatial resolution.

Finally, we look at the effect of completely suppressing all higher order modes, so that exactly one PM is excited. Intuitively, one may think that the best thing to do to minimize ISI is to suppress all higher order modes. However, our objective is to have a large eye opening. This does not necessarily translate to suppressing higher order modes. It turns out that if we allow some higher order modes to be excited, we more than compensate by having higher power in the lowest order mode. This is clearly demonstrated in Fig. 5, where we see the impulse responses of the optimal solution, and the solution involving selective excitation of the lowest order mode and no excitation of higher order modes, with the SLM at high spatial resolution. Comparing the eye diagrams, we see that one eye is clean, but small, while the other eye is less clean, but more open.

C. Comparison of Optical and Electrical Equalization

The proposed optical equalization scheme offers scalability advantages over electrical equalization schemes. One SLM can serve several wavelength-division-multiplexed channels [7], [12], unlike electrical equalizers, which must be implemented separately for each channel. The optical technique scales more easily to high bit rates and long fibers, because requirements on the SLM are independent of bit rate and fiber length, and depend only on the mode structure. By contrast, in maximum-likelihood sequence detection (MLSD) [5], the optimal electrical equalization scheme, complexity scales exponentially with the bit rate $\times$ length product. In the optical technique, once the SLM has been adapted, the steady-state power consumption can be very low. Electrical equalizers must process each received bit, which can lead to high power consumption.

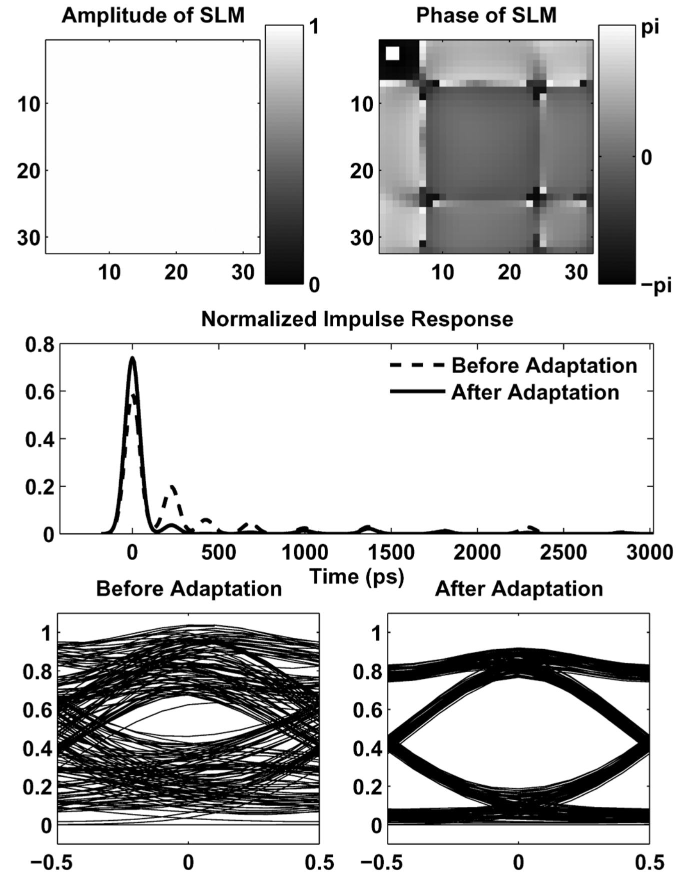

Fig. 4. Amplitude and phase of the SLM after adaptation—1-km, 50-μm-core graded-index MMF, with random mode coupling, center launch, SLM at high spatial resolution (32×32 blocks).

