Simultaneous Routing and Power Allocation in CDMA Wireless Data Networks

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***Abstract*— The optimal routing of data in a wireless network depends on the link capacities, which, in turn, are determined by the allocation of transmit powers across the network. Thus, the optimal network performance can only be achieved by simultaneous optimization of routing and power allocation. In this paper, we study this joint optimization problem in CDMA data networks using convex optimization techniques. Although link capacity constraints of CDMA systems are not jointly convex in rates and powers, we show that coordinate projections or transformations allow the simultaneous routing and power allocation problem to be formulated as (in systems with interference cancellation) or approximated by (in systems without interference cancellation) a convex optimization problem which can be solved very efficiently. We also propose a heuristic link-removal procedure based on the convex approximation to further improve the system performance.**

I. Introduction

To make efficient the use of scarce radio resources, it becomes important not only to optimize the operation of each layer in wireless data networks, but also to coordinate the operation of different layers (*e.g.*, [1]). In particular, the optimal routing problem in the network layer and resource allocation problem in the radio control layer are coupled through the link capacities, and the optimal performance can only be achieved by simultaneous optimization of routing and resource allocation.

In a previous paper [2], we formulated the simultaneous routing and resource allocation (SRRA) problem for wireless data networks. By assuming that the link capacity is a concave and increasing function of the communications resources allocated the link (this assumption holds for TDMA and FDMA systems with orthogonal channels) the SRRA problem is a convex optimization problem over the network flow variables and the communications variables. We exploited the separable structure of the SRRA problem via dual decomposition, and derived an efficient solution method which achieves the optimal coordination of data routing and resource allocation.

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In this paper, we generalize the SRRA formulation to include code-division multiple access (CDMA) systems. In CDMA systems, the capacity of a link depends not only on the power allocated to itself, but also the powers allocated to other links (due to interferences). Moreover, the capacity constraints are not jointly convex in communication rates and power allocations, so a straightforward formulation of the SRRA problem is not a convex optimization problem and generally very hard to solve. We will show, however, that by using a coordinate projection (that only considers the rate region) the SRRA problem using Gaussian broadcast channels with superposition coding and interference cancellation can be converted into an equivalent convex optimization problem. For practical CDMA systems without interference cancellation, we suggest an approximate capacity formula for relatively high signal to interference and noise ratio (SINR) and show how a coordinate transform yields a convex formulation. This approximate formulation is a restriction of the original problem, and its solution is guaranteed to be feasible for the original non-convex formulation. We propose a heuristic link-removal procedure to further improve the system performance based on this convex approximation.

II. Formulation of the SRRA problem

We briefly review the SRRA problem formulated in [2] and generalize the model to include the CDMA case.

A. Network flow model

Consider a connected communication network containing N nodes labeled $n = 1, \dots, N$ and L directed links labeled $l = 1, \dots, L$. The topology of the network is represented by a *node-link incidence matrix* $A \in \mathbf{R}^{N \times L}$ whose entry A_{nl} is associated with node n and link l via

$$A_{nl} = \begin{cases} 1, & \text{if } n \text{ is the start node of link } l \\ -1, & \text{if } n \text{ is the end node of link } l \\ 0, & \text{otherwise.} \end{cases}$$

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We define $\mathcal{O}(n)$ as the set of outgoing links from node n.

We use a multicommodity flow model for the routing of data flows, with average data rates in bits per second. We identify the flows by their destinations, labeled $d = 1, \ldots, D$, where $D \leq N$. For each destination d, we define a *source-sink vector* $s^{(d)} \in \mathbf{R}^N$, whose nth ($n \neq d$) entry $s_n^{(d)}$ denotes the non-negative amount of flow injected into the network at node n (the source) and destined for node d (the sink), where $s_d^{(d)} = -\sum_{n \neq d} s_n^{(d)}$. We also define $x^{(d)} \in \mathbf{R}_+^L$ as the *flow vector* for destination d, whose component $x_l^{(d)}$ is the amount of flow on each link l and destined for node d. Let t_l and c_l be the total traffic load and the capacity of link l respectively. Our network flow model imposes the following group of constraints on the network flow variables x, s and t:

$$\begin{array}{ll} Ax^{(d)} = s^{(d)}, & d = 1, \ldots, D \\ x^{(d)} \succeq 0, \quad s^{(d)} \succeq_d 0, & d = 1, \ldots, D \\ t_l = \sum_d x_l^{(d)}, & l = 1, \ldots, L \\ t_l \leq c_l, & l = 1, \ldots, L \end{array} \tag{1}$$

Here, $\succeq$ means component-wise inequality, and $\succeq_d$ means component-wise inequality except for the dth component. The first set of constraints are the flow conservation laws for each destination, while the last set of constraints are capacity constraints for each link.

