Operation and Configuration of a Storage Portfolio via Convex Optimization

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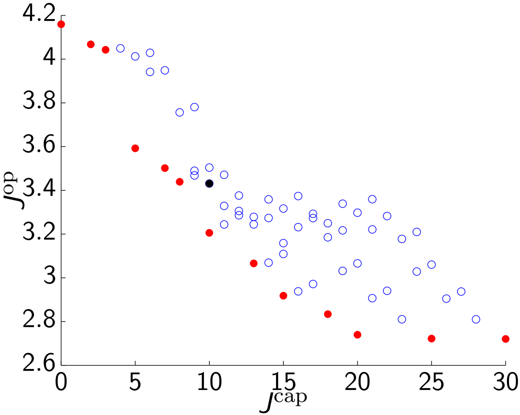
This talk

- a general framework for operating and configuring a **portfolio** of storage devices
- find **optimal trade-off** between operation cost (J^{op}) and capital construction cost (J^{cap})

Where we are going

- assume that J^{cap} is known for all candidate portfolios
- focus on evaluating J^{op} for each portfolio

The final result



Storage portfolio

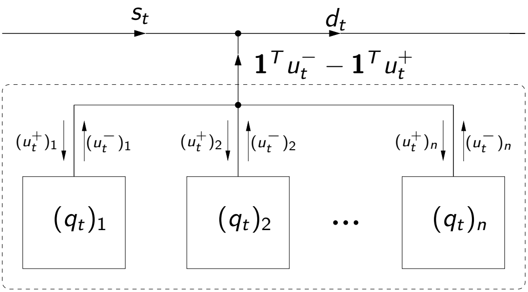
- portfolio of n different storage devices
- charge $q \in \mathbf{R}^n_+$, charging/discharging rates $u^+, u^- \in \mathbf{R}^n_+$
- maximum charge, charging/discharging rates $(Q, C, D) \in \mathbf{R}^{3n}$
- charge leakage, charging/discharging efficiencies $(\eta^l, \eta^c, \eta^d) \in (0, 1]^{3n}$
- exogenous input w
- discrete time state evolution:

$$q_{t+1} = \eta^l \circ q_t + \eta^c \circ u_t^+ - (1/\eta^d) \circ u_t^- + w_t, \quad t = 0, 1, \ldots$$

Storage portfolio

- pull energy s from source and deliver energy d to destination
- let $v_t = (d_t, s_t, u_t^+, u_t^-)$
- power balance:

$$(-1, 1, -\mathbf{1}, \mathbf{1})^T v_t = 0, \quad t = 0, 1, \ldots$$



Objective function

- decomposable objective function

$$\ell_t(v_t, q_t) = \phi_t^{\mathrm{sr}}(s_t) + \phi_t^{\mathrm{de}}(d_t) + \phi_t^{\mathrm{ch}}(u_t^+, u_t^-) + \phi_t^{\mathrm{st}}(q_t)$$

- functions not necessarily known ahead of time
- encode constraints by setting $\ell_t = +\infty$ if violated
- $\{\ell_t\}$ encodes all problem uncertainty other than $\{w_t\}$
- operation cost

$$J^{\mathrm{op}} \equiv \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \ell_t(v_t, q_t)$$

(we assume limit exists)

Example objective functions

- $\phi_t^{\mathrm{sr}}(s_t) = p_t s_t$
  - energy price p_t
  - with capacitation: $\phi_t^{\mathrm{sr}}(s_t) = \begin{cases} p_t s_t, & 0 \leq s_t \leq S^{\max} \\ \infty, & \text{otherwise} \end{cases}$
- $\phi_t^{\mathrm{de}}(d_t) = \alpha(r_t - d_t)_+$
  - energy requests r_t
  - typically $\alpha \gg p_t$
- $\phi_t^{\mathrm{ch}}(u_t^+, u_t^-) = \beta(\|u_t^+\|_1 + \|u_t^-\|_1)$
  - penalize frequent charging/discharging

Control policy

- $\hat{w}_{\tau|t}, \hat{\ell}_{\tau|t}$: estimates of exogenous input, objective function at time τ, based on information available at time t
- estimates can be obtained many ways
  - conditional expectation (if statistical model exists)
  - historical patterns
  - analyst predictions
  - futures market
- goal: pick v_t to minimize J^{op} and satisfy constraints, based on information available at time t

Control policy

- we use model predictive control (MPC)
- at time t, construct estimates $\hat{\ell}_{\tau|t}, \hat{w}_{\tau|t}$ for T steps into the future and solve

$$
\begin{array}{ll}
\text{minimize} & \frac{1}{T}\sum_{\tau=t}^{t+T-1} \hat{\ell}_{\tau|t}(\hat{v}_\tau, \hat{q}_\tau) \\
\text{subject to} & \hat{q}_{\tau+1} = \eta^{\mathrm{l}} \circ \hat{q}_\tau + \eta^{\mathrm{c}} \circ \hat{u}_\tau^+ - (1/\eta^{\mathrm{d}}) \circ \hat{u}_\tau^- + \hat{w}_{\tau|t}, \\
& \hat{d}_\tau - \hat{s}_\tau + \mathbf{1}^T \hat{u}_\tau^+ - \mathbf{1}^T \hat{u}_\tau^- = 0, \\
& 0 \leq \hat{q}_\tau \leq Q, \quad 0 \leq \hat{u}_\tau^+ \leq C, \\
& 0 \leq \hat{u}_\tau^- \leq D, \quad \tau = t, \ldots, t+T-1 \\
& \hat{q}_t = q_t, \quad \hat{q}_{t+T} = q_{\mathrm{final}}
\end{array}
$$