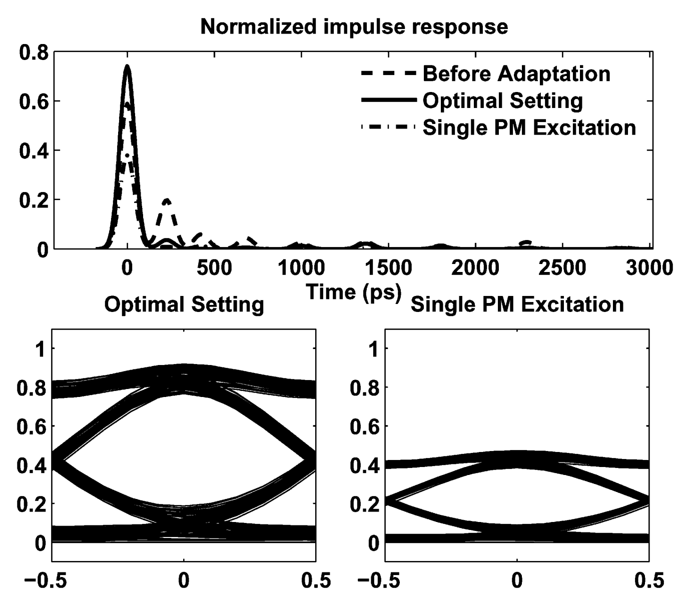

Fig. 5. Impulse responses before adaptation, after adaptation to optimal setting, and after adaptation to excite a single PM, and eye diagrams—1-km, 50-μm-core graded-index MMF, with random mode coupling, center launch, SLM at high spatial resolution (32×32 blocks), showing that selective excitation of a single mode does not maximize eye opening.

Fig. 6 gives a comparison of the BER performances of the best optical and electrical equalizers. The channel used is the

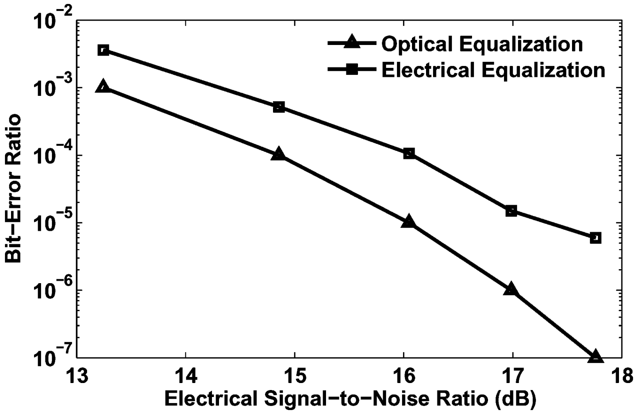

Fig. 6. Comparison of performance of optical equalization and electrical equalization using MLSD—1-km, 50-μm-core graded-index MMF, with random mode coupling, center launch, SLM at high spatial resolution (32×32 blocks), MLSD with 2^{10} states.

same as that in Fig. 4. We consider optical equalization with the SLM at high spatial resolution (32×32 blocks). This is compared to MLSD using 2^{10} states, with survivor sequences truncated to the 50 most recent symbols (no optical equalization is used). MLSD using a larger number of states is difficult to simulate and would be very difficult to implement in practice. For a fixed transmit power, the noise power is varied to change the electrical SNR. The electrical SNR is defined as the ratio of the square of the average signal photocurrent with a blank SLM to the variance of the noise power. We see that optical equalization outperforms MLSD. Other electrical equalizers, such as linear or decision-feedback equalizers, are known to perform worse than MLSD [5].

V. Conclusion

We have proposed an optimal technique to minimize ISI in MMF systems, using an SLM. A mathematical model of the system, incorporating light propagation in the presence of mode coupling and constraints on the SLM, is developed. Finding optimal SLM settings to minimize ISI is posed as an optimization problem. The globally optimal solution to this problem is found by solving an equivalent convex optimization problem, which can be solved with low computational complexity. The solution can be implemented using fast, low-complexity adaptive algorithms. We present simulation results, using parameters of commercially available fibers and SLMs, and show that this technique mitigates ISI and opens up an otherwise closed eye pattern. This work also shows that combining optimization and physical modeling leads to solutions that are not obvious from the physics of the problem. In particular, it suggests that mere suppression of higher order modes is not the best strategy to combat ISI.

Future work will involve development of custom adaptive algorithms designed to exploit structure in our problem, thereby ensuring fast convergence and robustness to noise. These will then be tested in experiment to quantify performance in presence of noise and system drift.

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