B. Communications model

In a wireless system, the capacities of individual links (channels) depend on the media access scheme and the allocation of *communications variables*, such as transmit powers, bandwidths or time-slot fractions, to the transmitters. We assume that the medium access, coding and modulation schemes are fixed, but that we can optimize over the communications variables r. We use the following generic model to relate the vector of total traffic t and the vector of communications variables r

$$\begin{array}{l} t_l \leq c_l = \phi_l(r), \quad l = 1, \ldots, L \\ Fr \preceq g, \quad r \succeq 0 \end{array} \tag{2}$$

Here, the formula $c_l = \phi_l(r)$ describes the dependence of the capacity of link l on the communications variables $r = (r_1, \ldots, r_L)$ and r_l is the sub-vector of resources allocated to link l. The linear inequality $Fr \preceq g$ describes resource limits, such as the total available transmit powers at each node. The constraint $r \succeq 0$ specifies that the communications variables are non-negative.

In this paper we consider two particular classes of CDMA systems that we now describe in some detail.

The Gaussian broadcast channel: The set of achievable rates t_i for an M-user Gaussian broadcast channel with noise powers $\sigma_1 \leq \sigma_2 \leq \ldots \leq \sigma_M$ in different links and total transmit power P_{tot} (assuming unit total bandwidth) is given by [3, 4]

$$\begin{array}{l} t_i \leq \log\left(1 + \dfrac{P_i}{\sigma_i + \sum_{j<i} P_j}\right), \; i = 1, \ldots, M \\ \sum_{i=1}^M P_i \leq P_{\mathrm{tot}}, \qquad P_i \geq 0, \; i = 1, \ldots, M \end{array} \tag{3}$$

where P_i is the transmit power allocated to link i. Here the communications variables are $r = (P_1, \ldots, P_M)$, and the constraints (3) are exactly in the generic form (2). This model can be used for links starting from the same node, with superposition coding and interference cancellation [3].

Interference-limited channels: Most CDMA systems in practice are designed without interference cancellation. Let $G \in \mathbf{R}_+^{L \times L}$ be the *channel gain matrix* across the whole network, whose entry G_{ij} is the power gain from the transmitter of link j to the receiver of link i. Only the diagonal terms G_{ii} are desired, and the off-diagonal terms G_{ij} ($i \neq j$) lead to interferences. Based on Shannon capacity, we have the link capacity constraints

$$t_l \leq \log\left(1 + \frac{G_{ll} P_l}{\sigma_l + \sum_{j \neq l} G_{lj} P_j}\right), \quad l = 1, \ldots, L. \tag{4}$$

The power limits can be specified for each link,

$$P_l \leq P_{l,\mathrm{tot}}, \quad l = 1, \ldots, L$$

or for each node (shared by its outgoing links)

$$\sum_{l \in \mathcal{O}(n)} P_l \leq P_{\mathrm{tot}}^{(n)}, \quad n = 1, \ldots, N. \tag{5}$$

Here the communications variables are the powers $P = (P_1, \ldots, P_L)$, and the above constraints have the form (2).

C. The generic SRRA formulation

Consider the operation of a wireless data network described by the network flow model (1) and the communications model (2) and suppose that the objective is to minimize a convex cost function $f(x, s, t, r)$ (or maximize a concave utility function). We have the following generic formulation of the SRRA problem:

$$\begin{array}{lll} \text{minimize} & f(x, s, t, r) & \\ \text{subject to} & Ax^{(d)} = s^{(d)}, & d = 1, \ldots, D \\ & x^{(d)} \succeq 0, \quad s^{(d)} \succeq_d 0, & d = 1, \ldots, D \\ & t_l = \sum_d x_l^{(d)}, & l = 1, \ldots, L \\ & t_l \leq \phi_l(r), & l = 1, \ldots, L \\ & Fr \preceq g, \quad r \succeq 0. & \end{array} \tag{6}$$

Here the optimization variables are the network flow variables x, s, t and the communications variables r. The SRRA problem is very general and it includes many important design problems for wireless data networks. For example, the objective can be maximum total utility, minimum total power (or bandwidth), minimax power among the nodes, and minimax link utilization [2].