- when $\hat{\ell}_{\tau|t}$ are convex, problem is convex and so easily solved

Numerical example

- time discretized into 30 minute intervals
- $T = 48$ (one day prediction horizon)
- $\ell_t(v_t, q_t) = p_t s_t + \alpha(r_t - d_t)_+$, with capacitated source
- r_t, p_t are log-normal stochastic process, with diurnal variation
- $\hat{r}_{\tau|t}$, $\hat{p}_{\tau|t}$ are conditional expectations

Portfolio configurations

- 3 types of devices: small (S), medium (M), large (L)

device	Q	C	D	η	η_c	η_d	$J^{\mathrm{cap}}/\mathrm{unit}$
L	5	0.75	0.75	0.98	0.8	0.8	5
M	2	0.5	0.5	0.99	0.9	0.9	3
S	1	0.5	0.5	0.995	1	1	2

- 64 configurations consisting of all combinations containing 0, 1, 2, or 3 units of each device type

MPC evaluation

- simulate MPC for each portfolio configuration for 365 days (17520 time periods)
- solve times on single core of 3.2 Ghz Intel i3
  - SDPT3: 3.23 s (15 hours total)

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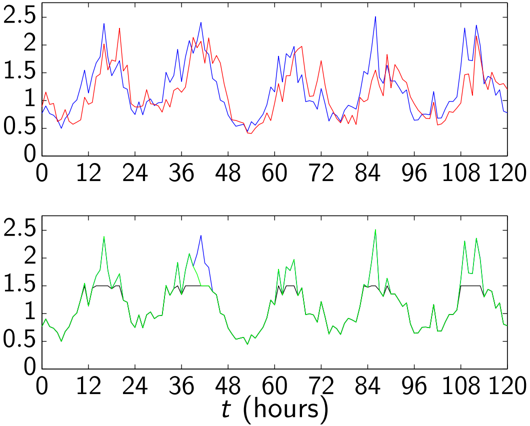
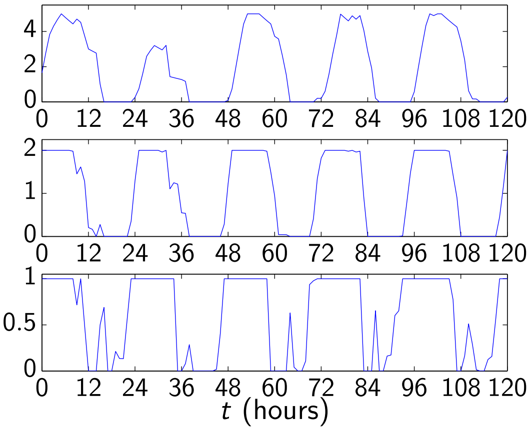
- solve times on single core of 3.2 Ghz Intel i3
  - SDPT3: 3.23 s (15 hours total)
  - CVXGEN: **6.56 ms** (under 2 minutes total)
- nearly **500**$\times$ speedup

Results



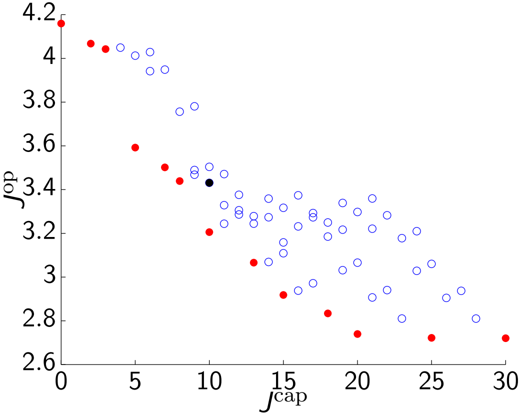
- top: r_t (blue), p_t (red)
- bottom: d_t with storage (green), without (black)

Results



- top: $(q_t)_L$, middle: $(q_t)_M$, bottom: $(q_t)_S$.

Results



- Pareto optimal portfolios (red)
- portfolio with one of each type of device (black)

Interpretation

- at low amounts of storage, well chosen additional devices allow for large decrease in operation cost
- at high amounts of storage, additional devices have minimal impact on operation cost of well chosen portfolios
- Pareto optimal portfolios tend to have mixtures of devices

Conclusion

- a well chosen portfolio of different storage devices can deliver better performance than a single type
- a storage device must be judged in the application context, with a good control policy
- while basic operation of a portfolio of storage devices is simple and intuitive, good operation requires optimization
- super fast solvers make possible substantial simulation-based analysis

Thank you