For TDMA and FDMA systems, the capacity constraints usually take the form $t_l \le \phi_l(r_l)$ (the links are orthogonal) and the functions ϕ_l are usually concave and monotone increasing in r_l. These properties imply that the capacity constraints are jointly convex in t and r, hence the SRRA problem is a convex optimization problem, which can be solved globally and efficiently by recently developed interior-point methods (*e.g.*, [5]). More effective methods can be developed for solving the SRRA problem by exploiting its structure via dual decomposition (see [2]).

For the CDMA systems described in section II-B, however, the capacity constraints are not jointly convex in the rates t_l and vector of transmit powers P. This means that the direct formulation of the SRRA problem as in (6) is not a convex optimization problem, and that it is generally very hard to find the global optimal solution. Nevertheless, we will show in sections III and IV that coordinate projections or transformations allow us to approach the SRRA problem effectively using convex optimization techniques.

III. SRRA in CDMA networks using Gaussian broadcast channels

In this section, we derive an equivalent convex formulation of the SRRA problem that uses Gaussian broadcast channel with interference-cancellation at each node.

A. An equivalent characterization of the rate region

Although the capacity constraints in (3) for the Gaussian broadcast channel is not jointly convex in the rates t_i and power vector P, it is a well-known fact that the achievable rate region $\mathcal{C}(P_{\rm tot}) = \{t \mid (3),\ t \succeq 0\}$ is a convex set. This is better seen from an equivalent characterization of the rate region [6]

$$\mathcal{C}(P_{\rm tot}) = \{t \mid p(t) \le P_{\rm tot},\ t \succeq 0\}$$

where the function p is defined as

$$p(t) = \sum_{i=1}^{M} (\sigma_i - \sigma_{i-1}) e^{\sum_{i \le j \le M} t_j} - \sigma_M \tag{7}$$

and $\sigma_0 = 0$. It is clear that the function p is convex, hence the rate region $\mathcal{C}(P_{\rm tot})$, the projection of the feasible set of (t, P) onto the t-coordinates, is a convex set.

Given any rate vector $t \in \mathcal{C}(P_{\rm tot})$, the corresponding power allocation is given by

$$P_k = \sum_{i=1}^{k} (\sigma_i - \sigma_{i-1}) e^{\sum_{i \le j < k} t_j} \left(e^{t_k} - 1\right) \tag{8}$$

for $k = 1, \ldots, M$. Note that the proof of these results, given in appendix, relies on the fact that the channel gains in the model (3) are all equal.

B. The convex formulation of the SRRA problem

Consider a wireless data network where each node uses the Gaussian broadcast channel to send information over its outgoing links. We assume that there is no interference among channels from different nodes (they use disjoint frequency bands). We denote the local total traffic vector by $t^{(n)} = \{t_l \mid l \in \mathcal{O}(n)\}$ and define a convex function $p^{(n)}(t^{(n)})$ similarly as $p(t)$ in (7) for each node n. Then we can formulate the convex SRRA problem

$$\begin{array}{lll}
\text{minimize} & f(x, s, t, r) & \\
\text{subject to} & A^{(d)} x^{(d)} = s^{(d)}, & d = 1, \ldots, D \\
 & x^{(d)} \succeq 0, \quad s^{(d)} \succeq_d 0, & d = 1, \ldots, D \\
 & t_l = \sum_{d=1}^{D} x_l^{(d)}, & l = 1, \ldots, L \\
 & p^{(n)}(t^{(n)}) \le r_n, & n = 1, \ldots, N \\
 & r_n \le P_{\rm tot}^{(n)}, & n = 1, \ldots, N
\end{array}$$

where the communications variable r_n is the total transmit power used at node n. Introducing r_n allows us to formulate the minimum total power and minimax power SRRA problems. This convex optimization problem is equivalent to (6) when the Gaussian broadcast channel model (3) is used at each node. After solving this convex problem, we can recover the power allocation P_l for each link using (8).

IV. SRRA in CDMA networks with interference-limited channels

In this section, we derive a convex approximation for the SRRA problem with interference-limited channels when the SINRs are relatively high. We then give a heuristic link-removal procedure to further improve the network performance based on the solution to the convex program.

A. A convex approximation

For CDMA systems with interference-limited channels described by (4), the SINRs are defined as

$$\gamma_l = \frac{G_{ll} P_l}{\sigma_l + \sum_{j \ne l} G_{lj} P_j}, \quad l = 1, \ldots, L.$$

When the SINRs are relatively high (*e.g.*, $\gamma_l \ge 5$ or 10), we use the approximation $\log(1 + \gamma_l) \approx \log \gamma_l$ and re-write

$$\begin{aligned}
\phi_l(P) &\approx \log\left(\frac{G_{ll} P_l}{\sigma_l + \sum_{j \ne l} G_{lj} P_j}\right) \\
&= -\log\left(\frac{\sigma_l + \sum_{j \ne l} G_{lj} P_j}{G_{ll} P_l}\right) \\
&= -\log\left(\frac{\sigma_l}{G_{ll}} P_l^{-1} + \sum_{j \ne l} \frac{G_{lj}}{G_{ll}} P_j P_l^{-1}\right).
\end{aligned}$$

Let $Q_l = \log(P_l)$ (*i.e.*, $P_l = e^{Q_l}$) for $l = 1, \ldots, L$ and define

$$\psi_l(Q) = \phi_l(P(Q)) = -\log\left(\frac{\sigma_l}{G_{ll}} e^{-Q_l} + \sum_{j \ne l} \frac{G_{lj}}{G_{ll}} e^{Q_j - Q_l}\right).$$

Note that the functions ψ_l are concave in the variable Q since "log-sum-exp" expressions are convex [5].

With the approximate capacity formula and the change of variables, we can formulate the following SRRA problem

$$\begin{array}{lll} \text{minimize} & f(x,s,t,r) & \\ \text{subject to} & A^{(d)}x^{(d)} = s^{(d)}, & d=1,\ldots,D \\ & x^{(d)} \succeq 0, \quad s^{(d)} \succeq_d 0, & d=1,\ldots,D \\ & t_l = \sum_{d=1}^{D} x_l^{(d)}, & l=1,\ldots,L \\ & t_l \leq \psi_l(Q), & l=1,\ldots,L \\ & \sum_{l\in\mathcal{O}(n)} e^{Q_l} \leq P_{\text{tot}}^{(n)}, & n=1,\ldots,N \end{array} \tag{9}$$

where the last constraint (which is convex) is (5) in the new variable Q. Here the capacity constraints $t_l \leq \psi_l(Q)$ are jointly convex in t and Q. This implies that (9) is a convex optimization problem, which can be solved globally and efficiently. We have the following remarks:

- Note that $\log\gamma_l \leq \log(1+\gamma_l)$, *i.e.*, we used an underestimate for the link capacity. This means that the solution to the (restricted) convex program (9) is always feasible to the original problem (6).
- The average throughput of many channels with bit-error-rate constraints (*e.g.*, variable-rate M-QAM in additive Gaussian noise or Rayleigh fading environment) can be well approximated (*e.g.*, [7, 8]) by

  $$\log(1+k\gamma) \approx \log k + \log\gamma$$

  provided that $k\gamma$ is relatively high. We can formulate the SRRA problem with these channels similarly.
- The change of variables that we have used is well-known in geometric programming (see, *e.g.*, [5]) and provides an explicit link between our work and the optimization of communication systems with quality of service constraints considered in [9, 10].

B. *A heuristic link-removal procedure*

Any solution to (9) must satisfy $\gamma_l \geq 1$ for all links, since we require $0 \leq t_l \leq \log\gamma_l$. The links with $\gamma_l = 1$ have zero capacity, but are allocated nonzero powers. We can safely remove these links, and still guarantee that the solution is feasible for the SRRA problem with the new (reduced) network topology. If we solve the SRRA problem for the new topology, the objective can only be improved (leading to larger total utility or less total power). Hence, we propose to use the following link-removal procedure

```
given a network topology and the SRRA problem (6)
repeat
      solve the convex SRRA formulation (9)
      remove links whose SINRs equal one (capacity zero)
until no links were removed (all SINRs greater than one)
```

This procedure usually takes very few iterations to stop. In many cases we can continue to remove links with very small SINRs, even if they are greater than one. This may result in sparser routing pattern and higher SINRs for the remaining links, but feasibility is no longer guaranteed and the objective may be degraded.

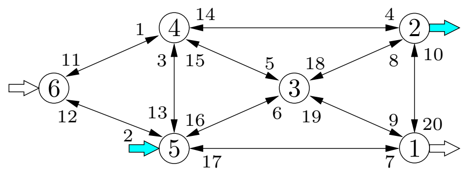

Fig. 1. A network with 6 nodes and 20 directed links.

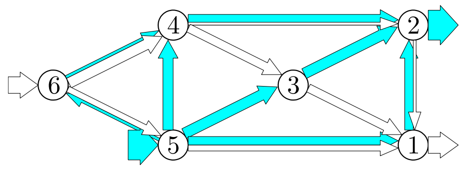

Fig. 2. Initial routing solution.

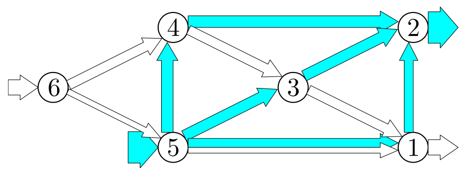

Fig. 3. Routing solution after the link-removal procedure.

V. A NUMERICAL EXAMPLE

Consider the network shown in figure 1, which has 6 nodes and 20 directed links (link labels are next to the arrows). Each node has total power $P_{\text{tot}}^{(n)} = 1$ to be shared by its outgoing links. All the receivers have the same noise power $\sigma = 0.001$. We use the interference-limited channel model (4). All diagonal entries of the channel gain matrix G are set to one, and the off-diagonal entries are generated randomly with a uniform distribution on [0, 0.01]. There are two data flows, one form node 6 to node 1 and the other form node 5 to node 2, to be supported by the network. The goal is to maximize the sum throughput of the two flows $s_6^{(1)} + s_5^{(2)}$. Following the link-removal procedure in section IV-B, we solved the problem in three steps:

(a) Solve the convex SRRA problem (9) (with 20 links). The solution is shown in figure 2 and table I. The total throughput is 12.89.

(b) Remove the 8 links with SINRs equal to one, and solve (9) with the new network topology having 12 links. The routing solution is roughly the same as before (see figure 2), and the total throughput is 13.06.

(c) Further remove link 12 and 20 which have very low SINRs, and solve (9) again for the resulting network with 10 links. The routing solution, shown in figure 3, achieves a total throughput of 13.00.

link	SINR			power allocation		
	(a)	(b)	(c)	(a)	(b)	(c)
1	21.2	22.2	14.2	0.276	0.275	0.171
2	5.2	5.3	4.9	0.076	0.074	0.064
3	17.1	17.8	25.6	0.165	0.163	0.211
4	34.8	37.1	25.6	0.310	0.308	0.218
5	10.4	10.6	14.2	0.129	0.127	0.160
6	19.6	20.1	20.4	0.227	0.224	0.213
7	55.9	59.1	59.5	0.597	0.603	0.576
8	19.6	20.1	20.4	0.149	0.142	0.136
9	10.4	10.6	14.2	0.075	0.073	0.089
10	10.6	11.1	12.0	0.157	0.160	0.153
11	1.0	0	0	0.012	0	0
12	1.1	1.1	0	0.012	0.011	0
13	1.0	0	0	0.009	0	0
14	1.0	0	0	0.013	0	0
15	1.0	0	0	0.010	0	0
16	1.0	0	0	0.013	0	0
17	1.0	0	0	0.012	0	0
18	1.0	0	0	0.013	0	0
19	1.0	0	0	0.011	0	0
20	1.8	1.8	0	0.013	0.013	0

TABLE I SINR AND POWER ALLOCATION.

The SINRs are all strictly greater than one after only one link-removal step (b), with a slight improvement in the objective value. Although the further removing of links with small SINRs in step (c) degrades the objective a little bit, it leads to a sparser routing pattern (cf. figure 2 and 3). In all three cases, the sum throughputs are very close.

VI. Conclusions

We have generalized the formulation of the SRRA problem in wireless data networks to include CDMA systems. Although the capacity constraints for CDMA systems are not jointly convex in rates and powers, we have shown that coordinate projection and transformation techniques allow the SRRA problem to be formulated as (with interference cancellation) or approximated by (without interference cancellation) a convex optimization problem that can be solved very efficiently. We have also proposed a heuristic link-removal procedure based on the convex approximation to further improve the system performance.

Distributed algorithms have been developed for power control problems in CDMA systems to achieve maxmin or specified SINRs (*e.g.*, [11, 12, 13]). It would be interesting to explore if these could be used to develop distributed algorithms for solving the SRRA problem in the dual decomposition framework [2], where distributed power control is done based on the pricing of the link capacities.

Appendix

The following derivation details the outline in [6].

To derive an equivalent characterization of the achievable rate region for the Gaussian broadcast channel (3), we solve the P_i's in terms of the rate vector t. We can rewrite the equalities in (3) into

$$\sum_{j\le i} P_j = e^{t_i}\sum_{j<i} P_j + e^{t_i}\sigma_i - \sigma_i, \quad i=1,\dots,M.$$

For a given k, we multiply the ith equality by $e^{\sum_{i<j\le k} t_j}$ for $1\le i\le k$. Then adding them together yields

$$\sum_{j\le k} P_j = \sum_{i=1}^k (\sigma_i-\sigma_{i-1})e^{\sum_{i\le j\le k} t_j} - \sigma_k.$$

where $\sigma_0 = 0$. We define the functions

$$p_k(t) = \sum_{j\le k} P_j = \sum_{i=1}^k (\sigma_i-\sigma_{i-1})e^{\sum_{i\le j\le k} t_j} - \sigma_k$$

for $k=1,\dots,M$. Note that $p_k(t)$ is non-negative and non-decreasing in k, since $p_1(t) = \sigma_1(e^{t_1}-1)\ge 0$ and

$$p_k(t) - p_{k-1}(t) = \sum_{i=1}^k (\sigma_i-\sigma_{i-1})e^{\sum_{i\le j<k} t_j}(e^{t_k}-1)\ge 0$$

for all $k>1$. Hence, given any $t\succeq 0$, provided that

$$p(t)\equiv p_M(t) = \sum_{i=1}^M (\sigma_i-\sigma_{i-1})e^{\sum_{i\le j\le M} t_j} - \sigma_M \le P_{\rm tot},$$

there must exist an allocation of powers P_k such that

$$P_k = p_k(t) - p_{k-1}(t)\ge 0, \quad k=1,\dots,M$$

where $p_0(t)=0$ and $\sum_{i=1}^M P_k \le P_{\rm tot}$, *i.e.*, t lies in the capacity region. Conversely, any rate vector satisfying (3) satisfy $p(t)\le P_{\rm tot}$, and the corresponding power allocation is given by $P_k = p_k(t) - p_{k-1}(t)$, *i.e.*, equation (8).

References

[1] N. Bambos. Toward power-sensitive network architectures in wireless communications: Concepts, issues, and design aspects. *IEEE Pers. Commun.*, 5(3):50–59, 1998.

[2] L. Xiao, M. Johansson, and S. P. Boyd. Simultaneous routing and resource allocation via dual decomposition. Submitted, July 2002. `http://www.stanford.edu/~boyd/srra.html`

[3] T. M. Cover. Broadcast channels. *IEEE Trans. Inform. Theory*, 18(1):2–14, 1972.

[4] P. P. Bergmans. Random coding theorem for broadcast channels with degraded components. *IEEE Trans. Inform. Theory*, 19:197–207, 1973.

[5] S. P. Boyd and L. Vandenberghe. *Course reader for EE364: Introduction to Convex Optimization with Engineering Applications.* Stanford University, 1998.

[6] D. N. Tse. Optimal power allocation over parallel Gaussian broadcast channels. Unpublished, a short summary published in Proc. of Int. Symp. Inform. Theory, Ulm, Germany, 1997.

[7] X. Qiu and K. Chawla. On the performance of adaptive modulation in cellular systems. *IEEE Trans. Commun.*, 47(6):884–895, June 1999.

[8] A. J. Goldsmith and S.-G. Chua. Variable-rate variable-power M-QAM for fading channels. *IEEE Trans. Commun.*, 45(10):1218–1230, October 1997.

[9] S. Kandukuri and S. P. Boyd. Optimal power control in interference limited fading wireless channels with outage-probability specifications. *IEEE Trans. Wireless Commun.*, 1(1):46–55, 2002.

[10] D. Julian, M. Chiang, D. O'Neill, and S. P. Boyd. QoS and fairness constrained convex optimization of resource allocation for wireless cellular and ad hoc networks. In *Proc. IEEE INFOCOM'02*, 2002.

[11] J. Zander. Distributed cochannel interference control in cellular radio systems. *IEEE Trans. Veh. Technol.*, 41(3):305–311, 1992.

[12] G. J. Foschini and Z. Miljanic. A simple distributed autonomous power control algorithm and its convergence. *IEEE Trans. Veh. Technol.*, 42(4):641–646, 1993.

[13] R. D. Yates. A framework for uplink power control in cellular radio systems. *IEEE J. Select. Areas Commun.*, 13(7):1341–1347, 1995.